



NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9–40)

- Attempt Questions 11–31
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 Which inequality gives the domain of $y = \sqrt{2x - 3}$?

A. $x < \frac{3}{2}$

B. $x > \frac{3}{2}$

C. $x \leq \frac{3}{2}$

D. $x \geq \frac{3}{2}$

2 The function $f(x) = x^3$ is transformed to $g(x) = (x - 2)^3 + 5$ by a horizontal translation of 2 units followed by a vertical translation of 5 units.

Which row of the table shows the directions of the translations?

	<i>Horizontal translation of 2 units</i>	<i>Vertical translation of 5 units</i>
A.	Left	Up
B.	Right	Up
C.	Left	Down
D.	Right	Down

- 3 John recently did a class test in each of three subjects. The class scores on each test were normally distributed.

The table shows the subjects and John's scores as well as the mean and standard deviation of the class scores on each test.

<i>Subject</i>	<i>John's score</i>	<i>Mean</i>	<i>Standard deviation</i>
French	82	70	8
Commerce	80	65	5
Music	74	50	12

Relative to the rest of the class, which row of the table below shows John's strongest subject and his weakest subject?

	<i>Strongest subject</i>	<i>Weakest subject</i>
A.	Commerce	French
B.	French	Music
C.	Music	French
D.	Commerce	Music

4 What is $\int e + e^{3x} dx$?

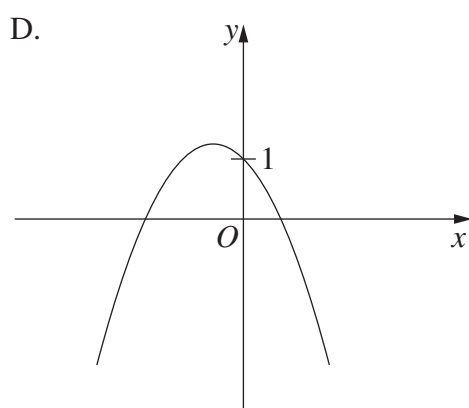
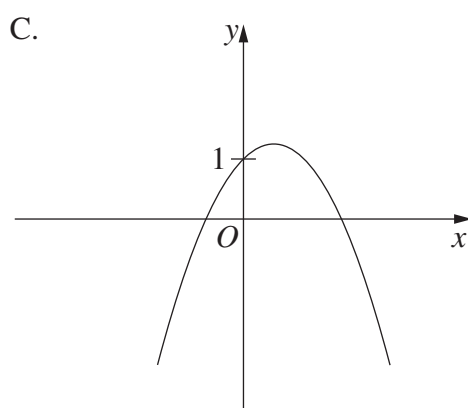
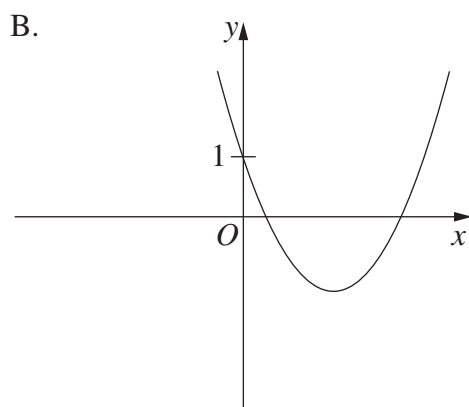
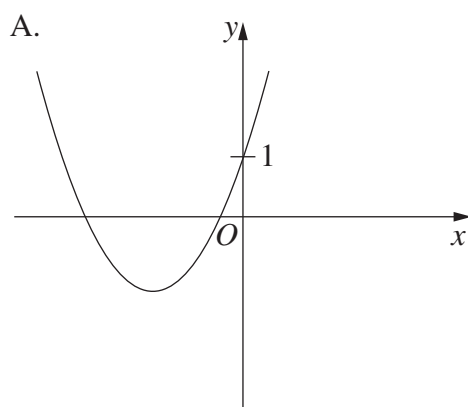
A. $ex + 3e^{3x} + c$

B. $ex + \frac{1}{3}e^{3x} + c$

C. $e + 3e^{3x} + c$

D. $e + \frac{1}{3}e^{3x} + c$

5 Which of the following could represent the graph of $y = -x^2 + bx + 1$, where $b > 0$?



6 Which interval gives the range of the function $y = 5 + 2\cos 3x$?

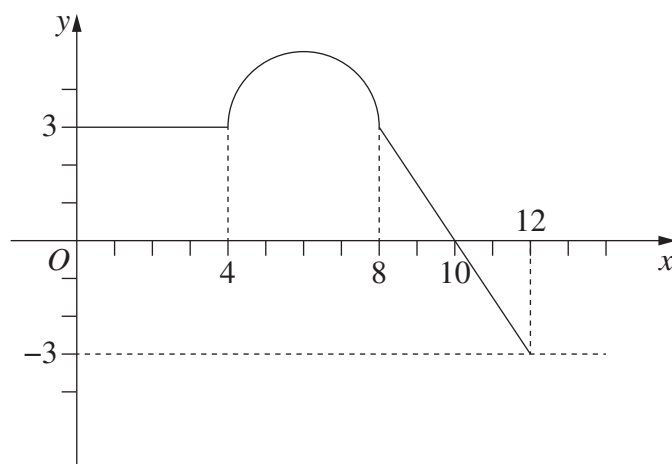
A. $[2, 8]$

B. $[3, 7]$

C. $[4, 6]$

D. $[5, 9]$

7 The diagram shows the graph $y = f(x)$, which is made up of line segments and a semicircle.



What is the value of $\int_0^{12} f(x) dx$?

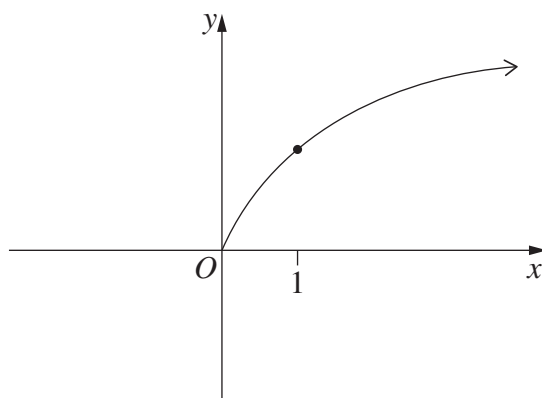
A. $24 + 2\pi$

B. $24 + 4\pi$

C. $30 + 2\pi$

D. $30 + 4\pi$

- 8 The graph of $y = f(x)$ is shown.

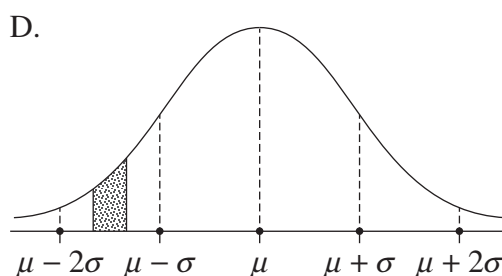
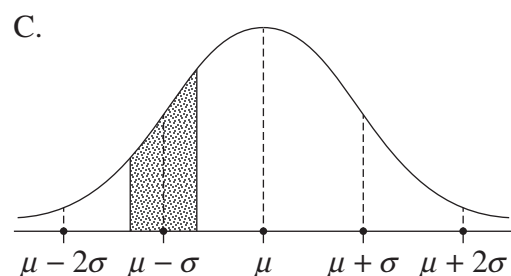
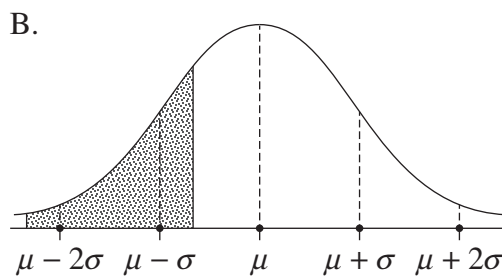
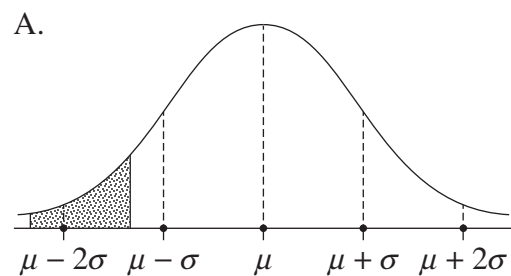


Which of the following inequalities is correct?

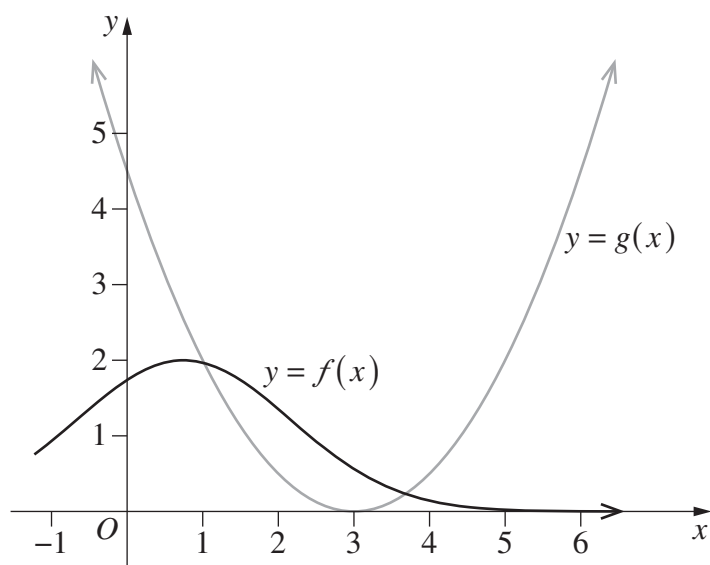
- A. $f''(1) < 0 < f'(1) < f(1)$
 B. $f''(1) < 0 < f(1) < f'(1)$
 C. $0 < f''(1) < f'(1) < f(1)$
 D. $0 < f''(1) < f(1) < f'(1)$
- 9 Suppose the weight of melons is normally distributed with a mean of μ and a standard deviation of σ .

A melon has a weight below the lower quartile of the distribution but NOT in the bottom 10% of the distribution.

Which of the following most accurately represents the region in which the weight of this melon lies?



- 10 The graph shows two functions $y = f(x)$ and $y = g(x)$.



Define $h(x) = f(g(x))$.

How many stationary points does $y = h(x)$ have for $1 \leq x \leq 5$?

- A. 0
- B. 1
- C. 2
- D. 3

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Centre Number

Mathematics Advanced

Section II Answer Booklet 1

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Student Number

Section II

90 marks**Attempt Questions 11–31****Allow about 2 hours and 45 minutes for this section****Booklet 1 — Attempt Questions 11–20 (32 marks)****Booklet 2 — Attempt Questions 21–31 (58 marks)**

Instructions

- Write your Centre Number and Student Number at the top of this page.
 - Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your responses should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided on pages 19–20 of Booklet 1. If you use this space, clearly indicate which question you are answering.
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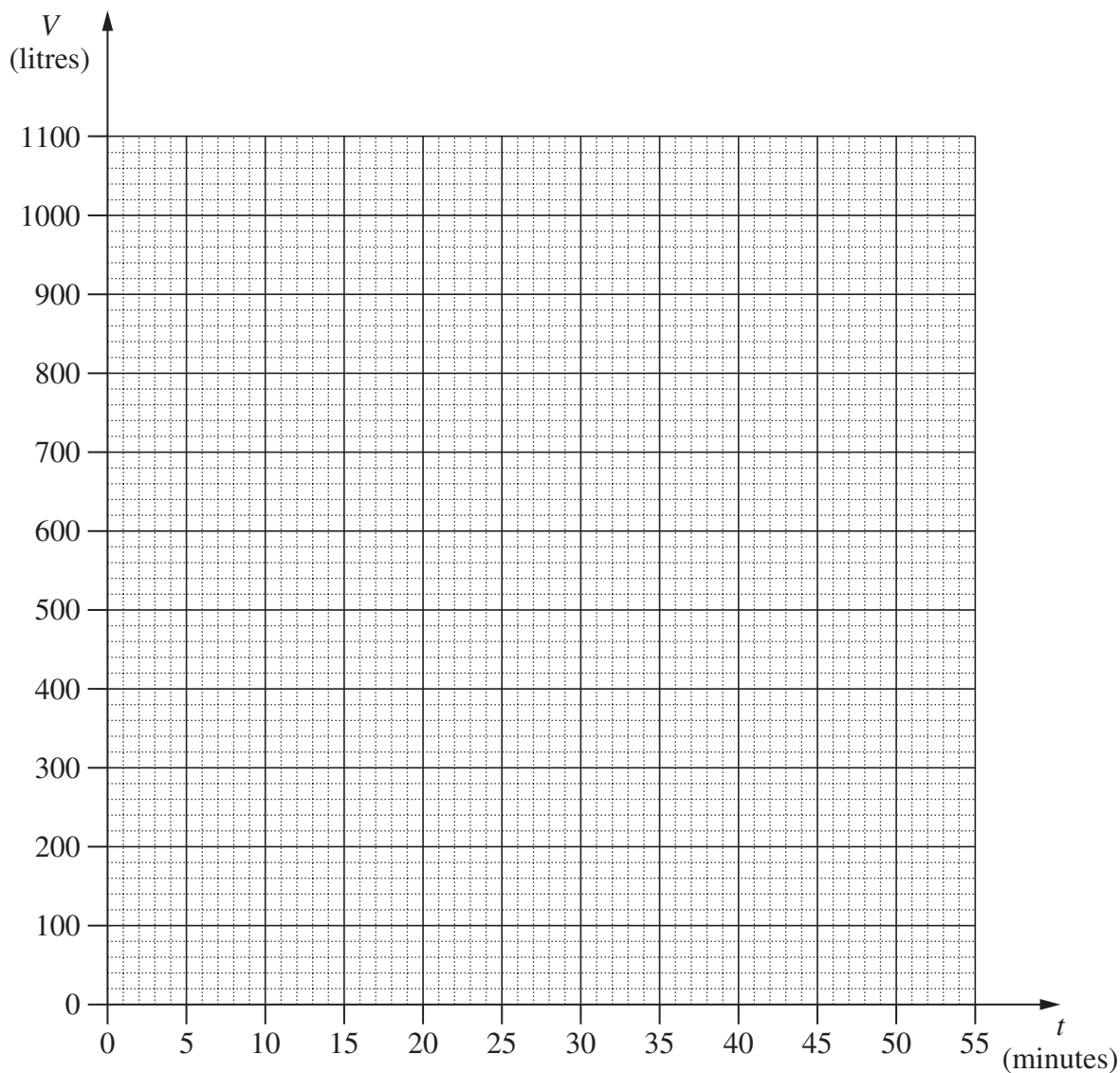
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Question 11 (4 marks)

There are two tanks on a property, Tank *A* and Tank *B*. Initially, Tank *A* holds 1000 litres of water and Tank *B* is empty.

- (a) Tank *A* begins to lose water at a constant rate of 20 litres per minute. The volume of water in Tank *A* is modelled by $V = 1000 - 20t$ where V is the volume in litres and t is the time in minutes from when the tank begins to lose water.

On the grid below, draw the graph of this model and label it as Tank *A*.



Question 11 continues on page 11

Question 11 (continued)

- (b) Tank *B* remains empty until $t = 15$ when water is added to it at a constant rate of 30 litres per minute. 2

By drawing a line on the grid on the previous page, or otherwise, find the value of t when the two tanks contain the same volume of water.

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- (c) Using the graphs drawn, or otherwise, find the value of t (where $t > 0$) when the total volume of water in the two tanks is 1000 litres. 1

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End of Question 11

Question 12 (3 marks)

Calculate the sum of the arithmetic series $4 + 10 + 16 + \cdots + 1354$.

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Question 13 (2 marks)

Evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x \, dx$.

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Question 14 (5 marks)

History and Geography are two of the subjects students may decide to study. For a group of 40 students, the following is known.

- 7 students study neither History nor Geography
- 20 students study History
- 18 students study Geography

- (a) A student is chosen at random. By using a Venn diagram, or otherwise, find the probability that the student studies both History and Geography. **2**

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- (b) A student is chosen at random. Given that the student studies Geography, what is the probability that the student does NOT study History? **1**

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- (c) Two different students are chosen at random, one after the other. What is the probability that the first student studies History and the second student does NOT study History? **2**

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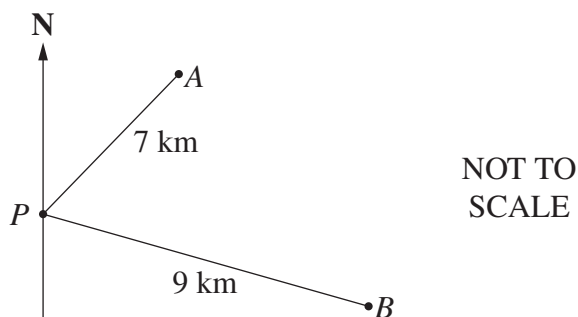
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Question 15 (5 marks)

Mr Ali, Ms Brown and a group of students were camping at the site located at P . Mr Ali walked with some of the students on a bearing of 035° for 7 km to location A . Ms Brown, with the rest of the students, walked on a bearing of 100° for 9 km to location B .



- (a) Show that the angle APB is 65° .

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- (b) Find the distance AB .

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- (c) Find the bearing of Ms Brown's group from Mr Ali's group. Give your answer correct to the nearest degree.

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Question 17 (2 marks)

Find $\int \frac{x}{4+x^2} dx$.

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Question 18 (3 marks)

(a) Differentiate $e^{2x}(2x+1)$.

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(b) Hence, find $\int (x+1)e^{2x} dx$.

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Question 19 (2 marks)

Prove that $\sec \theta - \cos \theta = \sin \theta \tan \theta$.

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Question 20 (2 marks)

Kenzo is driving his car along a road while his friend records the velocity of the car, $v(t)$, in km/h every minute over a 5-minute period. The table gives the velocity $v(t)$ at time t hours.

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t	0	$\frac{1}{60}$	$\frac{2}{60}$	$\frac{3}{60}$	$\frac{4}{60}$	$\frac{5}{60}$
$v(t)$	60	55	65	68	70	67

The distance covered by the car over the 5-minute period is given by

$$\int_0^{\frac{5}{60}} v(t) dt.$$

Use the trapezoidal rule and the velocity at each of the six time values to find the approximate distance in kilometres the car has travelled in the 5-minute period. Give your answer correct to one decimal place.

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Centre Number

Mathematics Advanced

Section II Answer Booklet 2

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Student Number

Booklet 2 — Attempt Questions 21–31 (58 marks)

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 38–40 of Booklet 2. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 21 (6 marks)

Hot tea is poured into a cup. The temperature of tea can be modelled by $T = 25 + 70(1.5)^{-0.4t}$, where T is the temperature of the tea, in degrees Celsius, t minutes after it is poured.

- (a) What is the temperature of the tea 4 minutes after it has been poured? 1

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- (b) At what rate is the tea cooling 4 minutes after it has been poured? 2

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- (c) How long after the tea is poured will it take for its temperature to reach 55°C? 3

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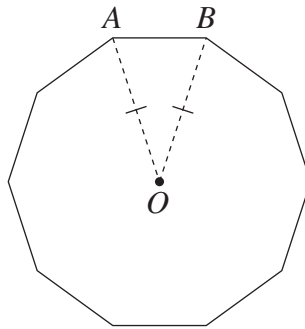
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Question 22 (4 marks)

The diagram shows a regular decagon (ten-sided shape with all sides equal and all interior angles equal). The decagon has centre O .

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The perimeter of the shape is 80 cm.

By considering triangle OAB , calculate the area of the ten-sided shape. Give your answer in square centimetres correct to one decimal place.

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Question 23 (4 marks)

A continuous random variable, X , has the following probability density function.

$$f(x) = \begin{cases} \sin x & \text{for } 0 \leq x \leq k \\ 0 & \text{for all other values of } x \end{cases}$$

- (a) Find the value of k .

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- (b) Find $P(X \leq 1)$. Give your answer correct to four decimal places.

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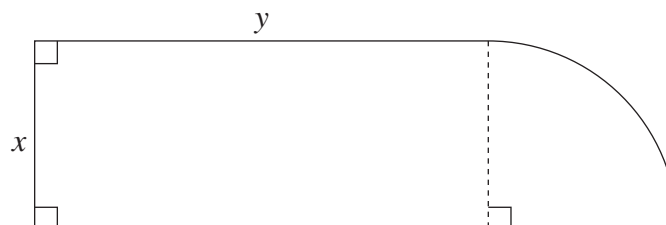
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Questions 11–23 are worth 46 marks in total

Question 25 (7 marks)

A landscape gardener wants to build a garden bed in the shape of a rectangle attached to a quarter-circle. Let x and y be the dimensions of the rectangle in metres, as shown in the diagram.



The garden bed is required to have an area of 36 m^2 and to have a perimeter which is as small as possible. Let P metres be the perimeter of the garden bed.

- (a) Show that $P = 2x + \frac{72}{x}$.

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Question 25 continues on page 27

Question 25 (continued)

- (b) Find the smallest possible perimeter of the garden bed, showing why this is the minimum perimeter.

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End of Question 25

Question 26 (7 marks)

Tina inherits \$60 000 and invests it in an account earning interest at a rate of 0.5% per month. Each month, immediately after the interest has been paid, Tina withdraws \$800.

The amount in the account immediately after the n th withdrawal can be determined using the recurrence relation

$$A_n = A_{n-1}(1.005) - 800,$$

where $n = 1, 2, 3, \dots$ and $A_0 = 60\,000$.

- (a) Use the recurrence relation to find the amount of money in the account immediately after the third withdrawal.

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Question 26 continues on page 29

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Question 26 (continued)

- (b) Calculate the amount of interest earned in the first three months.

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- (c) Calculate the amount of money in the account immediately after the 94th withdrawal.

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End of Question 26

Question 27 (5 marks)

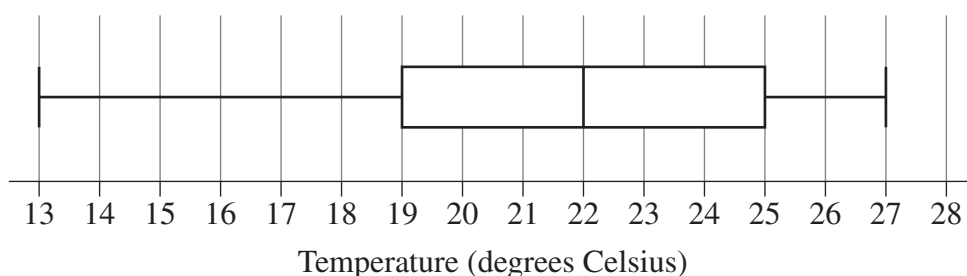
A cricket is an insect. The male cricket produces a chirping sound.

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A scientist wants to explore the relationship between the temperature in degrees Celsius and the number of cricket chirps heard in a 15-second time interval.

Once a day for 20 days, the scientist collects data. Based on the 20 data points, the scientist provides the information below.

- A box-plot of the temperature data is shown.



- The mean temperature in the dataset is 0.525°C below the median temperature in the dataset.
- A total of 684 chirps was counted when collecting the 20 data points.

The scientist fits a least-squares regression line using the data (x, y) , where x is the temperature in degrees Celsius and y is the number of chirps heard in a 15-second time interval. The equation of this line is

$$y = -10.6063 + bx,$$

where b is the slope of the regression line.

The least-squares regression line passes through the point $(\bar{x}, \bar{y})$ where $\bar{x}$ is the sample mean of the temperature data and $\bar{y}$ is the sample mean of the chirp data.

Question 27 continues on page 31

Question 27 (continued)

Calculate the number of chirps expected in a 15-second interval when the temperature is 19° Celsius. Give your answer correct to the nearest whole number.

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End of Question 27

Question 28 (5 marks)

In a particular country, the hourly rate of pay for adults who work is normally distributed with a mean of \$25 and a standard deviation of \$5.

- (a) Two adults who both work are chosen at random.

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Find the probability that at least one of them earns between \$15 and \$30 per hour.

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- (b) The number of adults who work is equal to three times the number of adults who do not work.

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One adult is chosen at random.

Find the probability that the chosen adult works and earns more than \$25 per hour.

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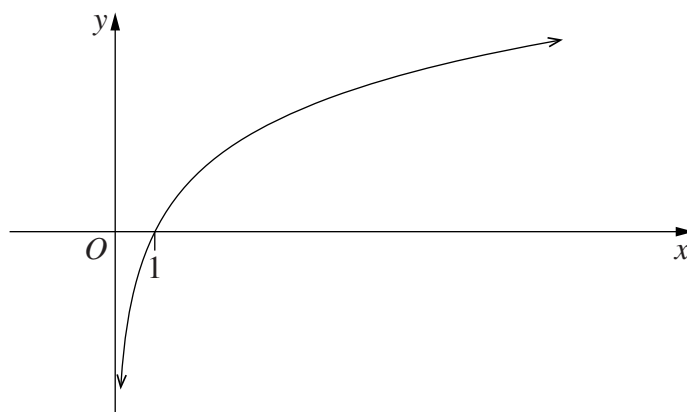
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Question 29 (4 marks)

The diagram shows the graph of $y = c \ln x$, $c > 0$.



- (a) Show that the equation of the tangent to $y = c \ln x$ at $x = p$, where $p > 0$, is 2

$$y = \frac{c}{p}x - c + c \ln p.$$

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- (b) Find the value of c such that the tangent from part (a) has a gradient of 1 and passes through the origin. 2

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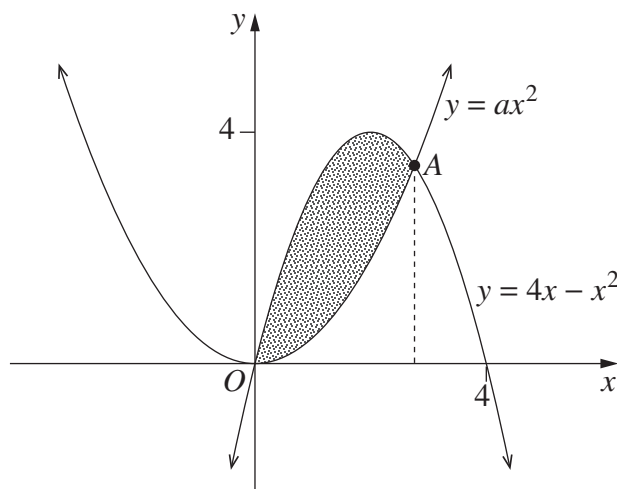
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Question 30 (6 marks)

The diagram shows two parabolas $y = 4x - x^2$ and $y = ax^2$, where $a > 0$. The two parabolas intersect at the origin, O , and at A .



- (a) Show that the x -coordinate of A is $\frac{4}{a+1}$.

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Question 30 continues on page 35

Question 30 (continued)

- (b) Find the value of a such that the shaded area is $\frac{16}{3}$.

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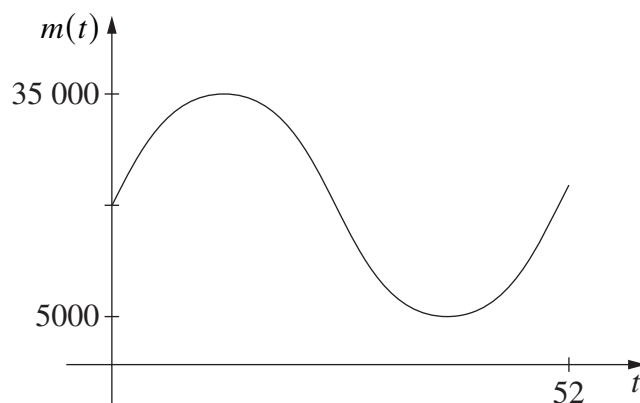
End of Question 30

Question 31 (7 marks)

The population of mice on an isolated island can be modelled by the function

$$m(t) = a \sin\left(\frac{\pi}{26}t\right) + b,$$

where t is the time in weeks and $0 \leq t \leq 52$. The population of mice reaches a maximum of 35 000 when $t = 13$ and a minimum of 5000 when $t = 39$. The graph of $m(t)$ is shown.



- (a) What are the values of a and b ?

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Question 31 continues on page 37

Question 31 (continued)

- (b) On the same island, the population of cats can be modelled by the function 3

$$c(t) = -80 \cos\left(\frac{\pi}{26}(t - 10)\right) + 120.$$

Consider the graph of $m(t)$ and the graph of $c(t)$.

Find the values of t , $0 \leq t \leq 52$, for which both populations are increasing.

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- (c) Find the rate of change of the mice population when the cat population reaches a maximum. 2

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Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

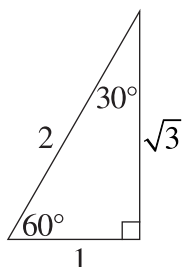
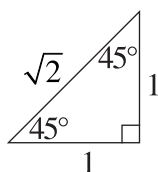
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1 + t^2}$$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2} (1 - \cos 2nx)$$

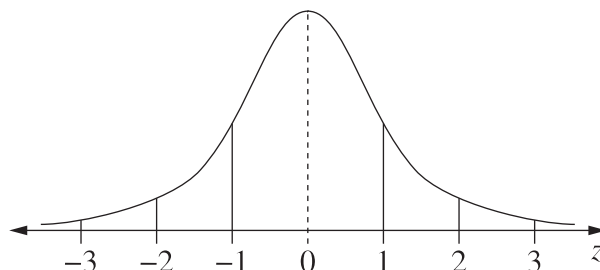
$$\cos^2 nx = \frac{1}{2} (1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z -scores between -1 and 1
- approximately 95% of scores have z -scores between -2 and 2
- approximately 99.7% of scores have z -scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function

Derivative

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

Integral Calculus

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \left\{ f(a) + f(b) + 2[f(x_1) + \cdots + f(x_{n-1})] \right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}_nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underline{u}| = |x\underline{i} + y\underline{j}| = \sqrt{x^2 + y^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1x_2 + y_1y_2,$$

$$\text{where } \underline{u} = x_1\underline{i} + y_1\underline{j}$$

$$\text{and } \underline{v} = x_2\underline{i} + y_2\underline{j}$$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos \theta + i \sin \theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos \theta + i \sin \theta)]^n &= r^n(\cos n\theta + i \sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2020 HSC Mathematics Advanced Marking Guidelines

Section I

Multiple-choice Answer Key

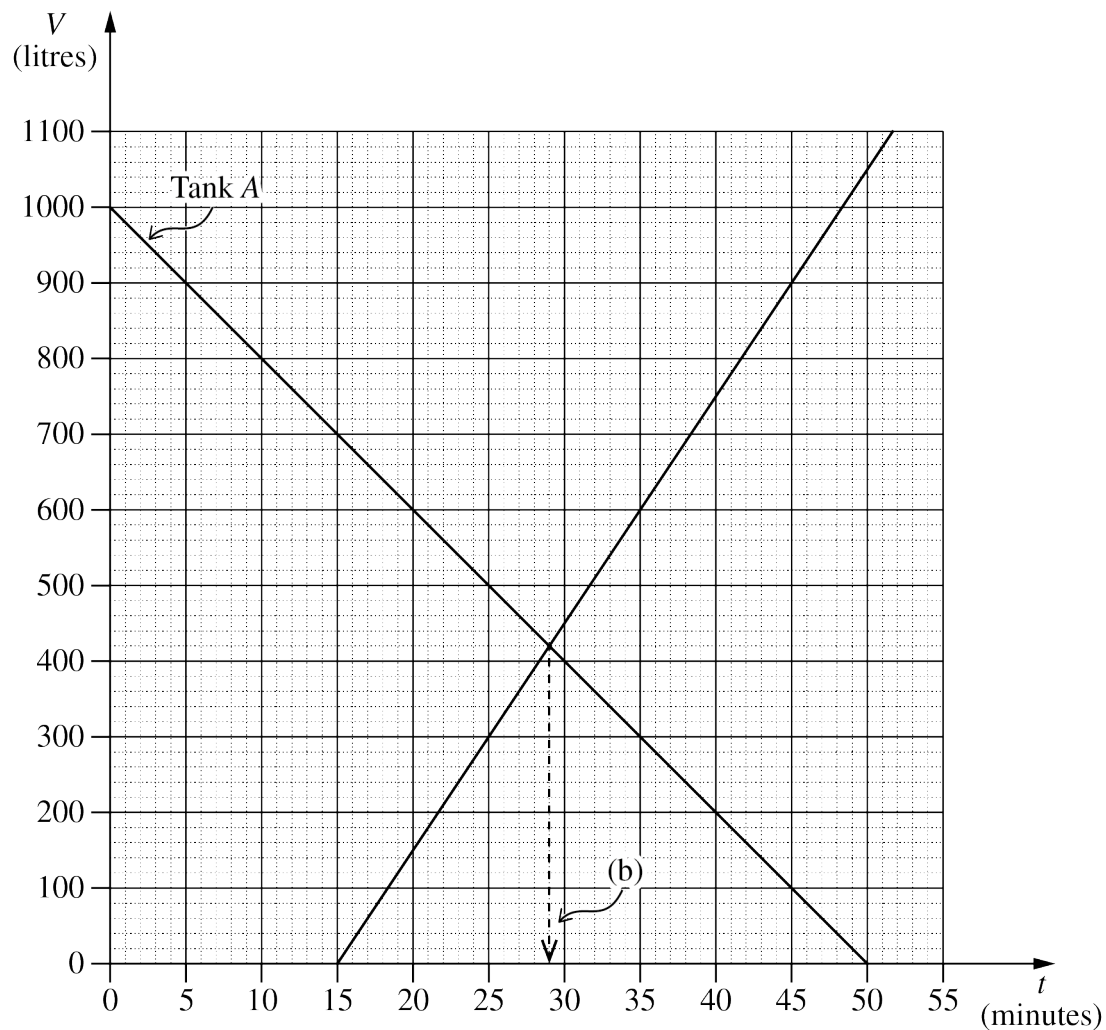
Question	Answer
1	D
2	B
3	A
4	B
5	C
6	B
7	A
8	A
9	C
10	D

Section II

Question 11 (a)

Criteria	Marks
Provides the correct solution	1

Sample answer:



Question 11 (b)

Criteria	Marks
• Provides the correct solution	2
• Draws the graph of volume for tank B , or equivalent merit	1

Sample answer:

29 minutes

Question 11 (c)

Criteria	Marks
• Provides the correct answer	1

Sample answer:

45 minutes

Question 12

Criteria	Marks
• Provides the correct solution	3
• Attempts to use the sum of an arithmetic series, or equivalent merit OR • Finds the number of terms in the series	2
• Finds the values of a and d , or equivalent merit	1

Sample answer:

$$4 + 10 + 16 + \dots + 1354$$

$$d = 6 \quad a = 4$$

$$1354 = 4 + (n - 1) \times 6$$

$$\frac{1350}{6} = n - 1$$

$$n = 226$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$= \frac{226}{2}(4 + 1354)$$

$$= 153\,454$$

Question 13

Criteria	Marks
• Provides the correct solution	2
• Finds the anti-derivative of $\sec^2 x$, or equivalent merit	1

Sample answer:

$$\int_0^{\frac{\pi}{4}} \sec^2 x \, dx$$

$$= \left[\tan x \right]_0^{\frac{\pi}{4}}$$

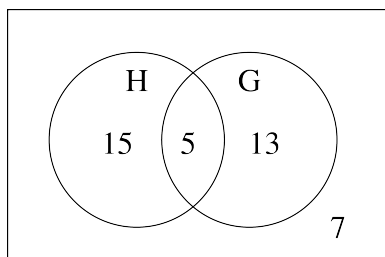
$$= \left[\tan \frac{\pi}{4} - \tan 0 \right]$$

$$= 1$$

Question 14 (a)

Criteria	Marks
• Provides the correct solution	2
• Finds the number of students who study both History and Geography, or equivalent merit	1

Sample answer:



$$40 - 7 = 33$$

$$20 + 18 - 33 = 5$$

$$\frac{5}{40} = \frac{1}{8}$$

Question 14 (b)

Criteria	Marks
• Provides the correct answer	1

Sample answer:

$$P(\bar{H}|G) = \frac{13}{18}$$

Question 14 (c)

Criteria	Marks
• Provides the correct solution	2
• Obtains the probability of a student studying History as $\frac{20}{40}$, or equivalent merit	1

Sample answer:

$$\frac{20}{40} \times \frac{20}{39} = \frac{10}{39}$$

Question 15 (a)

Criteria	Marks
• Provides the correct solution	1

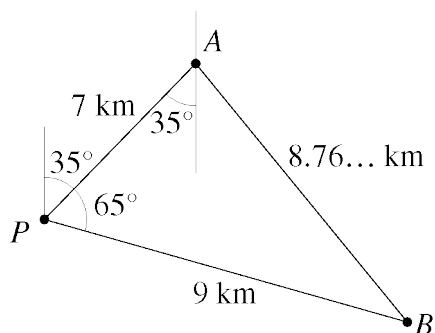
Sample answer:

$$100^\circ - 35^\circ = 65^\circ$$

Question 15 (b)

Criteria	Marks
• Provides the correct solution	2
• Attempts to use the cosine rule to find the distance AB	1

Sample answer:



$$\begin{aligned} AB^2 &= 7^2 + 9^2 - 2 \times 7 \times 9 \cos 65^\circ \\ &= 76.75... \end{aligned}$$

$$AB = 8.76 \text{ km}$$

Question 15 (c)

Criteria	Marks
• Provides the correct solution	2
• Finds another angle in $\triangle APB$, or equivalent merit	1

Sample answer:

$$\frac{\sin A}{9} = \frac{\sin 65^\circ}{8.76...}$$

$$A = 68.6...$$

$$\doteq 69^\circ$$

$$\begin{aligned} \text{Bearing} &= 180^\circ - (69^\circ - 35^\circ) \\ &= 146^\circ \end{aligned}$$

Question 16

Criteria	Marks
• Provides the correct solution	4
• Finds the stationary points and the point of inflection, or equivalent merit	3
• Finds the stationary points, or equivalent merit	2
• Finds the derivative of the function, or equivalent merit	1

Sample answer:

$$y = -x^3 + 3x^2 - 1$$

$$\begin{aligned}\frac{dy}{dx} &= -3x^2 + 6x \\ &= 3x(2 - x)\end{aligned}$$

When $\frac{dy}{dx} = 0$, we have $3x(2 - x) = 0$.

$$x = 0, \quad x = 2$$

$$\begin{aligned}y &= -1 & y &= -2^3 + 3(2)^2 - 1 \\ & & &= 3\end{aligned}$$

Stationary points are $(0, -1)$ and $(2, 3)$.

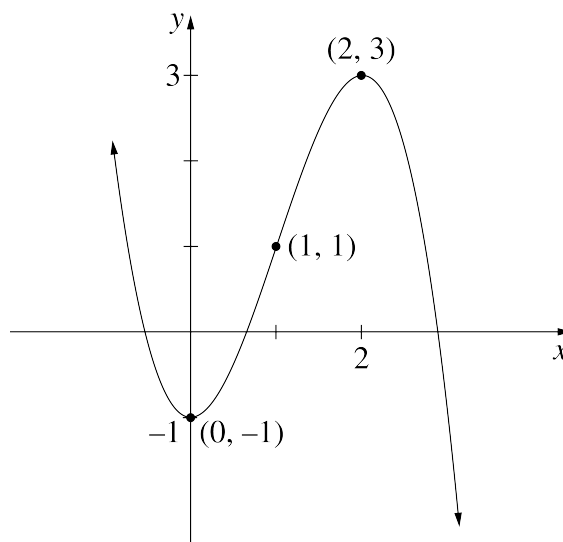
$$\frac{d^2y}{dx^2} = -6x + 6$$

When $\frac{d^2y}{dx^2} = 0$, we have $6(1 - x) = 0$.

$$x = 1$$

$$\begin{aligned}y &= -1^3 + 3 \times 1^2 - 1 \\ &= 1\end{aligned}$$

The point of inflection is $(1, 1)$.



Question 17

Criteria	Marks
• Provides the correct solution	2
• Writes an anti-derivative involving the log function, or equivalent merit	1

Sample answer:

$$\begin{aligned}\int \frac{x}{4+x^2} dx &= \frac{1}{2} \int \frac{2x}{4+x^2} dx \\ &= \frac{1}{2} \ln(4+x^2) + c\end{aligned}$$

Question 18 (a)

Criteria	Marks
• Provides the correct solution	2
• Attempts to use the product rule	1

Sample answer:

$$\begin{aligned}2e^{2x}(2x+1) + 2e^{2x} \\ &= 2e^{2x}(2x+1+1) \\ &= 2e^{2x}(2x+2) \\ &= 4e^{2x}(x+1)\end{aligned}$$

Question 18 (b)

Criteria	Marks
• Provides the correct answer	1

Sample answer:

$$\begin{aligned}\int (x+1)e^{2x} dx \\ &= \frac{1}{4} \int 4(x+1)e^{2x} dx \\ &= \frac{1}{4} e^{2x}(2x+1) + c\end{aligned}$$

Question 19

Criteria	Marks
• Provides the correct solution	2
• Shows that the LHS is equal to $\frac{1 - \cos^2 \theta}{\cos \theta}$, or equivalent merit	1

Sample answer:

$$\sec \theta - \cos \theta = \sin \theta \tan \theta$$

$$\text{LHS} = \frac{1}{\cos \theta} - \cos \theta$$

$$= \frac{1 - \cos^2 \theta}{\cos \theta}$$

$$= \frac{\sin^2 \theta}{\cos \theta}$$

$$= \sin \theta \times \frac{\sin \theta}{\cos \theta}$$

$$= \sin \theta \times \tan \theta$$

$$= \text{RHS}$$

Question 20

Criteria	Marks
• Provides the correct solution	2
• Attempts to use the trapezoidal rule	1

Sample answer:

$$\frac{\frac{5}{60} - 0}{2(5)} [60 + 67 + 2(55 + 65 + 68 + 70)]$$

$$= \frac{643}{120}$$

$$\div 5.4 \text{ km}$$

Question 21 (a)

Criteria	Marks
Provides the correct answer	1

Sample answer:

$$\begin{aligned}
 T &= 25 + 70(1.5)^{-0.4 \times 4} \\
 &= 61.5891\dots \\
 &\doteq 61.6^\circ
 \end{aligned}$$

Question 21 (b)

Criteria	Marks
Provides the correct solution	2
Finds the derivative of T, or equivalent merit	1

Sample answer:

$$\frac{dT}{dt} = 70(1.5)^{-0.4t}(-0.4)\ln(1.5)$$

When $t = 4$,

$$\begin{aligned}
 \frac{dT}{dt} &= 70(1.5)^{-1.6}(-0.4)\ln(1.5) \\
 &= -5.934\dots
 \end{aligned}$$

Therefore at $t = 4$ the tea is cooling at 5.9°C per minute.

Question 21 (c)

Criteria	Marks
• Provides the correct solution	3
• Uses the log function to solve the equation	2
• Makes $1.5^{-0.4t}$ the subject of the equation, or equivalent merit	1

Sample answer:

$$55 = 25 + 70(1.5)^{-0.4t}$$

$$\frac{30}{70} = 1.5^{-0.4t}$$

$$\ln\left(\frac{3}{7}\right) = -0.4t \ln(1.5)$$

$$t = -\frac{\ln\left(\frac{3}{7}\right)}{0.4 \ln(1.5)}$$

$$\doteq 5.22 \text{ minutes}$$

Question 22

Criteria	Marks
• Provides a correct solution	4
• Calculates area of $\triangle AOB$, or equivalent merit	3
• Finds OA , or equivalent merit	2
• Finds AB , or equivalent merit	1

Sample answer:

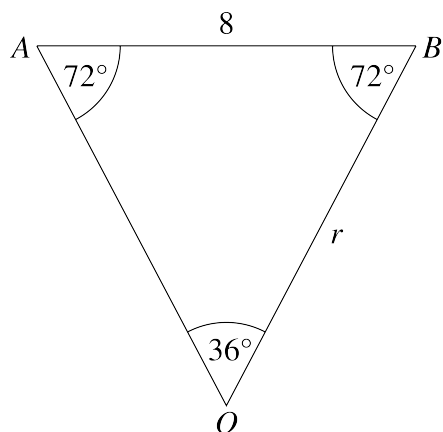
$$AB = 80 \div 10$$

$$= 8 \text{ cm}$$

$$\angle AOB = 360^\circ \div 10 = 36^\circ$$

$$\angle OAB = \frac{180^\circ - 36^\circ}{2}$$

$$= 72^\circ$$



$$\frac{r}{\sin 72^\circ} = \frac{8}{\sin 36^\circ}$$

$$r = 12.944\dots$$

$$\text{Area} = 10 \times \frac{1}{2} \times (12.944\dots)^2 \times \sin 36^\circ$$

$$\doteq 492.4 \text{ cm}^2$$

Question 23 (a)

Criteria	Marks
• Provides the correct solution	2
• Writes an equation involving a definite integral set equal to 1, or equivalent merit	1

Sample answer:

$$\int_0^k \sin x \, dx = 1$$

$$\left[-\cos x \right]_0^k = 1$$

$$-\cos k + \cos 0 = 1$$

$$\cos k = 0$$

$$k = \frac{\pi}{2}$$

Question 23 (b)

Criteria	Marks
• Provides the correct solution	2
• Writes the definite integral	1

Sample answer:

$$\int_0^1 \sin x \, dx$$

$$= \left[-\cos x \right]_0^1$$

$$= -\cos 1 + 1$$

$$\doteq 0.4597$$

Question 24

Criteria	Marks
• Provides the correct solution	3
• Finds the centre and the radius of the original circle or of the reflected circle, or equivalent merit	2
• Completes the square for the equation, or equivalent merit	1

Sample answer:

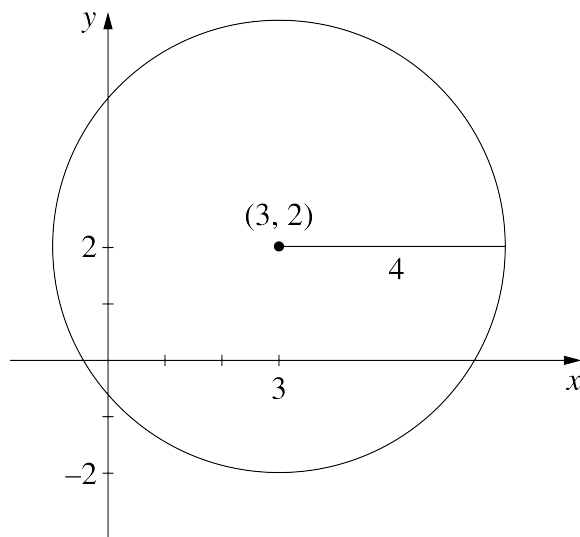
$$x^2 - 6x + y^2 + 4y - 3 = 0$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 = 3 + 9 + 4$$

$$(x - 3)^2 + (y + 2)^2 = 16$$

The original circle has centre $(3, -2)$ and radius 4.

The reflected circle has centre $(3, 2)$ and radius 4.



Question 25 (a)

Criteria	Marks
• Provides the correct solution	3
• Writes an expression for the perimeter in terms of x and y and finds a correct expression for y in terms of x , or equivalent merit	2
• Writes an expression for a perimeter in terms of x and y which includes the curved length of a quadrant, or equivalent merit	1

Sample answer:

$$xy + \frac{1}{4}\pi x^2 = 36$$

$$xy = 36 - \frac{1}{4}\pi x^2$$

$$y = \frac{36}{x} - \frac{1}{4}\pi x$$

$$P = 2y + 2x + \frac{1}{4} \times 2\pi x$$

$$= 2\left(\frac{36}{x} - \frac{1}{4}\pi x\right) + 2x + \frac{1}{2}\pi x$$

$$= \frac{72}{x} - \frac{1}{2}\pi x + 2x + \frac{1}{2}\pi x$$

$$= 2x + \frac{72}{x}$$

Question 25 (b)

Criteria	Marks
• Provides the correct solution	4
• Verifies that at $x = 6$ there is a minimum turning point, or equivalent merit	3
• Solves $\frac{dP}{dx} = 0$, or equivalent merit	2
• Finds $\frac{d^2P}{dx^2}$, or equivalent merit	1

Sample answer:

$$\frac{dP}{dx} = 2 - \frac{72}{x^2}$$

Setting $\frac{dP}{dx} = 0$, we have

$$2 = \frac{72}{x^2}$$

$$x^2 = 36$$

$$x = 6 \quad (\text{since } x > 0)$$

$$\frac{d^2P}{dx^2} = \frac{144}{x^3}$$

When $x = 6$,

$$\frac{d^2P}{dx^2} = \frac{144}{6^3} \text{ which is greater than zero.}$$

$\therefore P$ is a minimum when $x = 6$.

$$\text{The minimum perimeter} = 2 \times 6 + \frac{72}{6}$$

$$= 12 + 12$$

$$= 24 \text{ m}$$

Question 26 (a)

Criteria	Marks
• Provides the correct solution	2
• Finds the value of A_1 , or equivalent merit	1

Sample answer:

$$A_1 = 60\,000(1.005) - 800$$

$$= \$59\,500$$

$$A_2 = 59\,500(1.005) - 800$$

$$= \$58\,997.50$$

$$A_3 = 58\,997.50(1.005) - 800$$

$$\div \$58\,492.49$$

Question 26 (b)

Criteria	Marks
• Provides the correct solution	2
• Finds by how much the balance is reduced, or equivalent merit	1

Sample answer:

Total withdrawals after 3 months

$$= 800 \times 3$$

$$= \$2400$$

Balance is reduced by

$$= 60\,000 - 58\,492.49$$

$$= \$1507.51$$

Interest

$$= \$2400 - \$1507.51$$

$$= \$892.49$$

Question 26 (c)

Criteria	Marks
• Provides the correct solution	3
• Finds an expression for A_{94} , or equivalent merit	2
• Attempts to develop a formula for A_{94} , or equivalent merit	1

Sample answer:

$$A_1 = 60\,000(1.005) - 800$$

$$A_2 = 60\,000(1.005)^2 - 800(1.005) - 800$$

⋮

$$A_{94} = 60\,000(1.005)^{94} - 800(1 + 1.005 + \cdots + 1.005^{93})$$

$$= 60\,000(1.005)^{94} - 800 \frac{1.005^{94} - 1}{0.005}$$

$$= \$187.85$$

Question 27

Criteria	Marks
• Provides the correct solution	5
• Finds value of b , or equivalent merit	4
• Substitutes $\bar{x}$ and $\bar{y}$ into the regression equation, or equivalent merit	3
• Finds the value of $\bar{x}$ and $\bar{y}$, or equivalent merit	2
• Finds the value of $\bar{x}$ or $\bar{y}$ or the median of the temperature, or equivalent merit	1

Sample answer:

$$\bar{x} = 22 - 0.525$$

$$= 21.475$$

$$\bar{y} = \frac{684}{20}$$

$$= 34.2$$

Substituting $(\bar{x}, \bar{y})$ into the regression equation, we have

$$34.2 = -10.6063 + b(21.475)$$

$$\frac{44.8063}{21.475} = b$$

$$b = 2.08644\dots$$

When $x = 19$, $y = -10.6063 + 2.08644\dots \times 19$

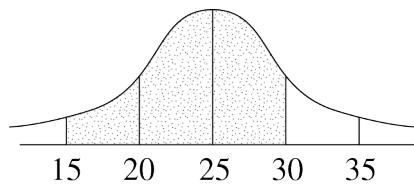
$$= 29.03606\dots$$

$\therefore$ 29 chirps are expected.

Question 28 (a)

Criteria	Marks
Provides the correct solution	3
Finds the percentage of adults earning between \$15 and \$30 and finds a probability using $1 - P(\text{neither earns ...})$, or equivalent merit	2
- Finds a probability using $1 - P(\text{neither earns ...})$, or equivalent merit OR - Finds the percentage of adults earning between \$15 and \$30, or equivalent merit	1

Sample answer:



$P(\$15 < \text{hourly rate of pay} < \$30)$

$$\begin{aligned}
 &= 68\% + \frac{95\% - 68\%}{2} \\
 &= 68\% + 13.5\% \\
 &= 81.5\%
 \end{aligned}$$

$P(\text{at least one earns between \$15 and \$30 per hour})$

$$\begin{aligned}
 &= 1 - P(\text{neither earns between \$15 and \$30 per hour}) \\
 &= 1 - (1 - 0.815)^2 \\
 &= 0.965775
 \end{aligned}$$

Question 28 (b)

Criteria	Marks
• Provides the correct solution	2
• Finds the probability of randomly choosing an adult who works or the probability of an adult earning more than \$25 per hour	1

Sample answer:

$$P(\text{chosen adult works}) = \frac{3}{4}$$

$$\begin{aligned} \therefore P(\text{chosen adult works and earns more than \$25 per hour}) &= \frac{3}{4} \times \frac{1}{2} \\ &= \frac{3}{8} \end{aligned}$$

Question 29 (a)

Criteria	Marks
• Provides the correct solution	2
• Finds the gradient of the tangent at $x = p$, or equivalent merit	1

Sample answer:

$$y = c \ln x$$

$$\frac{dy}{dx} = \frac{c}{x}$$

The gradient of the tangent at $x = p$ is $\frac{c}{p}$.

$$\text{At } x = p \quad y = c \ln p$$

$$\therefore y - c \ln p = \frac{c}{p}(x - p)$$

$$\therefore y - c \ln p = \frac{c}{p}x - c$$

$$\therefore y = \frac{c}{p}x - c + c \ln p$$

Question 29 (b)

Criteria	Marks
• Provides the correct solution	2
• States that $\frac{c}{p} = 1$ or $-c + c \ln p = 0$, or equivalent merit	1

Sample answer:

$$\frac{c}{p} = 1 \quad (\text{since the tangent has a gradient of 1})$$

$$\therefore c = p$$

$$-c + c \ln p = 0 \quad (\text{since the tangent passes through the origin})$$

$$-c + c \ln c = 0$$

$$c(-1 + \ln c) = 0$$

$$c = 0, \quad \ln c = 1$$

$$c = e \quad (\text{as } c > 0)$$

Question 30 (a)

Criteria	Marks
• Provides the correct solution	2
• Attempts to solve a quadratic equation to find the x-coordinate of A, or equivalent merit	1

Sample answer:

$$ax^2 = 4x - x^2$$

$$(a+1)x^2 - 4x = 0$$

$$x[(a+1)x - 4] = 0$$

$$x = 0, \quad (a+1)x - 4 = 0$$

$$(a+1)x = 4$$

$$x = \frac{4}{a+1}$$

Question 30 (b)

Criteria	Marks
• Provides the correct solution	4
• Correctly substitutes the limits into the anti-derivative, or equivalent merit	3
• Finds the anti-derivative, or equivalent merit	2
• Writes an expression for the area involving an integral of the difference between two functions, or equivalent merit	1

Sample answer:

$$\int_0^{\frac{4}{a+1}} (4x - x^2 - ax^2) dx = \frac{16}{3}$$

$$\therefore \int_0^{\frac{4}{a+1}} (4x - (a+1)x^2) dx = \frac{16}{3}$$

$$\left[2x^2 - \frac{(a+1)x^3}{3} \right]_0^{\frac{4}{a+1}} = \frac{16}{3}$$

$$\frac{2 \times 16}{(a+1)^2} - \frac{(a+1)}{3} \times \frac{64}{(a+1)^3} - 0 = \frac{16}{3}$$

$$\frac{32}{(a+1)^2} - \frac{64}{3(a+1)^2} = \frac{16}{3}$$

$$\frac{96 - 64}{3(a+1)^2} = \frac{16}{3}$$

$$\frac{32}{3(a+1)^2} = \frac{16}{3}$$

$$\therefore (a+1)^2 = 2$$

$$a = \sqrt{2} - 1 \quad (\text{since } a > 0)$$

Question 31 (a)

Criteria	Marks
• Provides the correct values for a and b	2
• Provides the correct value of a or the correct value of b , or equivalent merit	1

Sample answer:

$$a = \frac{35\,000 - 5\,000}{2}$$

$$= 15\,000$$

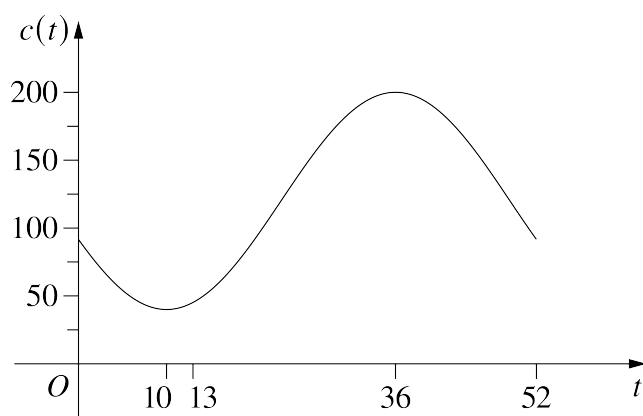
$$b = \frac{35\,000 + 5\,000}{2}$$

$$= 20\,000$$

Question 31 (b)

Criteria	Marks
• Provides the correct solution	3
• Finds the values of t when the population of mice increases and the values of t when the population of cats increases	2
• Finds the values of t when the population of mice increases, or equivalent merit	1

Sample answer:



The population of mice increases for $0 < t < 13$ and $39 < t < 52$.

The population of cats increases for $10 < t < 36$.

Therefore both populations are increasing for $10 < t < 13$.

Question 31 (c)

Criteria	Marks
• Provides the correct solution	2
• Correctly differentiates $m(t)$, or equivalent merit	1

Sample answer:

The cat population reaches a maximum at $t = 36$.

$$m'(t) = \frac{\pi}{26} \times 15\,000 \cos\left(\frac{\pi}{26}t\right)$$

$$\begin{aligned} m'(36) &= \frac{\pi}{26} \times 15\,000 \cos\left(\frac{\pi}{26} \times 36\right) \\ &= -642.7... \end{aligned}$$

Therefore the mice population is decreasing at 643 mice per week.

2020 HSC Mathematics Advanced Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	MA-F1 Working with Functions	MA11-1
2	1	MA-F2 Graphing Techniques	MA12-1
3	1	MA-S1 Probability and Discrete Probability Distributions	MA12-8
4	1	MA-C4 Integral Calculus	MA12-7
5	1	MA-F1 Working with Functions	MA11-2
6	1	MA-T3 Trigonometric Functions	MA12-5
7	1	MA-C4 Integral Calculus	MA12-7
8	1	MA-C3 Applications of Differentiation	MA12-3
9	1	MA-S3 Random Variables	MA12-8
10	1	MA-C2 Differential Calculus	MA12-6

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	1	MA-F1 Working with Functions	MA11-2
11 (b)	2	MA-F1 Working with Functions	MA11-1
11 (c)	1	MA-F1 Working with Functions	MA11-2
12	3	MA-M1 Modelling Financial Situations	MA12-4
13	2	MA-C4 Integral Calculus	MA12-7
14 (a)	2	MA-S1 Probability and Discrete Probability Distributions	MA11-9
14 (b)	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
14 (c)	2	MA-S1 Probability and Discrete Probability Distributions	MA11-7
15 (a)	1	MA-T1 Trigonometry and Measuring Angles	MA11-1
15 (b)	2	MA-T1 Trigonometry and Measuring Angles	MA11-3
15 (c)	2	MA-T1 Trigonometry and Measuring Angles	MA11-3
16	4	MA-C3 Applications of Differentiation	MA12-3, 6, 10
17	2	MA-C4 Integral Calculus	MA12-7
18 (a)	2	MA-C2 Differential Calculus	MA12-6
18 (b)	1	MA-C4 Integral Calculus	MA12-3
19	2	MA-T2 Trigonometric Functions and Identities	MA11-4

Question	Marks	Content	Syllabus outcomes
20	2	MA-C4 Integral Calculus	MA12-7
21 (a)	1	MA-E1 Exponentials and Logarithms	MA11-6
21 (b)	2	MA-C3 Applications of Differentiation	MA12-6
21 (c)	3	MA-E1 Exponentials and Logarithms	MA11-6
22	4	MA-T1 Trigonometry and Measuring Angles	MA11-9
23 (a)	2	MA-S3 Random Variables	MA12-8
23 (b)	2	MA-S3 Random Variables	MA12-8
24	3	MA-F2 Graphing Techniques	MA12-1
25 (a)	3	MA-F1 Working with Functions	MA11-1
25 (b)	4	MA-C3 Applications of Differentiation	MA12-10
26 (a)	2	MA-M1 Modelling Financial Situations	MA12-2
26 (b)	2	MA-M1 Modelling Financial Situations	MA12-2
26 (c)	3	MA-M1 Modelling Financial Situations	MA12-4
27	5	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
28 (a)	3	MA-S3 Random Variables	MA12-8
28 (b)	2	MA-S3 Random Variables	MA12-8
29 (a)	2	MA-C2 Differential Calculus	MA12-6
29 (b)	2	MA-C2 Differential Calculus	MA12-6
30 (a)	2	MA-F1 Working with Functions	MA11-1
30 (b)	4	MA-C4 Integral Calculus	MA12-7
31 (a)	2	MA-T3 Trigonometric Functions	MA12-5
31 (b)	3	MA-T3 Trigonometric Functions	MA12-5
31 (c)	2	MA-C3 Applications of Differentiation	MA12-6