



NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9–40)

- Attempt Questions 11–34
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 Which of the following is equivalent to $\sin^2 5x$?

- A. $1 + \cos^2 5x$
- B. $1 - \cos^2 5x$
- C. $-1 + \cos^2 5x$
- D. $-1 - \cos^2 5x$

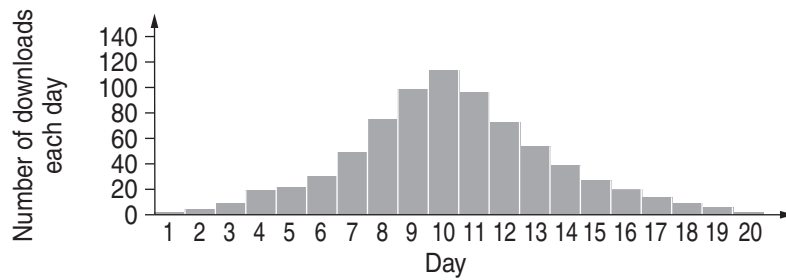
2 The probability distribution table for a discrete random variable X is shown.

x	$P(X = x)$
1	0.6
2	0.3
3	0.1

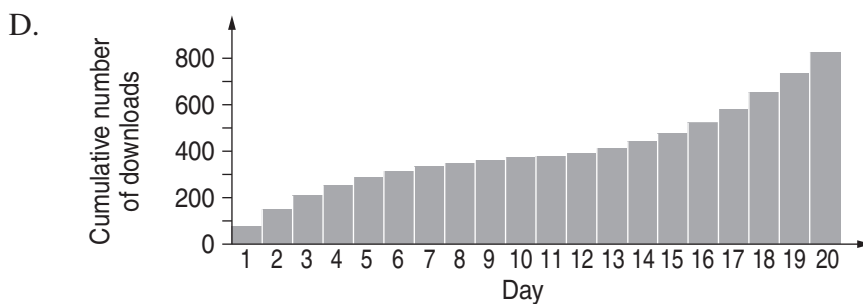
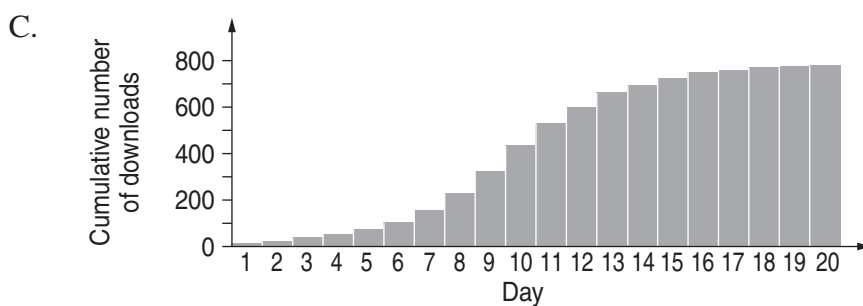
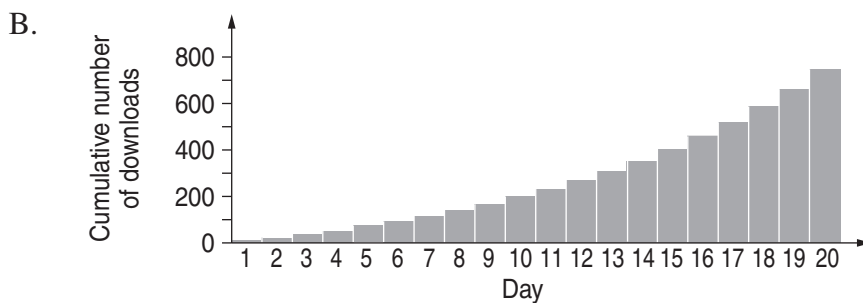
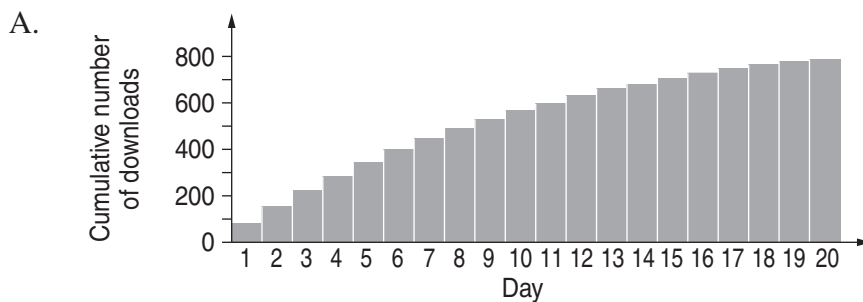
What is the expected value of X ?

- A. 0.6
 - B. 1.0
 - C. 1.5
 - D. 2.0
- 3 Which of the following represents the domain of the function $f(x) = \ln(1 - x)$?
- A. $[1, \infty)$
 - B. $(1, \infty)$
 - C. $(-\infty, 1]$
 - D. $(-\infty, 1)$

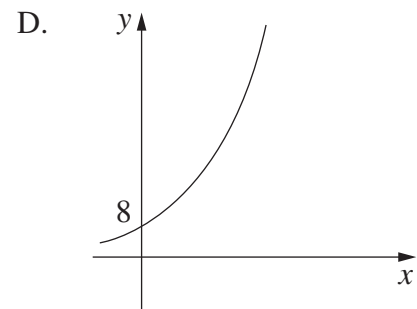
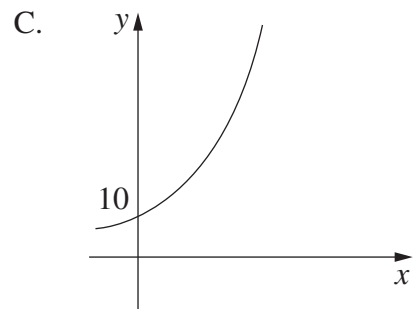
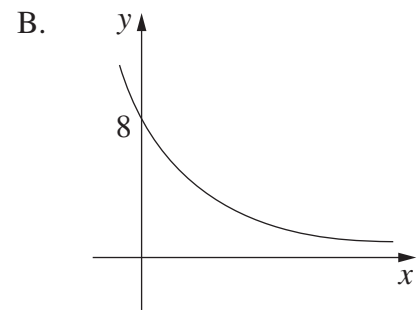
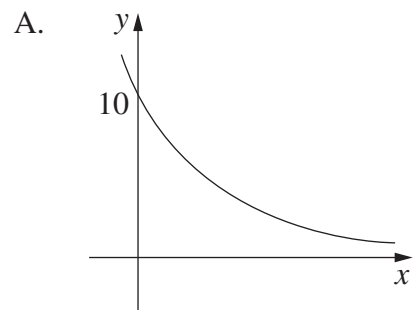
- 4 The number of downloads of a song on each of twenty consecutive days is shown in the following graph.



Which of the following graphs best shows the cumulative number of downloads up to and including each day?



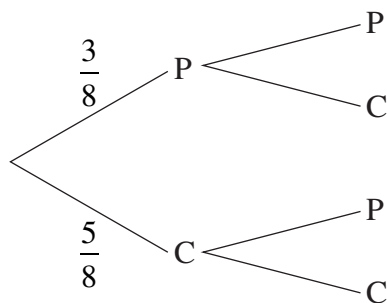
5 Which of the following best represents the graph of $y = 10(0.8)^x$?



- 6 There are 8 chocolates in a box. Three have peppermint centres (P) and five have caramel centres (C).

Kim randomly chooses a chocolate from the box and eats it. Sam then randomly chooses and eats one of the remaining chocolates.

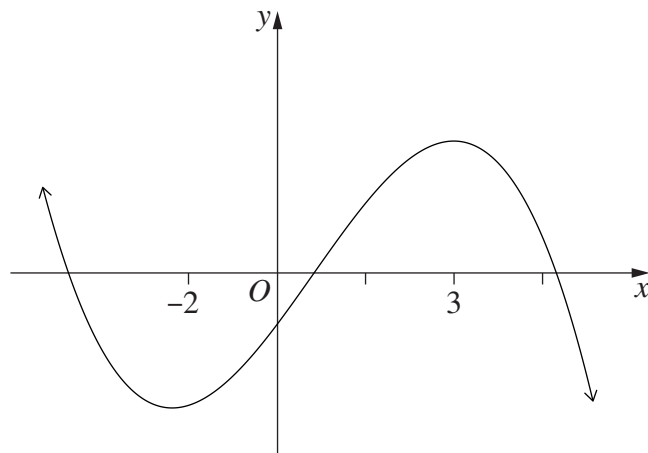
A partially completed probability tree is shown.



What is the probability that Kim and Sam choose chocolates with different centres?

- A. $\frac{15}{64}$
- B. $\frac{15}{56}$
- C. $\frac{15}{32}$
- D. $\frac{15}{28}$

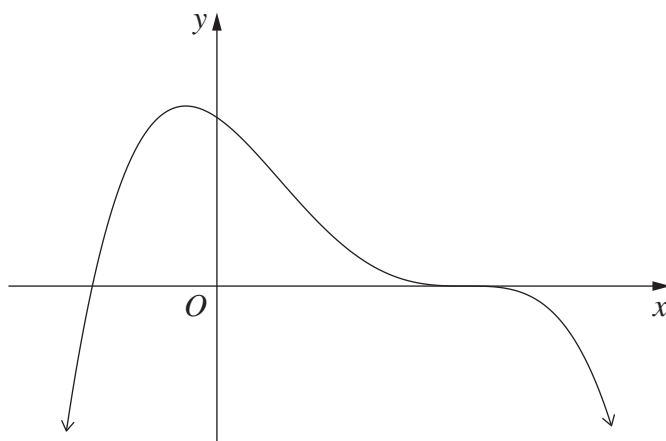
- 7 The diagram shows part of $y = f(x)$ which has a local minimum at $x = -2$ and a local maximum at $x = 3$.



Which of the following shows the correct relationship between $f''(-2)$, $f(0)$ and $f'(3)$?

- A. $f(0) < f'(3) < f''(-2)$
- B. $f(0) < f''(-2) < f'(3)$
- C. $f''(-2) < f'(3) < f(0)$
- D. $f''(-2) < f(0) < f'(3)$

- 8 The graph of $y = f(x)$ is shown.



Which of the following could be the equation of this graph?

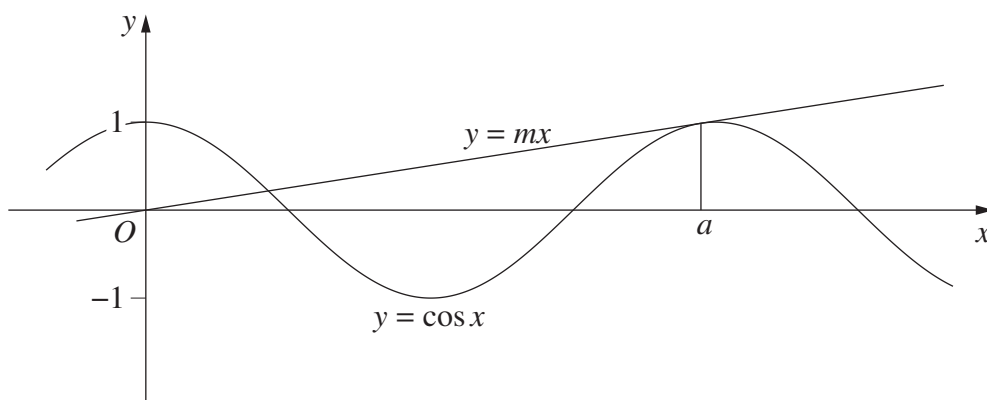
- A. $y = (1 - x)(2 + x)^3$
 - B. $y = (x + 1)(x - 2)^3$
 - C. $y = (x + 1)(2 - x)^3$
 - D. $y = (x - 1)(2 + x)^3$
- 9 Let $h(x) = f(g(x))$ where the function $f(x)$ is an odd function and the function $g(x)$ is an even function.

The tangent to $y = h(x)$ at $x = k$, where $k > 0$, has the equation $y = mx + c$.

What is the equation of the tangent to $y = h(x)$ at $x = -k$?

- A. $y = mx + c$
- B. $y = -mx + c$
- C. $y = mx - c$
- D. $y = -mx - c$

- 10 The line $y = mx$ is a tangent to the curve $y = \cos x$ at the point where $x = a$, as shown in the diagram.



Which of the following statements is true?

- A. $m < \frac{1}{a} < \frac{1}{2\pi}$
- B. $\frac{1}{2\pi} < m < \frac{1}{a}$
- C. $\frac{1}{2\pi} < \frac{1}{a} < m$
- D. $m < \frac{1}{2\pi} < \frac{1}{a}$

--	--	--	--	--

Centre Number

Mathematics Advanced

Section II Answer Booklet 1

--	--	--	--	--	--	--	--	--

Student Number

Section II

90 marks

Attempt Questions 11–34

Allow about 2 hours and 45 minutes for this section

Booklet 1 — Attempt Questions 11–28 (64 marks)

Booklet 2 — Attempt Questions 29–34 (26 marks)

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 28. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

Solve $x + \frac{x-1}{2} = 9$.

2

.....

.....

.....

.....

.....

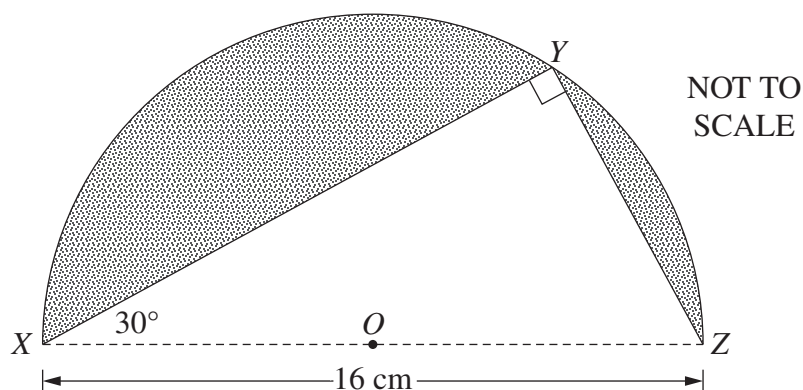
.....

.....

Do NOT write in this area.

Question 12 (5 marks)

A right-angled triangle XYZ is cut out from a semicircle with centre O . The length of the diameter XZ is 16 cm and $\angle YXZ = 30^\circ$, as shown on the diagram.



- (a) Find the length of XY in centimetres, correct to two decimal places.

2

.....

.....

.....

.....

.....

- (b) Hence, find the area of the shaded region in square centimetres, correct to one decimal place.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 13 (3 marks)

Find the exact gradient of the tangent to the curve $y = x \tan x$ at the point where $x = \frac{\pi}{3}$.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 14 (2 marks)

The first term of an arithmetic sequence is 5. The sum of the first 43 terms is 2021.

2

What is the common difference of the sequence?

.....

.....

.....

.....

.....

.....

.....

.....

Do NOT write in this area.

Question 15 (2 marks)

Evaluate $\int_{-2}^0 \sqrt{2x+4} \, dx$.

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 16 (3 marks)

For what values of x is $f(x) = x^2 - 2x^3$ increasing?

3

.....

.....

.....

.....

.....

.....

.....

.....

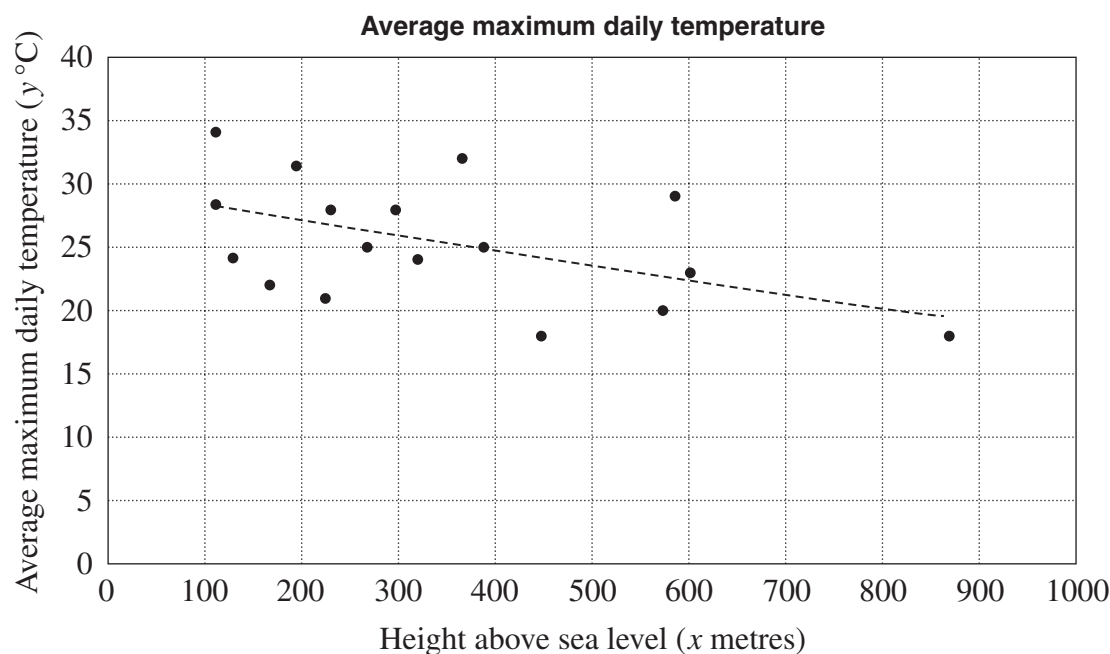
.....

.....

Question 17 (4 marks)

For a sample of 17 inland towns in Australia, the height above sea level, x (metres), and the average maximum daily temperature, y ($^{\circ}\text{C}$), were recorded.

The graph shows the data as well as a regression line.



The equation of the regression line is $y = 29.2 - 0.011x$.

The correlation coefficient is $r = -0.494$.

- (a) (i) By using the equation of the regression line, predict the average maximum daily temperature, in degrees Celsius, for a town that is 540 m above sea level. Give your answer correct to one decimal place.

1

.....

.....

- (ii) The gradient of the regression line is -0.011 . Interpret the value of this gradient in the given context.

2

.....

.....

.....

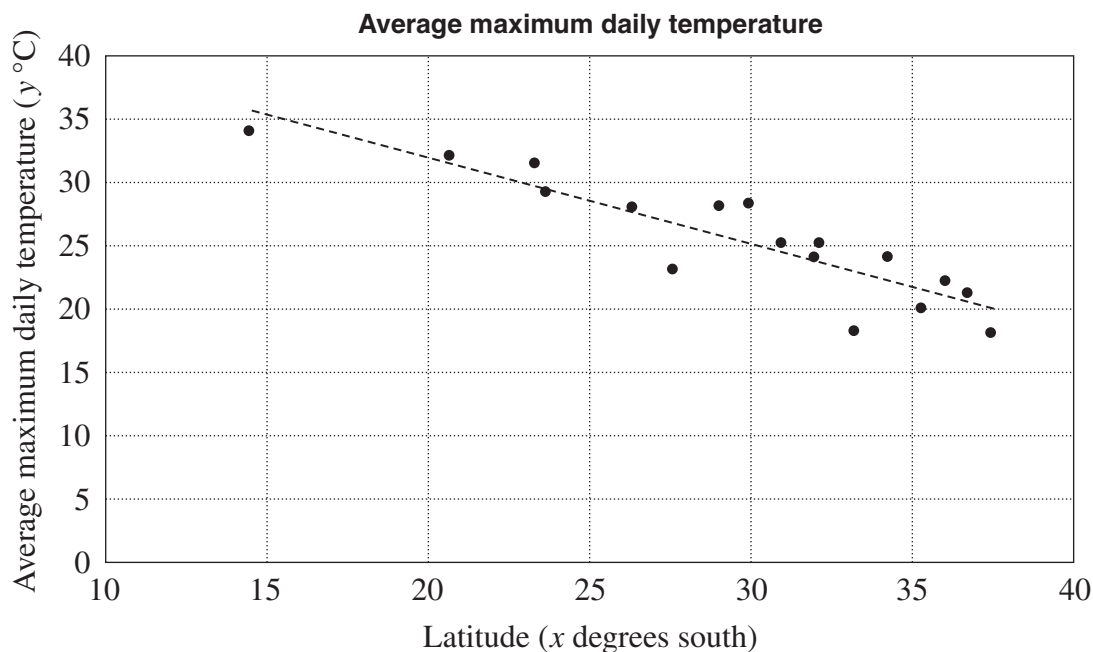
.....

Question 17 continues on page 15

Question 17 (continued)

- (b) The graph below shows the relationship between the latitude, x (degrees south), and the average maximum daily temperature, y ($^{\circ}\text{C}$), for the same 17 towns, as well as a regression line.

1



The equation of the regression line is $y = 45.6 - 0.683x$.

The correlation coefficient is $r = -0.897$.

Another inland town in Australia is 540 m above sea level. Its latitude is 28 degrees south.

Which measurement, height above sea level or latitude, would be better to use to predict this town's average maximum daily temperature? Give a reason for your answer.

.....

.....

.....

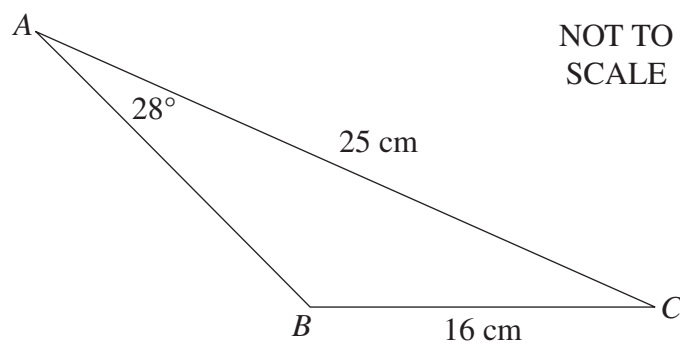
.....

End of Question 17

Question 18 (3 marks)

The diagram shows a triangle ABC where $AC = 25$ cm, $BC = 16$ cm, $\angle BAC = 28^\circ$ and angle ABC is obtuse.

3



Find the size of the obtuse angle ABC correct to the nearest degree.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Do NOT write in this area.

Question 19 (3 marks)

Without using calculus, sketch the graph of $y = 2 + \frac{1}{x+4}$, showing the asymptotes and the x and y intercepts.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 20 (2 marks)

For what values of x , in the interval $0 \leq x \leq \frac{\pi}{4}$, does the line $y = 1$ intersect the graph of $y = 2 \sin 4x$?

2

.....

.....

.....

.....

.....

.....

.....

.....

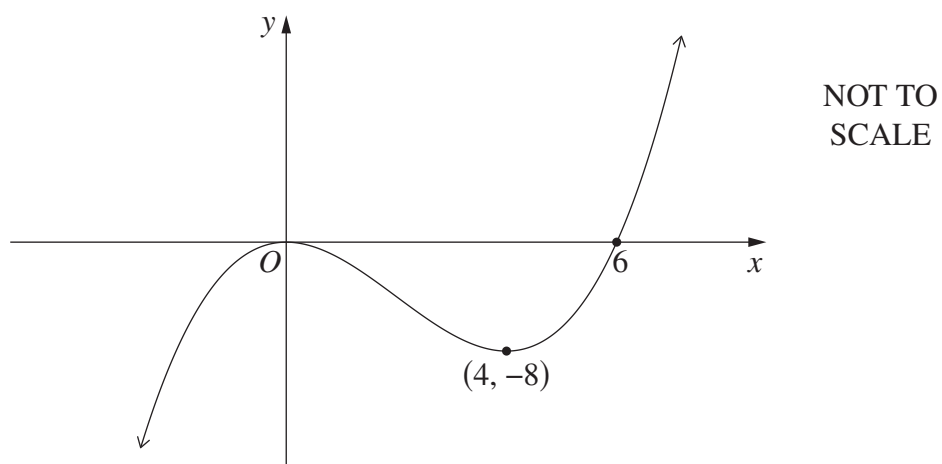
.....

.....

Question 21 (2 marks)

Consider the graph of $y = f(x)$ as shown.

2



Sketch the graph of $y = 4f(2x)$ showing the x -intercepts and the coordinates of the turning points.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

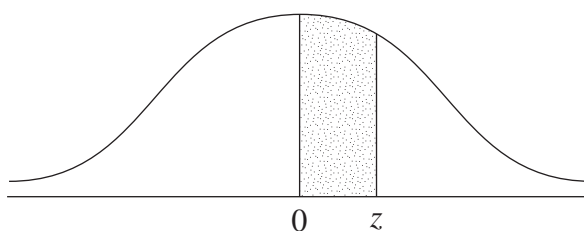
Do NOT write in this area.

Question 22 (4 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table gives the probability that this random variable lies between 0 and z for different values of z .

z	0.1	0.2	0.3	0.4	0.5	0.6
Probability	0.0398	0.0793	0.1179	0.1554	0.1915	0.2257

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



- (a) Using the table, find the probability that a value from a random variable that is normally distributed with mean 0 and standard deviation 1 lies between 0.1 and 0.5. 1

.....

.....

.....

.....

- (b) Birth weights are normally distributed with a mean of 3300 grams and a standard deviation of 570 grams. By first calculating a z -score, find how many babies, out of 1000 born, are expected to have a birth weight greater than 3528 grams. 3

.....

.....

.....

.....

.....

.....

.....

.....

Question 23 (4 marks)

A population, P , which is initially 5000, varies according to the formula

4

$$P = 5000b^{\frac{-t}{10}},$$

where b is a positive constant and t is time in years, $t \geq 0$.

The population is 1250 after 20 years.

Find the value of t , correct to one decimal place, for which the instantaneous rate of decrease is 30 people per year.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

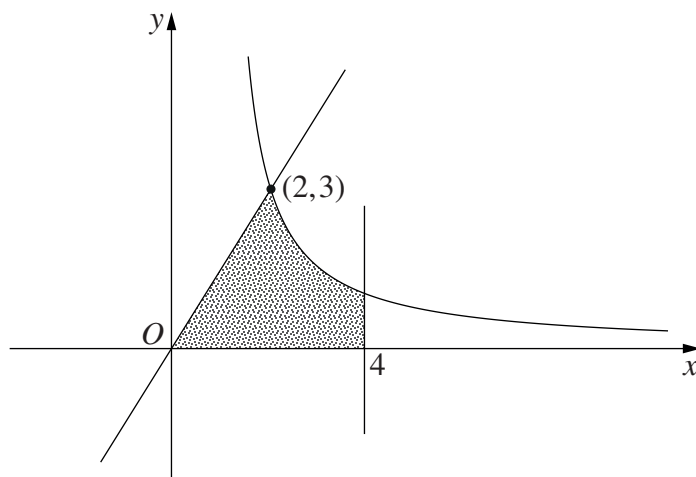
Do NOT write in this area.

Question 24 (3 marks)

The curve $y = \frac{3}{x-1}$ intersects the line $y = \frac{3}{2}x$ at the point $(2, 3)$.

3

The region bounded by the curve $y = \frac{3}{x-1}$, the line $y = \frac{3}{2}x$, the x -axis and the line $x = 4$ is shaded in the diagram.



Find the exact area of the shaded region.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 25 (3 marks)

A table of future value interest factors for an annuity of \$1 is shown.

3

Table of future value interest factors

<i>Number of periods</i>	<i>Interest rate per period</i>				
	0.25%	0.5%	0.75%	1%	1.25%
2	2.0025	2.0050	2.0075	2.0100	2.0125
4	4.0150	4.0301	4.0452	4.0604	4.0756
6	6.0376	6.0755	6.1136	6.1520	6.1907
8	8.0704	8.1414	8.2132	8.2857	8.3589
10	10.1133	10.2280	10.3443	10.4622	10.5817

Simone deposits \$1000 into a savings account at the end of each year for 8 years. The interest rate for these 8 years is 0.75% per annum, compounded annually.

After the 8th deposit, Simone stops making deposits but leaves the money in the savings account. The money in her savings account then earns interest at 1.25% per annum, compounded annually, for a further two years.

Find the amount of money in Simone's savings account at the end of ten years.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Do NOT write in this area.

Question 26 (5 marks)

A particle is shot vertically upwards from a point 100 metres above ground level. The position of the particle, y metres above the ground after t seconds, is given by $y(t) = -5t^2 + 70t + 100$.

- (a) Find the maximum height above ground level reached by the particle.

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the velocity of the particle, in metres per second, immediately before it hits the ground, leaving your answer in the form $a\sqrt{b}$, where a and b are integers.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Questions 11–26 are worth 50 marks in total

Question 27 (8 marks)

Kenzo has a solar powered phone charger. Its power, P , can be modelled by the function

$$P(t) = 400 \sin\left(\frac{\pi}{12}t\right), 0 \leq t \leq 12,$$

where t is the number of hours after sunrise.

- (a) Sketch the graph of P for $0 \leq t \leq 12$.

2

.....

.....

.....

.....

.....

.....

.....

Power is the rate of change of energy. Hence the amount of energy, E units, generated by the solar powered phone charger from $t = a$ to $t = b$, where $0 \leq a \leq b \leq 12$ is given by

$$E = \int_a^b P(t) dt.$$

- (b) Show that $E = \frac{4800}{\pi} \left(\cos \frac{a\pi}{12} - \cos \frac{b\pi}{12} \right)$.

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 27 continues on page 25

Question 27 (continued)

- (c) To make a phone call, a phone battery needs at least 300 units of energy. Kenzo woke up 3 hours after sunrise and found that his phone battery had no units of energy. He immediately began to use his solar powered charger to charge his phone battery. 3

Find the least amount of time he needed to wait before he could make a phone call. Give your answer correct to the nearest minute.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (d) The next day, Kenzo woke up 6 hours after sunrise and again found that his phone battery had no units of energy. He immediately began to use his solar powered charger to charge his phone battery. 1

Would it take more time or less time or the same amount of time, compared to the answer in part (c), to charge his phone battery in order to make a phone call? Explain your answer by referring to the graph drawn in part (a).

.....

.....

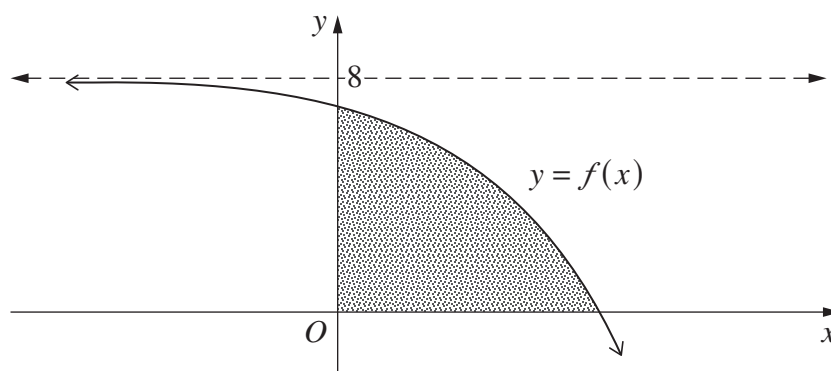
.....

.....

End of Question 27

Question 28 (6 marks)

The region bounded by the graph of the function $f(x) = 8 - 2^x$ and the coordinate axes is shown.



- (a) Show that the exact area of the shaded region is given by $24 - \frac{7}{\ln 2}$.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 28 continues on page 27

Question 28 (continued)

- (b) A new function $g(x)$ is found by taking the graph of $y = -f(-x)$ and translating it by 5 units to the right.

2

Sketch the graph of $y = g(x)$ showing the x -intercept and the asymptote.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Hence, find the exact value of $\int_2^5 g(x) dx$.

1

.....

.....

.....

End of Question 28

Proceed to Booklet 2 for Questions 29–34

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Do NOT write in this area.

--	--	--	--	--

Centre Number

Mathematics Advanced

Section II Answer Booklet 2

--	--	--	--	--	--	--	--	--

Student Number

Booklet 2 — Attempt Questions 29–34 (26 marks)

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 39–40. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 29 (5 marks)

- (a) On the day that Megan was born, her grandfather deposited \$5000 into an account earning 3% per annum compounded annually. On each birthday after this, her grandfather deposited \$1000 into the same account, making his final deposit on Megan's 17th birthday. That is, a total of 18 deposits were made.

2

Let A_n be the amount in the account on Megan's n th birthday, after the deposit is made.

Show that $A_3 = \$8554.54$.

.....

.....

.....

.....

.....

.....

.....

.....

Question 29 continues on page 31

Do NOT write in this area.

Do NOT write in this area.

- 3

How many withdrawals of \$2000 will Megan be able to make?

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

– 31 –

Question 30 (2 marks)

The number of hours for which light bulbs will work before failing can be modelled by the random variable X with cumulative distribution function

2

$$F(x) = \begin{cases} 1 - e^{-0.01x}, & x \geq 0 \\ 0, & x < 0 \end{cases}.$$

Jane sells light bulbs and promises that they will work for longer than exactly 99% of all light bulbs.

Find how long, according to Jane's promise, a light bulb bought from her should work. Give your answer in hours, rounded to two decimal places.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Do NOT write in this area.

Do NOT write in this area.

4

[illegible]

Question 32 (4 marks)

In a particular city, the heights of adult females and the heights of adult males are each normally distributed.

4

Information relating to two females from that city is given in Table 1.

Table 1

Height	Gender	Percentage of females in this city shorter than this person
175 cm	Female	97.5%
160.6 cm	Female	16%

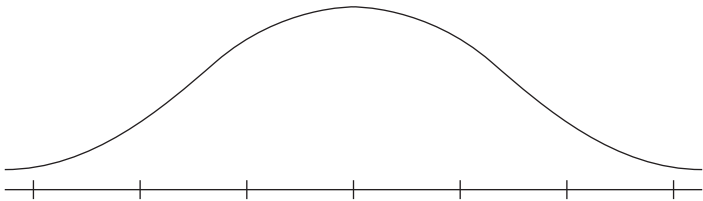
The means and standard deviations of adult females and males, in centimetres, are given in Table 2.

Table 2

	Mean	Standard deviation
Females	μ	σ
Males	1.05μ	1.1σ

A selected male is taller than 84% of the population of adult males in this city.

By first labelling the normal distribution curve below with the heights of the two females given in Table 1, calculate the height of the selected male, in centimetres, correct to two decimal places.



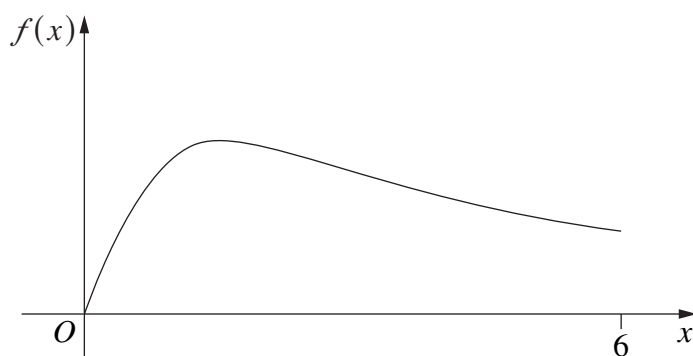
Do NOT write in this area.

Question 33 (8 marks)

People are given a maximum of six hours to complete a puzzle. The time spent on the puzzle, in hours, can be modelled using the continuous random variable X which has probability density function

$$f(x) = \begin{cases} \frac{Ax}{x^2 + 4}, & \text{for } 0 \leq x \leq 6, \text{ (where } A > 0) \\ 0, & \text{for all other values of } x \end{cases}$$

The graph of the probability density function is shown below. The graph has a local maximum.



- (a) Show that $A = \frac{2}{\ln 10}$.

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 33 continues on page 36

Question 33 (continued)

- (b) Show that the mode of X is two hours.

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Show that $P(X < 2) = \log_{10} 2$.

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 33 continues on page 37

Question 33 (continued)

- (d) The Intelligence Quotient (IQ) scores of people are normally distributed with a mean of 100 and standard deviation of 15.

2

It has been observed that the puzzle is generally completed more quickly by people with a high IQ.

It is known that 80% of people with an IQ greater than 130 can complete the puzzle in less than two hours.

A person chosen at random can complete the puzzle in less than two hours.

What is the probability that this person has an IQ greater than 130? Give your answer correct to three decimal places.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

End of Question 33

Please turn over

3

3



End of paper

Do NOT write in this area.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Do NOT write in this area.

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

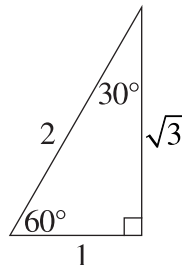
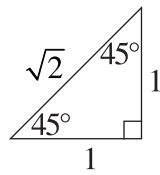
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1 + t^2}$$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2} (1 - \cos 2nx)$$

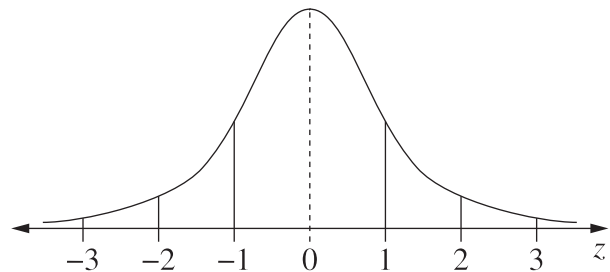
$$\cos^2 nx = \frac{1}{2} (1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z -scores between -1 and 1
- approximately 95% of scores have z -scores between -2 and 2
- approximately 99.7% of scores have z -scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function

Derivative

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

Integral Calculus

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \left\{ f(a) + f(b) + 2[f(x_1) + \cdots + f(x_{n-1})] \right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}_nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underline{u}| = |x\underline{i} + y\underline{j}| = \sqrt{x^2 + y^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1x_2 + y_1y_2,$$

$$\text{where } \underline{u} = x_1\underline{i} + y_1\underline{j}$$

$$\text{and } \underline{v} = x_2\underline{i} + y_2\underline{j}$$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos \theta + i \sin \theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos \theta + i \sin \theta)]^n &= r^n(\cos n\theta + i \sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2021 HSC Mathematics Advanced Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	B
2	C
3	D
4	C
5	A
6	D
7	A
8	C
9	B
10	B

Section II

Question 11

Criteria	Marks
• Provides correct solution	2
• Writes fraction using a common denominator, or equivalent merit	1

Sample answer:

$$x + \frac{x-1}{2} = 9$$

$$2x + x - 1 = 18$$

$$3x = 19$$

$$x = \frac{19}{3}$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	2
• Uses correct trigonometric ratio, or equivalent merit	1

Sample answer:

In $\triangle XYZ$

$$\cos 30^\circ = \frac{XY}{16}$$

$$XY = 13.86 \text{ cm (two decimal places)}$$

Question 12 (b)

Criteria	Marks
• Provides correct solution	3
• Calculates the area of $\triangle XYZ$ and the semicircle, or equivalent merit	2
• Calculates the area of a semicircle, or equivalent merit	1

Sample answer:

$$\text{Shaded area} = \frac{1}{2}\pi(8)^2 - \frac{1}{2} \times 13.86 \times 16 \times \sin 30^\circ$$

$$= 45.1 \text{ cm}^2 \text{ (one decimal place)}$$

Question 13

Criteria	Marks
• Provides correct solution	3
• Finds the derivative and attempts to calculate gradient	2
• Applies the product rule, or equivalent merit	1

Sample answer:

$$y = x \tan x$$

$$\frac{dy}{dx} = x \sec^2 x + \tan x$$

$$\begin{aligned} \text{Gradient of tangent} &= \frac{\pi}{3} \sec^2\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{3}\right) \\ &= \frac{\pi}{3} \frac{1}{\left(\frac{1}{2}\right)^2} + \sqrt{3} \\ &= \frac{4\pi}{3} + \sqrt{3} \end{aligned}$$

Question 14

Criteria	Marks
• Provides correct solution	2
• Substitutes into sum of an arithmetic series formula	1

Sample answer:

$$a = 5, S_{43} = 2021$$

$$\therefore 2021 = \frac{43}{2}(2 \times 5 + (43 - 1)d)$$

$$94 = 10 + 42d$$

$$d = 2$$

Question 15

Criteria	Marks
• Provides correct solution	2
• Finds the anti-derivative of $\sqrt{2x+4}$, or equivalent merit	1

Sample answer:

$$\begin{aligned}
 \int_{-2}^0 \sqrt{2x+4} \, dx &= \int_{-2}^0 (2x+4)^{\frac{1}{2}} \, dx \\
 &= \frac{1}{2} \cdot \frac{2}{\frac{3}{2}} \left[(2x+4)^{\frac{3}{2}} \right]_{-2}^0 \\
 &= \frac{1}{3} \left(4^{\frac{3}{2}} - 0 \right) \\
 &= \frac{8}{3}
 \end{aligned}$$

Question 16

Criteria	Marks
• Provides correct solution	3
• States $2x - 6x^2 > 0$, or equivalent merit	2
• Finds the derivative, or equivalent merit	1

Sample answer:

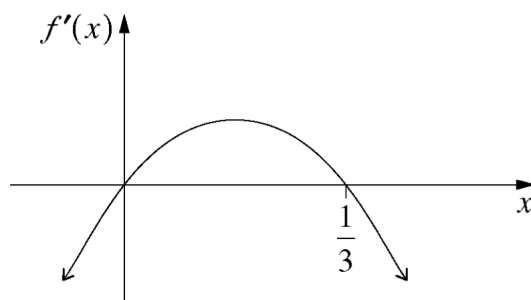
$$f(x) = x^2 - 2x^3$$

$$f'(x) = 2x - 6x^2$$

For $f(x)$ to be increasing, we need $f'(x) > 0$.

$$2x - 6x^2 > 0$$

$$2x(1 - 3x) > 0$$



$$\therefore 0 < x < \frac{1}{3}$$

Question 17 (a) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$y = 29.2 - 0.011 \times 540$$

$$= 23.3^{\circ}\text{C}$$

Question 17 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Provides a correct relevant statement relating temperature to height above sea level, or equivalent merit	1

Sample answer:

For a one metre increase in the height above sea level, we expect a 0.011°C drop in average maximum daily temperature.

Question 17 (b)

Criteria	Marks
• Provides correct solution	1

Sample answer:

It would be better to use the latitude. From the information given, the average maximum daily temperature has a stronger correlation with latitude than with height above sea level.

Question 18

Criteria	Marks
• Provides correct solution	3
• Calculates 47.18° , or equivalent merit	2
• Substitutes into the sine rule correctly, or equivalent merit	1

Sample answer:

$$\frac{\sin \hat{A}BC}{25} = \frac{\sin 28^\circ}{16}$$

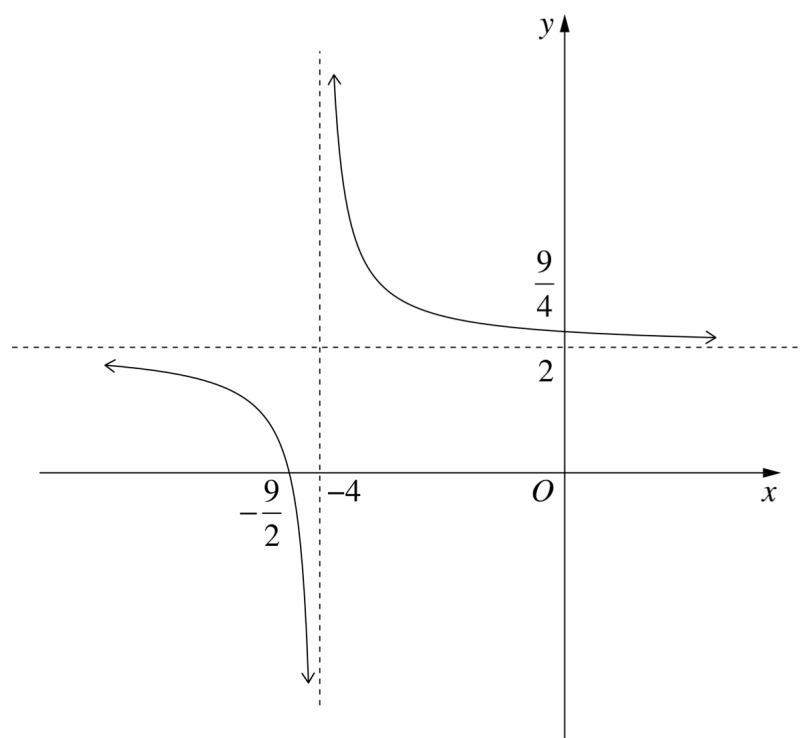
$$\therefore \sin \hat{A}BC = 0.733\dots$$

$$\begin{aligned}\therefore \hat{A}BC &= 180^\circ - 47.18\dots^\circ \\ &= 133^\circ \text{ (nearest degree)}\end{aligned}$$

Question 19

Criteria	Marks
• Provides correct solution	3
• Provides correct graph without x and y intercepts, or equivalent merit	2
• Draws a hyperbola, or equivalent merit	1

Sample answer:



$$\begin{aligned} \text{y-intercept: } y &= 2 + \frac{1}{4} \\ &= \frac{9}{4} \end{aligned}$$

$$\begin{aligned} \text{x-intercept: } 0 &= 2 + \frac{1}{x+4} \\ x &= -\frac{9}{2} \end{aligned}$$

Question 20

Criteria	Marks
• Provides correct solution	2
• Provides $\frac{\pi}{6}$, or equivalent merit	1

Sample answer:

$$0 \leq x \leq \frac{\pi}{4} \Rightarrow 0 \leq 4x \leq \pi$$

$$2 \sin 4x = 1$$

$$\sin 4x = \frac{1}{2}$$

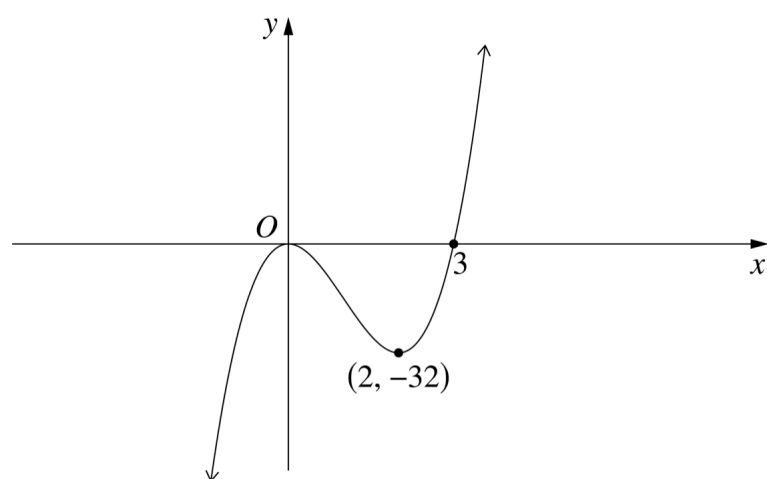
$$4x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \frac{\pi}{24}, \frac{5\pi}{24}$$

Question 21

Criteria	Marks
• Provides correct graph	2
• Provides graph with correct x intercepts, or equivalent merit	1

Sample answer:



Question 22 (a)

• Provides correct solution	1
-----------------------------	---

Sample answer:

$$\begin{aligned}\text{Probability} &= 0.1915 - 0.0398 \\ &= 0.1517\end{aligned}$$

Question 22 (b)

Criteria	Marks
• Provides correct solution	3
• Finds associated probability, or equivalent merit	2
• Calculates z-score, or equivalent merit	1

Sample answer:

$$\begin{aligned}z &= \frac{3528 - 3300}{570} \\ &= 0.4\end{aligned}$$

$$P(Z > 0.4) = 0.5 - 0.1554 = 0.3446$$

$\therefore$ Expect $1000 \times 0.3446 = 344.6$ babies born to have a birth weight greater than 3528 grams.

Question 23

Criteria	Marks
• Provides correct solution	4
• Equates -30 to a correct derivative, with $b = 2$, or equivalent merit	3
• Finds b and provides correct derivative, or equivalent merit	2
• Provides correct derivative, or equivalent merit	1

Sample answer:

$$P = 5000b^{-\frac{t}{10}}$$

$$t = 20, P = 1250$$

$$1250 = 5000b^{-2}$$

$$b^{-2} = \frac{1}{4}$$

$$b = 2 \ (b > 0)$$

$$\therefore P = 5000(2)^{-\frac{t}{10}}$$

$$\frac{dP}{dt} = 5000 \times \ln 2 \times \frac{-1}{10} \times 2^{-\frac{t}{10}}$$

$$\frac{dP}{dt} = -30 \ t = ?$$

$$-30 = -500 \times \ln 2 \times 2^{-\frac{t}{10}}$$

$$2^{\frac{t}{10}} = \frac{500 \ln 2}{30}$$

$$2^{\frac{t}{10}} = 11.55 \dots$$

$$\frac{t}{10} \ln 2 = \ln 11.55 \dots$$

$$t = 35.3$$

Question 24

Criteria	Marks
• Provides the correct solution	3
• Evaluates $\int_2^4 \frac{3}{x-1} dx$, or equivalent merit	2
• Calculates the area of the triangle, or equivalent merit	1

Sample answer:

$$\begin{aligned}
 \text{Area} &= \frac{2 \times 3}{2} + \int_2^4 \frac{3}{x-1} dx \\
 &= 3 + 3[\ln(x-1)]_2^4 \\
 &= (3 + 3\ln 3) \text{ units}^2
 \end{aligned}$$

Question 25

Criteria	Marks
• Provides correct solution	3
• Finds the correct future value of the annuity and attempts to apply the compound interest formula, or equivalent merit	2
• Identifies the correct interest factor from the table, or equivalent merit	1

Sample answer:

$$\begin{aligned}
 \text{Money in account after 8 years} &= 1000 \times 8.2132 \\
 &= \$8213.20
 \end{aligned}$$

$$\begin{aligned}
 \text{Amount after 2 more years} &= 8213.20 (1.0125)^2 \\
 &= \$8419.81
 \end{aligned}$$

Question 26 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the time it takes to reach maximum height, or equivalent merit	1

Sample answer:

$$y(t) = -5t^2 + 70t + 100$$

$$y'(t) = -10t + 70$$

$$y'(t) = 0 \quad \therefore -10t + 70 = 0$$

$$t = 7$$

$$\begin{aligned} \text{At } t = 7, \quad y &= -5(7)^2 + 70(7) + 100 \\ &= 345 \text{ m} \end{aligned}$$

Question 26 (b)

Criteria	Marks
• Provides correct solution	3
• Finds the correct value of t , or equivalent merit	2
• Attempts to solve to find the time $y(t) = 0$, or equivalent merit	1

Sample answer:

To find when the particle hits the ground, we solve

$$-5t^2 + 70t + 100 = 0$$

$$t^2 - 14t - 20 = 0$$

$$t = \frac{14 \pm \sqrt{196 + 80}}{2}$$

$$= 7 \pm \sqrt{69}$$

$$t = 7 + \sqrt{69}, \text{ since } t > 0$$

$$\begin{aligned} y'(7 + \sqrt{69}) &= -10(7 + \sqrt{69}) + 70 \\ &= -10\sqrt{69} \end{aligned}$$

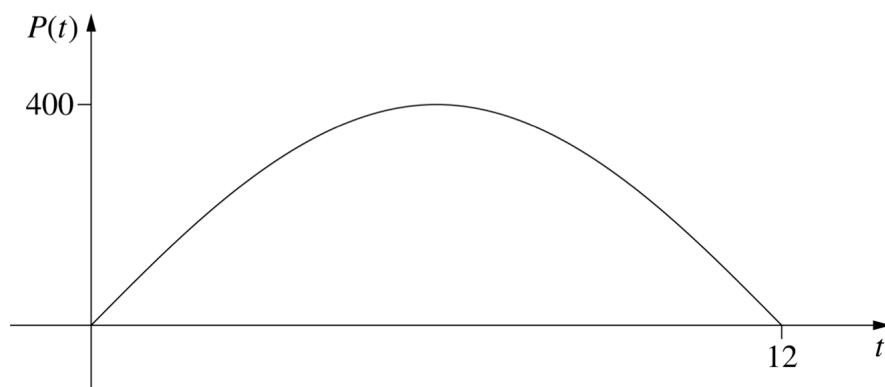
$$\therefore a = -10 \text{ and } b = 69$$

Question 27 (a)

Criteria	Marks
• Provides correct sketch	2
• Provides a sine graph with the correct amplitude, or equivalent merit	1

Sample answer:

$$\begin{aligned}\text{Period} &= \frac{2\pi}{\frac{\pi}{12}} \\ &= 24 \text{ hours}\end{aligned}$$



Question 27 (b)

Criteria	Marks
• Provides correct solution	2
• Provides correct anti-derivative of $\sin\left(\frac{\pi}{12}t\right)$, or equivalent merit	1

Sample answer:

$$\begin{aligned}E &= \int_a^b P(t) dt \\ &= \int_a^b 400 \sin\left(\frac{\pi}{12}t\right) dt \\ &= -\frac{400(12)}{\pi} \left[\cos\left(\frac{\pi}{12}t\right) \right]_a^b \\ &= -\frac{4800}{\pi} \left(\cos \frac{b\pi}{12} - \cos \frac{a\pi}{12} \right) \\ &= \frac{4800}{\pi} \left(\cos \frac{a\pi}{12} - \cos \frac{b\pi}{12} \right)\end{aligned}$$

Question 27 (c)

Criteria	Marks
• Provides correct solution	3
• Calculates the value of b , or equivalent merit	2
• Substitutes into E given in part (b), or equivalent merit	1

Sample answer:

$$300 = \frac{4800}{\pi} \left(\cos \frac{3\pi}{12} - \cos \frac{b\pi}{12} \right)$$

$$\frac{\pi}{16} = \frac{1}{\sqrt{2}} - \cos \frac{b\pi}{12}$$

$$\cos \frac{b\pi}{12} = \frac{1}{\sqrt{2}} - \frac{\pi}{16}$$

$$b = \frac{12}{\pi} \cos^{-1} \left(\frac{1}{\sqrt{2}} - \frac{\pi}{16} \right)$$

$$b = 3.95$$

$\therefore$ He needs to wait $0.95 \times 60 = 57$ minutes (to the nearest minute)

Question 27 (d)

Criteria	Marks
• Provides correct solution	1

Sample answer:

It would take less time to charge his phone battery.

By starting to charge the battery six hours after sunrise, he is charging the phone at the time when the power is greatest.

Question 28 (a)

Criteria	Marks
• Provides correct solution	3
• Finds the correct anti-derivative, or equivalent merit	2
• Writes the correct definite integral, or equivalent merit	1

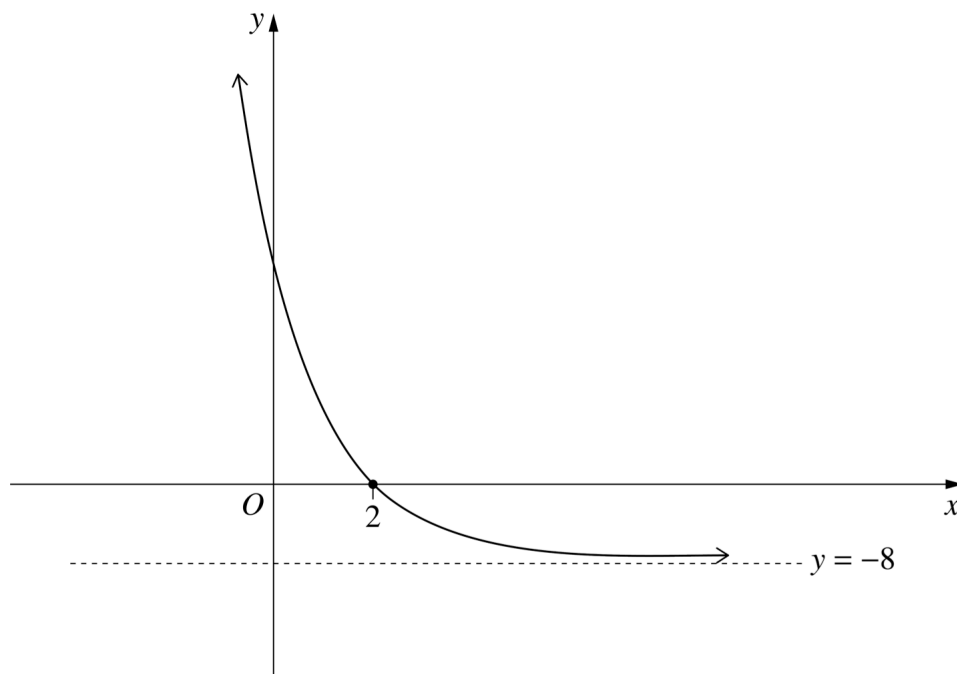
Sample answer:

$$\begin{aligned}
 & \int_0^3 8 - 2^x dx \\
 &= \left[8x - \frac{2^x}{\ln 2} \right]_0^3 \\
 &= \left(24 - \frac{8}{\ln 2} \right) - \left(0 - \frac{1}{\ln 2} \right) \\
 &= 24 - \frac{8}{\ln 2} + \frac{1}{\ln 2} \\
 &= 24 - \frac{7}{\ln 2}
 \end{aligned}$$

Question 28 (b)

Criteria	Marks
• Provides correct graph	2
• Draws the graph $y = f(-x)$, or equivalent merit	1

Sample answer:



Question 28 (c)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\int_2^5 g(x) dx = \frac{7}{\ln 2} - 24$$

Question 29 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the correct value of A_1 , or equivalent merit	1

Sample answer:

$$A_0 = \$5000$$

$$\begin{aligned} A_1 &= 5000(1.03) + 1000 \\ &= \$6150 \end{aligned}$$

$$\begin{aligned} A_2 &= 6150(1.03) + 1000 \\ &= \$7334.50 \end{aligned}$$

$$\begin{aligned} A_3 &= 7334.50(1.03) + 1000 \\ &= \$8554.54 \end{aligned}$$

Question 29 (b)

Criteria	Marks
• Provides correct solution	3
• Finds the correct expression for the amount after n withdrawals using the sum of GP formula, or equivalent merit	2
• Attempts to find an expression for the amount after n withdrawals, or equivalent merit	1

Sample answer:

Let B_n = the amount in the account after n withdrawals

Then $B_0 = 30\,025.83$

$$B_1 = 30\,025.83(1.03) - 2000$$

$$B_2 = 30\,025.83(1.03)^2 - 2000(1.03) - 2000$$

$$B_n = 30\,025.83(1.03)^n - 2000[1 + (1.03) + \dots + (1.03)^{n-1}]$$

We want $B_n < 0$.

Find n so that $B_n = 0$.

$$30\,025.83(1.03)^n = 2000 \left[\frac{(1.03)^n - 1}{1.03 - 1} \right]$$

$$\frac{0.03 \times 30\,025.83}{2000}(1.03)^n = (1.03)^n - 1$$

$$0.45039(1.03)^n = (1.03)^n - 1$$

$$0.54961(1.03)^n = 1$$

$$(1.03)^n = 1.8194\dots$$

$$n \ln(1.03) = \ln(1.8194)$$

$$n = \frac{\ln(1.8194)}{\ln(1.03)}$$

$$n \cong 20.25$$

$\therefore$ Megan makes 20 withdrawals of \$2000.

Question 30

Criteria	Marks
• Provides correct solution	2
• Writes $1 - e^{-0.01x} = 0.99$, or equivalent merit	1

Sample answer:

$$F(x) = 0.99$$

$$1 - e^{-0.01x} = 0.99$$

$$e^{-0.01x} = 0.01$$

$$-0.01x = \ln(0.01)$$

$$x = 460.517\dots$$

$$= 460.52 \text{ hours (two decimal places)}$$

Question 31

Criteria	Marks
• Provides correct solution	4
• Substitute $(3, -8)$ into the equation of the tangent at $x = a$, or equivalent merit	3
• Finds the equation of the tangent at $x = a$, or equivalent merit	2
• Finds the gradient of the tangent at $x = a$, or equivalent merit	1

Sample answer:

$$y = x^2 - 1$$

$$y' = 2x$$

Gradient of tangent at $x = a$ is $2a$.

$\therefore$ Equation of tangent at $(a, a^2 - 1)$ is

$$y - (a^2 - 1) = 2a(x - a)$$

$$y - a^2 + 1 = 2ax - 2a^2$$

$$y = 2ax - a^2 - 1$$

Since the tangent passes through $(3, -8)$ then

$$-8 = 6a - a^2 - 1$$

$$a^2 - 6a - 7 = 0$$

$$(a - 7)(a + 1) = 0$$

$$a = 7, \quad a = -1$$

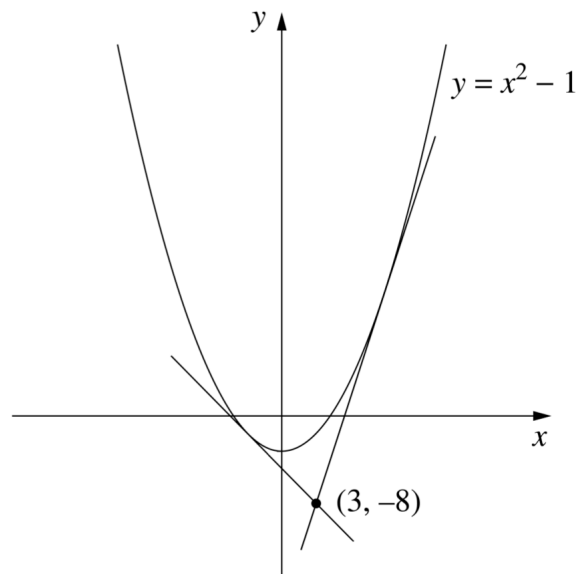
$\therefore$ The equations of the two tangents are

$$\text{When } a = 7, \quad y = 2(7)x - 7^2 - 1$$

$$y = 14x - 50$$

$$\text{and when } a = -1, \quad y = 2(-1)x - (-1)^2 - 1$$

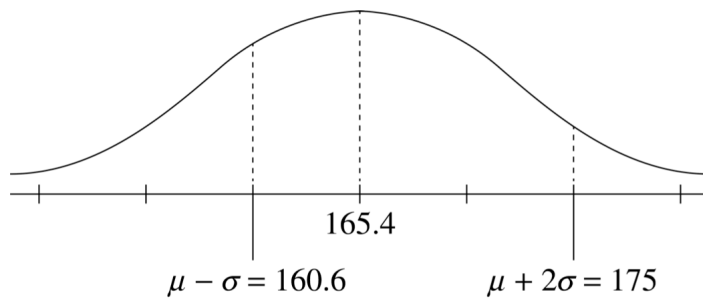
$$y = -2x - 2$$



Question 32

Criteria	Marks
• Provides correct solution	4
• Finds the mean and standard deviation for the heights of adult males, or equivalent merit	3
• Finds σ , or equivalent merit	2
• Labels the diagram, or equivalent merit	1

Sample answer:



$$\sigma = \frac{175 - 160.6}{3}$$

$$= 4.8$$

$$\mu = 160.6 + 4.8$$

$$= 165.4$$

Selected male is taller than 84% of population

$\therefore$ Selected male is 1 standard deviation above mean.

$$\text{Mean height for males} = 1.05 \times 165.4 = 173.67$$

$$\text{Standard deviation of height for males} = 4.8 \times 1.1 = 5.28$$

$$\therefore \text{Height} = 1.05\mu + 1.1\sigma$$

$$= 173.67 + 5.28$$

$$= 178.95 \text{ cm}$$

Question 33 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the anti-derivative of $\frac{Ax}{x^2 + 4}$, or equivalent merit	1

Sample answer:

$$\int_0^6 \frac{Ax}{x^2 + 4} dx = 1$$

$$\frac{A}{2} [\ln(x^2 + 4)]_0^6 = 1$$

$$\frac{A}{2} (\ln 40 - \ln 4) = 1$$

$$\frac{A}{2} \left(\ln \frac{40}{4} \right) = 1$$

$$\frac{A}{2} \ln 10 = 1$$

$$A = \frac{2}{\ln 10}$$

Question 33 (b)

Criteria	Marks
• Provides correct solution	2
• Finds the correct derivative of $f(x)$, or equivalent merit	1

Sample answer:

$$f'(x) = \frac{A(x^2 + 4) - Ax(2x)}{(x^2 + 4)^2}$$

$$= \frac{Ax^2 + 4A - 2Ax^2}{(x^2 + 4)^2}$$

$$= \frac{4A - Ax^2}{(x^2 + 4)^2} = 0$$

$$\therefore 4A - Ax^2 = 0$$

When $Ax^2 = 4A$

$$x^2 = 4$$

$$x = 2, \quad x > 0$$

 $\therefore$ The mode of X is 2.

Question 33 (c)

Criteria	Marks
• Provides correct solution	2
• Provides correct definite integral, or equivalent merit	1

Sample answer:

$$\begin{aligned}P(X < 2) &= \int_0^2 \frac{Ax}{x^2 + 4} dx \\&= \frac{A}{2} \left[\ln(x^2 + 4) \right]_0^2 \\&= \frac{A}{2} (\ln 8 - \ln 4) \\&= \frac{A}{2} \ln 2 \\&= \frac{2}{\ln 10} \times \frac{1}{2} \times \ln 2 \\&= \frac{\ln 2}{\ln 10} \\&= \log_{10} 2\end{aligned}$$

Question 33 (d)

Criteria	Marks
• Provides correct solution	2
• Attempts to find the conditional probability, or equivalent merit	1

Sample answer:

$$P(\text{IQ} > 130)$$

$$= P\left(\frac{\text{IQ} - 100}{15} > \frac{130 - 100}{15}\right)$$

$$= P(Z > 2)$$

$$= 2.5\%$$

$$P\left(\text{IQ} > 130 \mid \begin{array}{c} \text{Time to} \\ \text{complete} \end{array} < 2\right)$$

$$= \frac{P\left(\text{IQ} > 130 \cap \begin{array}{c} \text{Time to} \\ \text{complete} \end{array} < 2\right)}{P\left(\begin{array}{c} \text{Time to} \\ \text{complete} \end{array} < 2\right)}$$

$$= \frac{2.5\% \times 80\%}{\log_{10} 2}$$

$$= 0.06643\dots$$

$$= 0.066 \text{ (three decimal places)}$$

Question 34

Criteria	Marks
• Provides correct solution	3
• Shows $E(X) = nr^{n+1}$ and attempts to find the sum of probabilities using sum of GP formula, or equivalent merit	2
• Finds correct expression for $E(X)$ or writes the sum of probabilities equal to 1	1

Sample answer:

$$r + r^2 + \dots + r^n = 1$$

Sum of GP with $a = r$, common ratio $= r$, number of terms $= n$

$$\begin{aligned} \therefore \frac{r(r^n - 1)}{r - 1} &= 1 \\ r^{n+1} - r &= r - 1 \\ r^{n+1} &= 2r - 1 \quad \text{—————} \quad (*) \end{aligned}$$

$$\begin{aligned} E(X) &= \sum xP(X = x) \\ &= r \times r^n + r^2 \times r^{n-1} + \dots + r^n \times r \\ &= r^{n+1} + r^{n+1} + \dots + r^{n+1} \\ &= r^{n+1} \times n \\ &= nr^{n+1} \\ &= n(2r - 1) \quad \text{from } (*) \end{aligned}$$

2021 HSC Mathematics Advanced Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	MA-T2 Trigonometric Functions and Identities	MA11-1
2	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
3	1	MA-E1 Logarithms and Exponentials	MA12-1
4	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
5	1	MA-E1 Logarithms and Exponentials	MA12-1
6	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
7	1	MA-C3 Applications of Differentiation	MA12-3
8	1	MA-F1 Working with Functions	MA11-1
9	1	MA-C1 Introduction to Differentiation	MA11-1
10	1	MA-C3 Applications of Differentiation	MA12-10

Section II

Question	Marks	Content	Syllabus outcomes
11	2	MA-F1 Working with Functions	MA11-1
12 (a)	2	MA-T1 Trigonometry and Measure of Angles	MA11-3
12 (b)	3	MA-T1 Trigonometry and Measure of Angles	MA11-3
13	3	MA-C2 Differential Calculus	MA12-3
14	2	MA-M1 Modelling Financial Situations	MA12-4
15	2	MA-C4 Integral Calculus	MA12-3
16	3	MA-C3 Applications of Differentiation	MA12-6
17 (a) (i)	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
17 (a) (ii)	2	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
17 (b)	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
18	3	MA-T1 Trigonometry and Measure of Angles	MA11-3
19	3	MA-F2 Graphing Techniques	MA12-1
20	2	MA-T3 Trigonometric Functions and Graphs	MA12-5
21	2	MA-F2 Graphing Techniques	MA12-1

Question	Marks	Content	Syllabus outcomes
22 (a)	1	MA-S3 Random Variables	MA12-8
22 (b)	3	MA-S3 Random Variables	MA12-8
23	4	MA-C3 Applications of Differentiation	MA12-3
24	3	MA-C4 Integral Calculus	MA12-7
25	3	MA-M1 Modelling Financial Situations	MA12-2
26 (a)	2	MA-C3 Applications of Differentiation	MA12-6
26 (b)	3	MA-C3 Applications of Differentiation	MA12-6
27 (a)	2	MA-T3 Trigonometric Functions and Graphs	MA12-5
27 (b)	2	MA-C4 Integral Calculus	MA12-3
27 (c)	3	MA-T3 Trigonometric Functions and Graphs	MA12-5
27 (d)	1	MA-C4 Integral Calculus	MA12-10
28 (a)	3	MA-C4 Integral Calculus	MA12-7
28 (b)	2	MA-F2 Graphing Techniques	MA12-1
28 (c)	1	MA-C4 Integral Calculus	MA12-7
29 (a)	2	MA-M1 Modelling Financial Situations	MA12-4
29 (b)	3	MA-M1 Modelling Financial Situations	MA12-2
30	2	MA-S3 Random Variables	MA12-8
31	4	MA-C3 Applications of Differentiation	MA12-10
32	4	MA-S3 Random Variables	MA12-8
33 (a)	2	MA-S3 Random Variables	MA12-8
33 (b)	2	MA-S3 Random Variables	MA12-8
33 (c)	2	MA-S1 Random Variables	MA 12-8
33 (d)	2	MA-S1 Probability and Discrete Probability Distributions	MA11-7
34	3	MA-S1 Probability and Discrete Probability Distributions	MA11-7