NSW Education Standards Authority

2022 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9–36)

- Attempt Questions 11–32
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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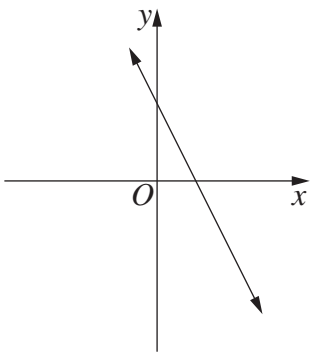
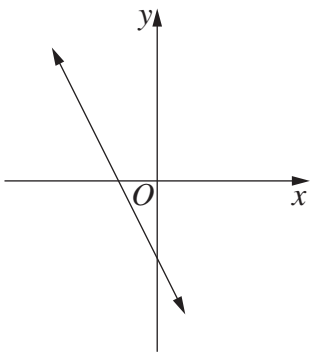
1 Which of the following could be the graph of $y = -2x + 2$?

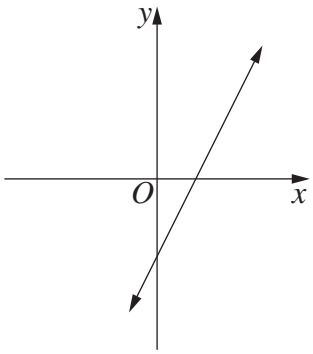
A.



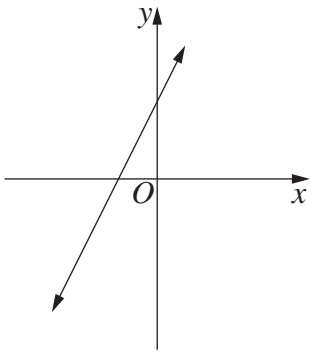
B.



C.



D.



2 Consider the following dataset.

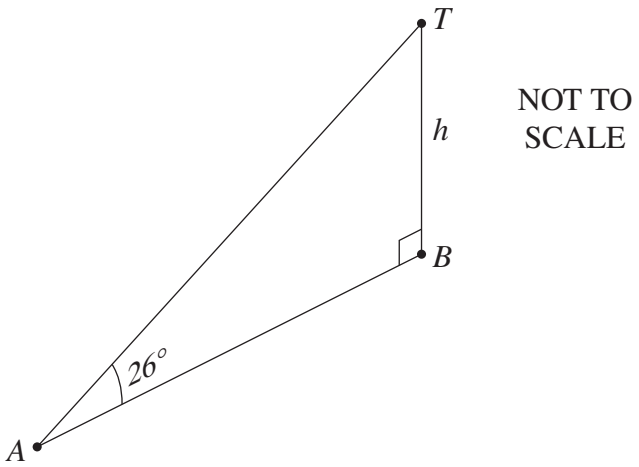
13 16 17 17 21 24

Which row of the table shows how the median and mean are affected when a score of 5 is added to the dataset?

	Median	*Mean*
A.	Changes	Changes
B.	Stays the same	Stays the same
C.	Changes	Stays the same
D.	Stays the same	Changes

3 A tower BT has height h metres.

From point A, the angle of elevation to the top of the tower is 26° as shown.



Which of the following is the correct expression for the length of AB?

A. $h \tan 26°$

B. $h \cot 26°$

C. $h \sin 26°$

D. $h \operatorname{cosec} 26°$

4 Which of the following is the range of the function $f(x) = x^2 - 1$?

A. $[-1, \infty)$

B. $(-\infty, 1]$

C. $[-1, 1]$

D. $(-\infty, \infty)$

5 Let $h(x) = \dfrac{f(x)}{g(x)}$, where

$f(1) = 2 \qquad f'(1) = 4$

$g(1) = 8 \qquad g'(1) = 12.$

What is the gradient of the tangent to the graph of $y = h(x)$ at $x = 1$?

A. -8

B. 8

C. $-\dfrac{1}{8}$

D. $\dfrac{1}{8}$

6 What is $\displaystyle\int \frac{1}{(2x+1)^2}\, dx$?

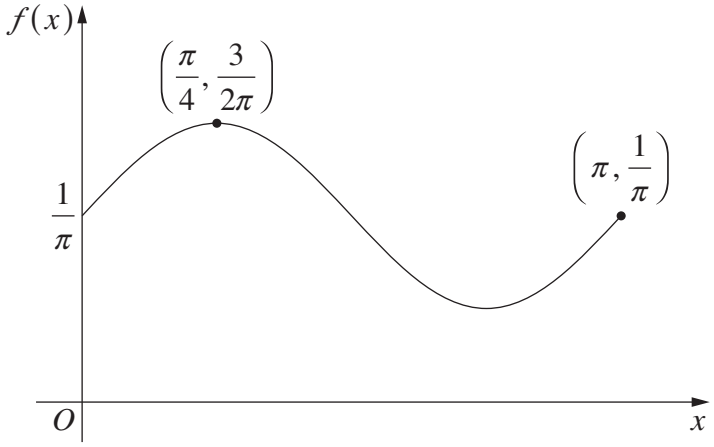
A. $\dfrac{-2}{2x+1} + C$

B. $\dfrac{-1}{2(2x+1)} + C$

C. $2\ln(2x+1) + C$

D. $\dfrac{1}{2}\ln(2x+1) + C$

7 Consider the following graph of a probability density function $f(x)$.



NOT TO SCALE

What is the value of the mode?

A. $\frac{1}{\pi}$

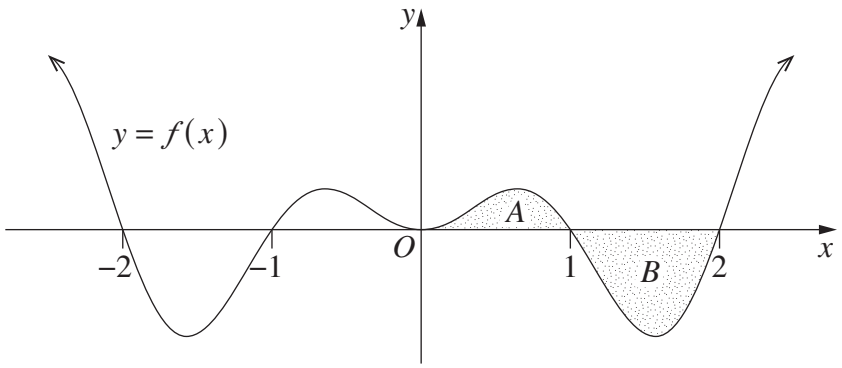
B. $\frac{3}{2\pi}$

C. $\frac{\pi}{4}$

D. π

8 The graph of the even function $y = f(x)$ is shown.

The area of the shaded region A is $\frac{1}{2}$ and the area of the shaded region B is $\frac{3}{2}$.



NOT TO SCALE

What is the value of $\int_{-2}^{2} f(x)\,dx$?

A. 4

B. 2

C. −2

D. −4

9 Liam is playing two games. He is equally likely to win each game. The probability that Liam will win at least one of the games is 80%.

Which of the following is closest to the probability that Liam will win both games?

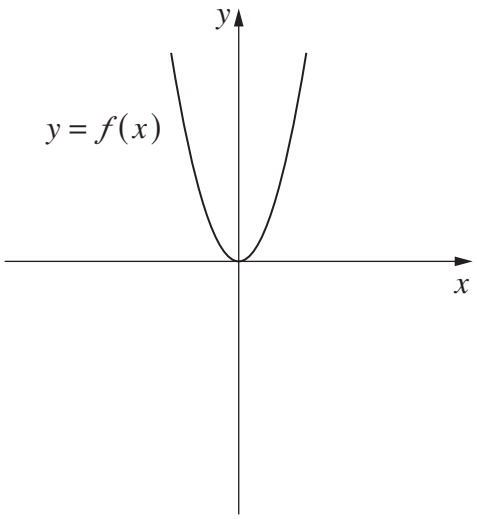
A. 31%

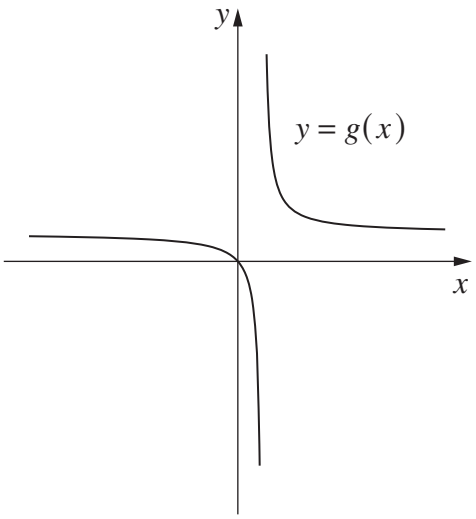
B. 40%

C. 55%

D. 64%

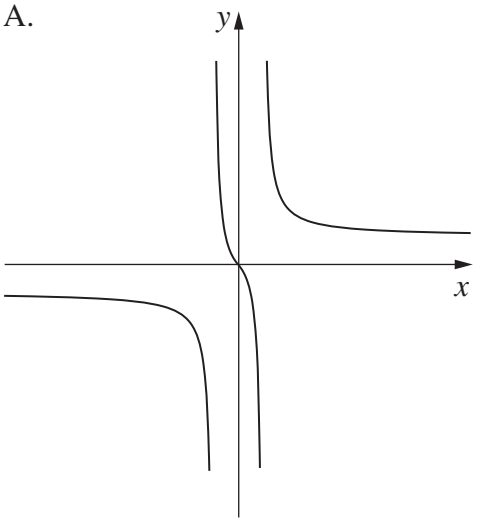
10 The graphs of $y = f(x)$ and $y = g(x)$ are shown.





Which graph best represents $y = g(f(x))$?

A.



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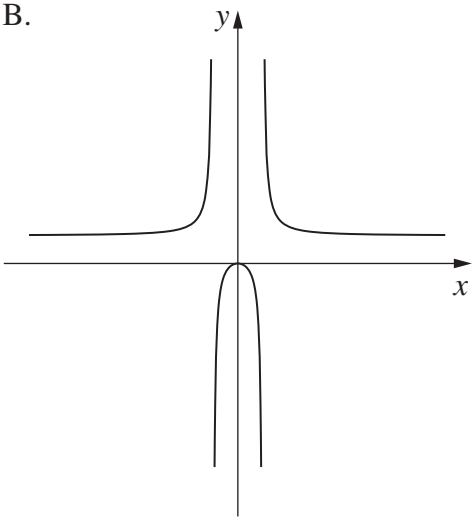
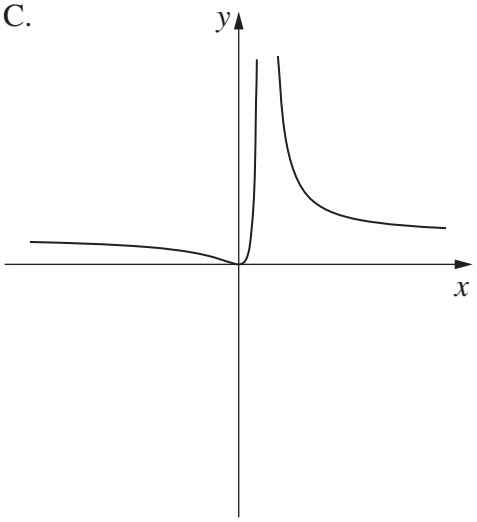
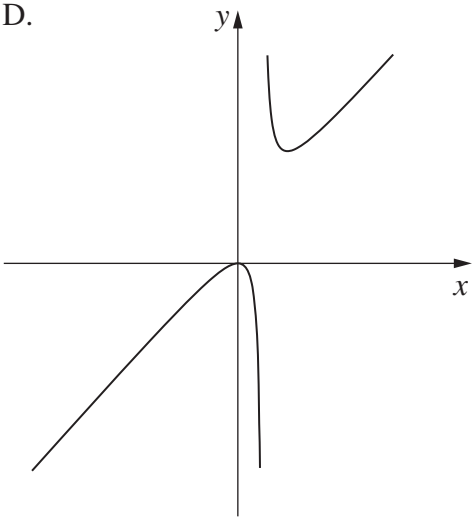


C.



D.



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2022 HIGHER SCHOOL CERTIFICATE EXAMINATION

Centre Number

Student Number

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Office Use Only – Do NOT write anything, or make any marks below this line.

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Question 11 (3 marks)

The table shows the types of customer complaints received by an online business in a month.

Type of complaint	*Frequency*	*Cumulative frequency*	*Cumulative percentage*
Stock shortage	98	98	49
Delivery fee	62	***A***	80
Delivery time	24	184	92
Damaged item	8	192	***B***
Returns policy	6	198	99
Product information	2	200	100
Total	200		

(a) What are the values of ***A*** and ***B***? **2**

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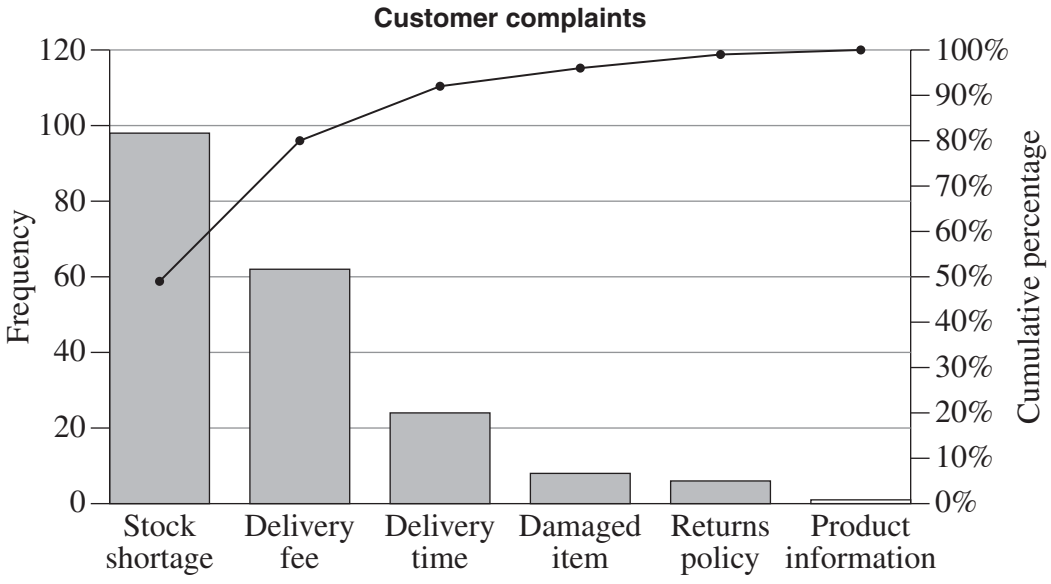
(b) The data from the table are shown in the following Pareto chart. **1**



The manager will address 80% of the complaints.

Which types of complaints will the manager address?

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Question 12 (4 marks)

A student believes that the time it takes for an ice cube to melt (M minutes) varies inversely with the room temperature (T°C). The student observes that at a room temperature of 15°C it takes 12 minutes for an ice cube to melt.

(a) Find the equation relating M and T. **2**

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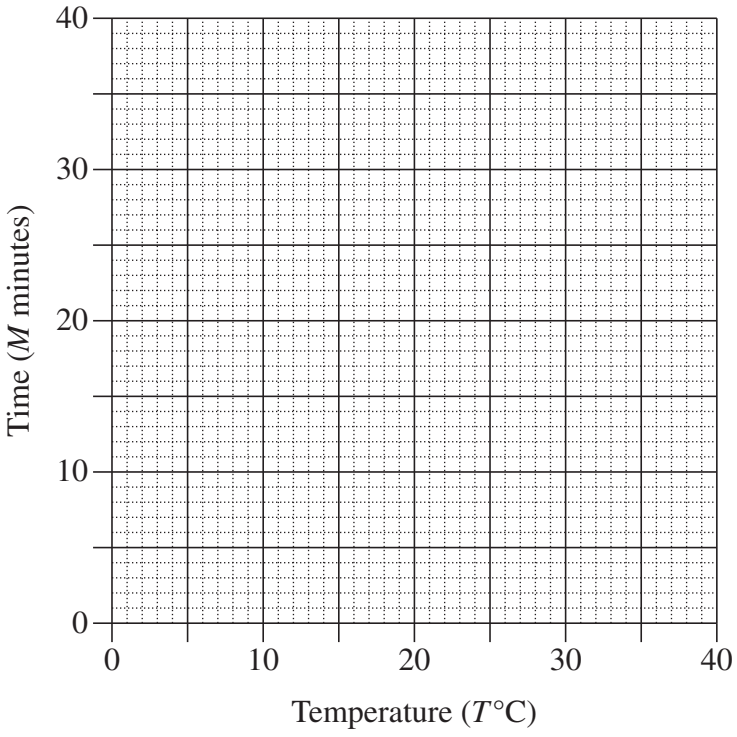
(b) By first completing this table of values, graph the relationship between temperature and time from $T = 5$°C to $T = 30$°C. **2**

T	5	15	30
M			



Question 13 (2 marks)

Use two applications of the trapezoidal rule to find an approximate value of $\int_0^2 \sqrt{1+x^2}\,dx$. Give your answer correct to 2 decimal places. **2**

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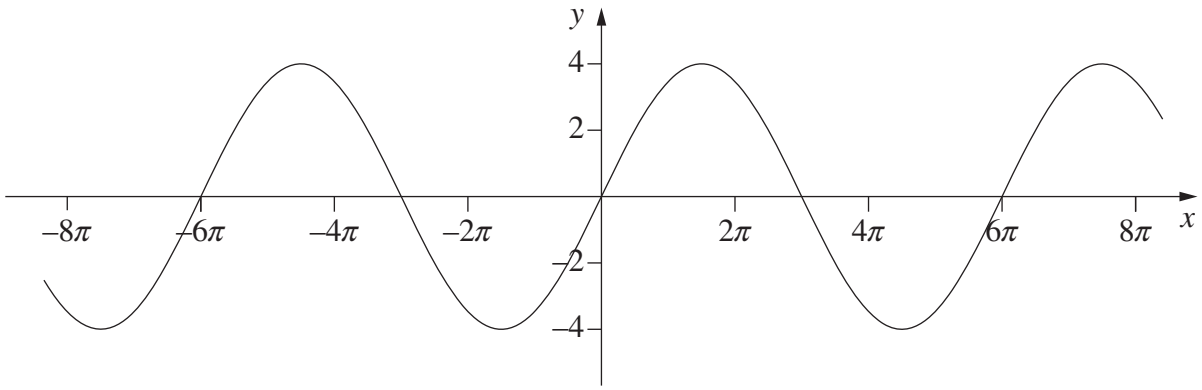
Question 14 (2 marks)

The graph of $y = k\sin(ax)$ is shown. **2**



What are the values of k and a?

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Question 15 (2 marks)

In a bag there are 3 six-sided dice. Two of the dice have faces marked 1, 2, 3, 4, 5, 6. The other is a special die with faces marked 1, 2, 3, 5, 5, 5.

One die is randomly selected and tossed.

(a) What is the probability that the die shows a 5? **1**

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(b) Given that the die shows a 5, what is the probability that it is the special die? **1**

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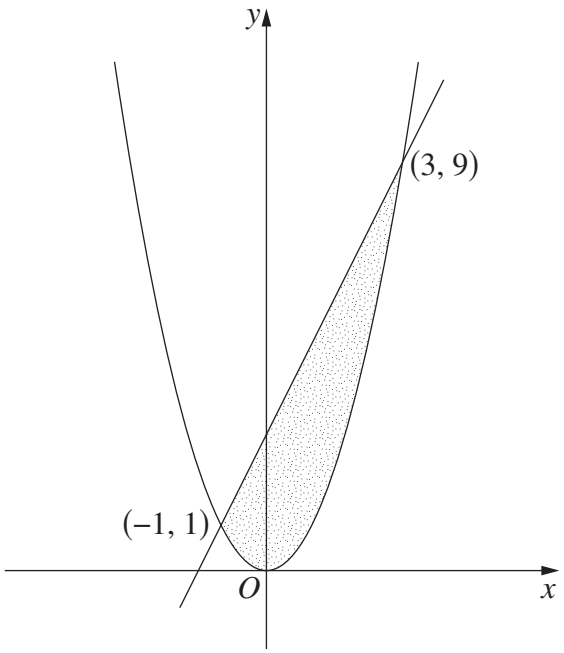
Question 16 (3 marks)

The parabola $y = x^2$ meets the line $y = 2x + 3$ at the points $(-1, 1)$ and $(3, 9)$ as shown in the diagram. **3**



Find the area enclosed by the parabola and the line.

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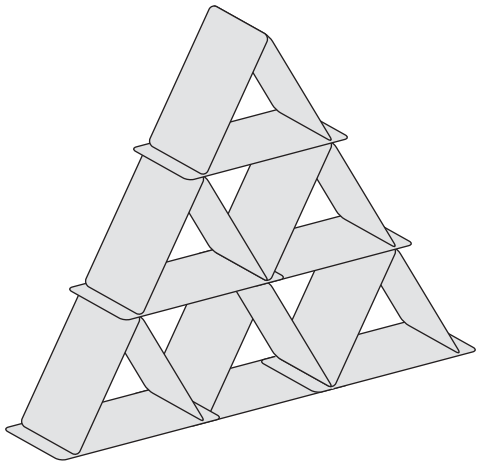
Question 17 (5 marks)

Cards are stacked to build a 'house of cards'. A house of cards with 3 rows is shown.

A house of cards requires 3 cards in the top row, 6 cards in the next row, and each successive row has 3 more cards than the previous row.



(a) Show that a house of cards with 12 rows has a total of 234 cards. **2**

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(b) Another house of cards has a total of 828 cards. **3**

How many rows are in this house of cards?

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Question 18 (3 marks)

(a) Differentiate $y = \left(x^2 + 1\right)^4$. **2**

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(b) Hence, or otherwise, find $\int x\left(x^2 + 1\right)^3 dx$. **1**

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Question 19 (3 marks)

The graph of the function $f(x) = x^2$ is translated m units to the right, dilated vertically by a scale factor of k and then translated 5 units down. The equation of the transformed function is $g(x) = 3x^2 - 12x + 7$. **3**

Find the values of m and k.

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Question 20 (4 marks)

A scientist is studying the growth of bacteria. The scientist models the number of bacteria, N, by the equation

$$N(t) = 200e^{0.013t},$$

where t is the number of hours after starting the experiment.

(a) What is the initial number of bacteria in the experiment? **1**

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(b) What is the number of bacteria 24 hours after starting the experiment? **1**

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(c) What is the rate of increase in the number of bacteria 24 hours after starting the experiment? **2**

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Question 21 (4 marks)

Eli is choosing between two investment options.

Option 1: Depositing a single amount of $40 000 today, earning interest of 1.2% per annum, compounded monthly.

Option 2: Depositing $1000 at the end of each quarter, earning interest of 2.4% per annum, compounded quarterly.

A table of future value interest factors for an annuity of $1 is shown.

r / *N*	*Interest rate per period as a decimal*					
	0.002	0.006	0.020	0.024	0.060	0.240
10	10.09048	10.27437	10.94972	11.15211	13.18079	31.64344
20	20.38460	21.18211	24.29737	25.28909	36.78559	303.60062
30	30.88646	32.76227	40.56808	43.20983	79.05819	2640.91639
40	41.60026	45.05630	60.40198	65.92708	154.76197	22 728.80260

(a) What is the value of Eli's investment after 10 years using Option 1? **2**

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(b) What is the difference between the future values after 10 years using Option 1 and Option 2? **2**

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Question 22 (4 marks)

Find the global maximum and minimum values of $y = x^3 - 6x^2 + 8$, where $-1 \leq x \leq 7$. **4**

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Question 23 (6 marks)

The depth of water in a bay rises and falls with the tide. On a particular day the depth of the water, d metres, can be modelled by the equation

$$d = 1.3 - 0.6\cos\left(\frac{4\pi}{25}t\right),$$

where t is the time in hours since low tide.

(a) Find the depth of water at low tide and at high tide. **2**

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(b) What is the time interval, in hours, between two successive low tides? **1**

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(c) For how long between successive low tides will the depth of water be at least 1 metre? **3**

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Questions 11–23 are worth 45 marks in total

Question 24 (4 marks)

Jo is researching the relationship between the ages of teenage characters in television series and the ages of actors playing these characters. **4**

After collecting the data, Jo finds that the correlation coefficient is 0.4564.

A scatterplot showing the data is drawn. The line of best fit with equation $y = -7.51 + 1.85x$, is also drawn.

Describe and interpret the data and other information provided, with reference to the context given.

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Question 25 (3 marks)

Let $f(x) = \sin(2x)$. **3**

Find the value of x, for $0 < x < \pi$, for which $f'(x) = -\sqrt{3}$ AND $f''(x) = 2$.

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Question 26 (3 marks)

The life span of batteries from a particular factory is normally distributed with a mean of 840 hours and a standard deviation of 80 hours. **3**

It is known from statistical tables that for this distribution approximately 60% of the batteries have a life span of less than 860 hours.

What is the approximate percentage of batteries with a life span between 820 and 920 hours?

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Question 27 (7 marks)

Let $f(x) = xe^{-2x}$.

It is given that $f'(x) = e^{-2x} - 2xe^{-2x}$.

(a) Show that $f''(x) = 4(x-1)e^{-2x}$. **2**

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(b) Find any stationary points of $f(x)$ and determine their nature. **2**

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Question 27 continues on page 25

Question 27 (continued)

(c) Sketch the curve $y = xe^{-2x}$, showing any stationary points, points of inflection and intercepts with the axes. **3**

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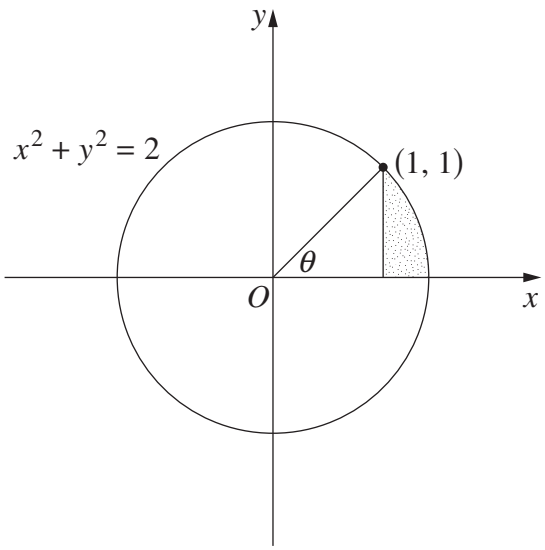
End of Question 27

Question 28 (7 marks)

The graph of the circle $x^2 + y^2 = 2$ is shown.

The interval connecting the origin, O, and the point $(1, 1)$ makes an angle θ with the positive x-axis.



NOT TO SCALE

(a) By considering the value of θ, find the exact area of the shaded region, as shown on the diagram. **2**

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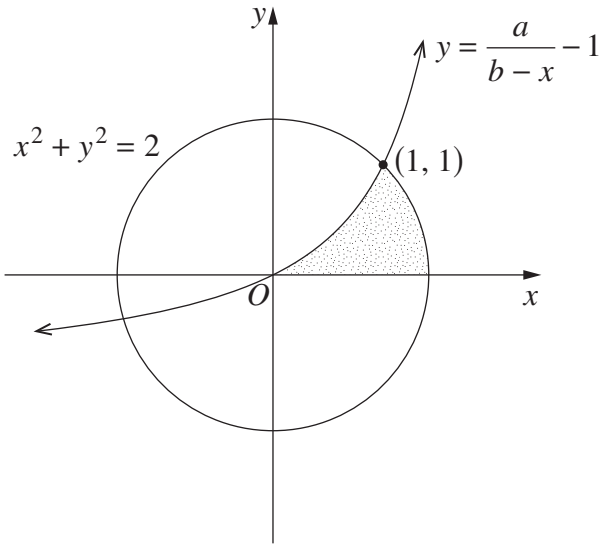
Question 28 continues on page 27

Question 28 (continued)

Part of the hyperbola $y = \dfrac{a}{b - x} - 1$ which passes through the points $(0, 0)$ and $(1, 1)$ is drawn with the circle $x^2 + y^2 = 2$ as shown.



NOT TO SCALE

(b) Show that $a = b = 2$. **2**

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(c) Using parts (a) and (b), find the exact area of the region bounded by the hyperbola, the positive x-axis and the circle as shown on the diagram. **3**

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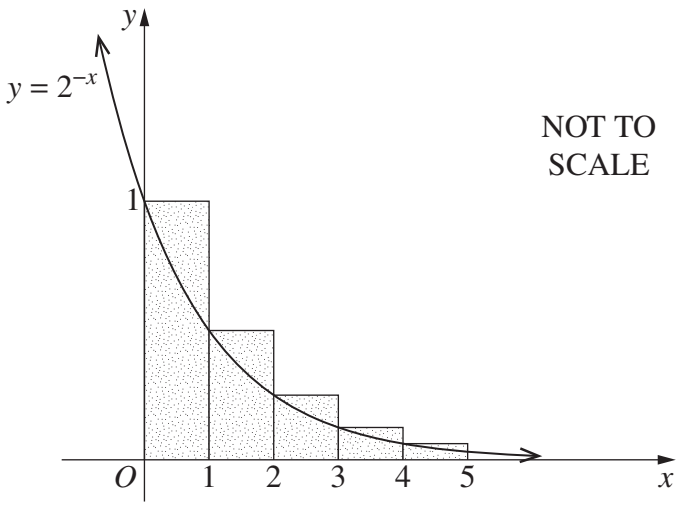
End of Question 28

Question 29 (5 marks)

(a) The diagram shows the graph of $y = 2^{-x}$. Also shown on the diagram are the first 5 of an infinite number of rectangular strips of width 1 unit and height $y = 2^{-x}$ for non-negative integer values of x. For example, the second rectangle shown has width 1 and height $\frac{1}{2}$. **1**



The sum of the areas of the rectangles forms a geometric series.

Show that the limiting sum of this series is 2.

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(b) Show that $\int_0^4 2^{-x}\,dx = \frac{15}{16\ln 2}$. **2**

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Question 29 continues on page 29

Question 29 (continued)

(c) Use parts (a) and (b) to show that $e^{15} < 2^{32}$. **2**

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Question 30 (3 marks)

A continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 1 & x > e^3 \\ \frac{1}{k}\ln x & 1 \le x \le e^3 \\ 0 & x < 1 \end{cases}.$$

(a) Show that $k = 3$. **1**

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(b) Given that $P(X < c) = 2P(X > c)$, find the exact value of c. **2**

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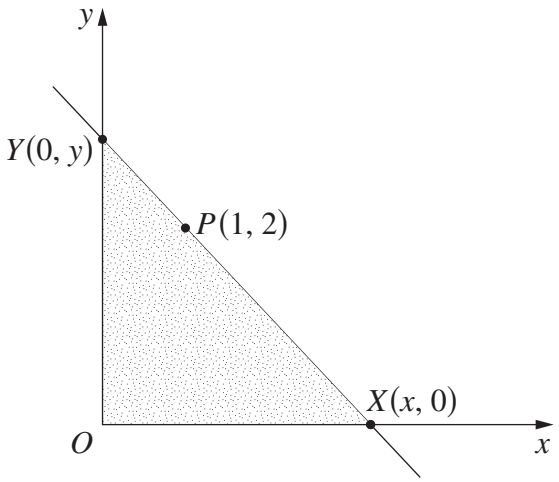
Question 31 (6 marks)

A line passes through the point $P(1, 2)$ and meets the axes at $X(x, 0)$ and $Y(0, y)$, where $x > 1$.



NOT TO SCALE

(a) Show that $y = \dfrac{2x}{x-1}$. **2**

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Question 31 continues on page 31

Question 31 (continued)

(b) Find the minimum value of the area of triangle XOY. **4**

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End of Question 31

Question 32 (7 marks)

In a reducing-balance loan, an amount \$$P$ is borrowed for a period of n months at an interest rate of 0.25% per month, compounded monthly. At the end of each month, a repayment of \$$M$ is made. After the nth repayment has been made, the amount owing, \$$A_n$, is given by

$$A_n = P(1.0025)^n - M\left(1 + (1.0025)^1 + (1.0025)^2 + \cdots + (1.0025)^{n-1}\right).$$

(Do NOT prove this.)

(a) Jane borrows \$200 000 in a reducing-balance loan as described. **2**

The loan is to be repaid in 180 monthly repayments.

Show that $M = 1381.16$, when rounded to the nearest cent.

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Question 32 continues on page 33

Question 32 (continued)

(b) After 100 repayments of $1381.16 have been made, the interest rate changes to 0.35% per month. **3**

At this stage, the amount owing to the nearest dollar is $100 032. (Do NOT prove this.)

Jane continues to make the same monthly repayments.

For how many more months will Jane need to make full monthly payments of $1381.16?

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(c) The final repayment will be less than $1381.16. **2**

How much will Jane need to pay in the final payment in order to pay off the loan?

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Section II extra writing space

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NSW Education Standards Authority

2022 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(a + b\right)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$\left(x - h\right)^2 + \left(y - k\right)^2 = r^2$$

Financial Mathematics

$$A = P\left(1 + r\right)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r} = \frac{a\left(r^n - 1\right)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

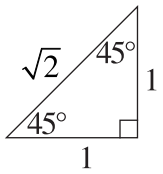
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

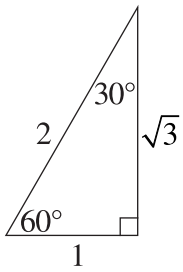
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A},\ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

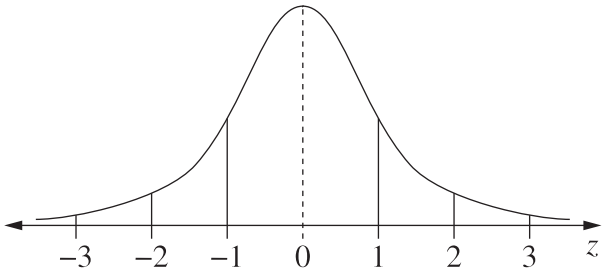
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow\ P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x},\ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

2022 HSC Mathematics Advanced Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	A
2	D
3	B
4	A
5	D
6	B
7	C
8	C
9	A
10	B

Section II

Question 11 (a)

Criteria	Marks
• Finds the correct values of *A* and *B*	2
• Finds the correct value of *A* or *B*	1

Sample answer:

$\boldsymbol{A} = 98 + 62 = 160$

$\boldsymbol{B} = \dfrac{192}{200} = \dfrac{96}{100} = 96\%$

Question 11 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Stock shortage and delivery fee

Question 12 (a)

Criteria	Marks
• Provides correct solution	2
• Writes down an equation representing inverse variation, or equivalent merit	1

Sample answer:

$$M = \frac{k}{T}$$

When $T = 15 \qquad M = 12$

$$12 = \frac{k}{15}$$

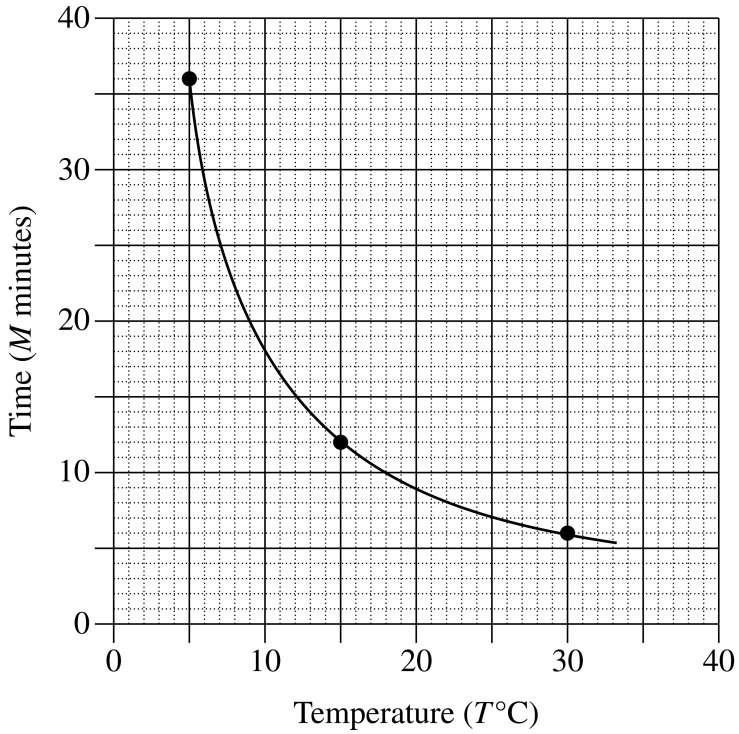
$$k = 15 \times 12 = 180$$

$$M = \frac{180}{T}$$

Question 12 (b)

Criteria	Marks
• Provides correct solution	2
• Completes the table, or equivalent merit	1

Sample answer:



T	5	15	30
M	36	12	6

Question 13

Criteria	Marks
• Provides correct solution	2
• Finds function values at 0 and 2, or equivalent merit	1

Sample answer:

$$\int_0^2 \sqrt{1+x^2}\, dx$$

x	0	1	2
$\sqrt{1+x^2}$	1	1.414	2.236

$$\int_0^2 \sqrt{1+x^2}\, dx = \frac{1}{2}[1+1.414]+\frac{1}{2}[1.414+2.236]$$

$$= 3.03 \text{ (2 decimal places)}$$

Question 14

Criteria	Marks
• Provides the values of k and a	2
• Finds the value of k or a, or equivalent merit	1

Sample answer:

$k = \text{amplitude} = 4$

$$\text{Period} = \frac{2\pi}{a}$$

$$6\pi = \frac{2\pi}{a}$$

$$\therefore \quad a = \frac{1}{3}$$

Question 15 (a)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$P(5) = \frac{2}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{3}{6}$$

$$= \frac{5}{18}$$

Question 15 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$P(\text{special die} \mid 5) = \frac{P(\text{special die} \cap 5)}{P(5)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{6}}{\frac{5}{18}}$$

$$= \frac{3}{5}$$

Question 16

Criteria	Marks
• Provides correct solution	3
• Provides correct anti-derivative, or equivalent merit	2
• Provides an integral expression for area, or equivalent merit	1

Sample answer:

$$A = \int_{-1}^{3} 2x + 3 - x^2 \, dx$$

$$= \left[x^2 + 3x - \frac{x^3}{3} \right]_{-1}^{3}$$

$$= (9 + 9 - 9) - \left(1 - 3 + \frac{1}{3} \right)$$

$$\text{Area} = 10\frac{2}{3} \text{ units}^2$$

Question 17 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the values of *a* and *d*, or equivalent merit	1

Sample answer:

Number of cards $= 3 + 6 + 9 + \cdots$ (12 terms)

$$S_{12} = \frac{12}{2}(2\times 3 + 11\times 3) \qquad \left(\text{arithmetic series} \quad S_n = \frac{n}{2}(2a + (n-1)d) \quad a = 3 \quad d = 3\right)$$

$$= 234$$

Question 17 (b)

Criteria	Marks
• Provides correct solution	3
• Finds correct quadratic expression	2
• Correctly substitutes *a*, *d* and 828 into formula for S_n, or equivalent merit	1

Sample answer:

Let n = number of rows

$$\frac{n}{2}(2\times 3 + (n-1)3) = 828$$

$$n(3n+3) = 1656$$

$$3n(n+1) = 1656$$

$$n^2 + n = 552$$

$$n^2 + n - 552 = 0$$

$$n = \frac{-1 \pm \sqrt{1^2 - 4\times -552}}{2}$$

$$= \frac{-1 \pm 47}{2}$$

$$= 23 \text{ (taking positive solution)}$$

There are 23 rows.

Question 18 (a)

Criteria	Marks
• Provides correct derivative	2
• Attempts to use the chain rule, or equivalent merit	1

Sample answer:

$$y = \left(x^2 + 1\right)^4$$

$$\frac{dy}{dx} = 4\left(x^2 + 1\right)^3 \times 2x$$

$$= 8x\left(x^2 + 1\right)^3$$

Question 18 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\int x\left(x^2+1\right)^3 dx$$

$$= \frac{1}{8}\int 8x\left(x^2 + 1\right)^3 dx$$

$$= \frac{1}{8}\left(x^2 + 1\right)^4 + c \qquad \text{from part (a)}$$

Question 19

Criteria	Marks
• Provides correct solution	3
• Writes $g(x)$ in terms of m and k, and finds value of k, or equivalent merit	2
• Writes $g(x)$ in terms of m and k, or equivalent merit	1

Sample answer:

$$\begin{aligned} g(x) &= k(x-m)^2 - 5 \\ &= k\left(x^2 - 2xm + m^2\right) - 5 \\ &= kx^2 - 2mkx + m^2k - 5 \end{aligned}$$

By inspection $k = 3$

By equating coefficient of x

$$\begin{aligned} 2mk &= 12 \\ 6m &= 12 \\ m &= 2 \end{aligned}$$

ALTERNATE SOLUTION

$$\begin{aligned} g(x) &= k(x-m)^2 - 5 \\ g(x) &= 3x^2 - 12x + 7 \\ &= 3\left(x^2 - 4x\right) + 7 \\ &= 3\left(x^2 - 4x + 4\right) + 7 - 12 \\ &= 3(x-2)^2 - 5 \end{aligned}$$

So $k = 3$ and $m = 2$

Question 20 (a)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$N(0) = 200e^0$

$= 200$

Question 20 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$N(24) = 200e^{0.013 \times 24}$

$= 273.23\ldots$

≈ 273 bacteria

Question 20 (c)

Criteria	Marks
• Provides correct answer	2
• Attempts to find $\frac{dN}{dt}$	1

Sample answer:

$\frac{dN}{dt} = 200 \times 0.013 \times e^{0.013t}$

When $t = 24$ $\frac{dN}{dt} = 0.013 \times 273$ from part (b)

$= 3.55$

Rate of increase = 3.55 bacteria/hour.

Question 21 (a)

Criteria	Marks
• Provides correct solution	2
• Writes an expression for the future value with some correct element(s), or equivalent merit	1

Sample answer:

$P = 40\,000, \quad r = 1.2\%$ pa compounded monthly, $\quad N = 10 \times 12 = 120$

$$FV = 40\,000\left(1 + \frac{0.012}{12}\right)^{120}$$
$$= \$45\,097.17$$

Question 21 (b)

Criteria	Marks
• Provides correct solution	2
• Calculates the future value using the interest factor table, or equivalent merit	1

Sample answer:

Option 2

10 years quarterly $\implies$ $N = 10 \times 4 = 40$

$$r = \frac{2.4\%}{4} = \frac{0.024}{4} = 0.006$$

From table: 45.05630

So $FV = 45.05630 \times 1000$

$= 45\,056.30$

As Option 1 = \$45 097.17, the difference is \$40.87 .

Question 22

Criteria	Marks
• Provides correct solution	4
• Finds the *y* values for at least 3 relevant points	3
• Solves the quadratic to find *x* coordinates of stationary points	2
• Finds the derivative, or equivalent merit	1

Sample answer:

Global maximum and minimum values occur at local maximum and minimum or end points of the interval.

$y = x^3 - 6x^2 + 8$

$\dfrac{dy}{dx} = 3x^2 - 12x$

$\dfrac{dy}{dx} = 0 \implies 3x^2 - 12x = 0$

$3x(x - 4) = 0$

Stationary points at $x = 0$ and $x = 4$

When $x = 0$ $y = 8$

When $x = 4$ $y = -24$

End points of interval

When $x = -1$ $y = 1$

When $x = 7$ $y = 57$

Global minimum value = -24

Global maximum value = 57

Question 23 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the depth of water at either high or low tide, or equivalent merit	1

Sample answer:

$$d = 1.3 - 0.6\cos\left(\frac{4\pi}{25}t\right)$$

Maximum depth (*high tide*) $= 1.3 + 0.6 = 1.9$ m

Minimum depth (*low tide*) $= 1.3 - 0.6 = 0.7$ m

Question 23 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\text{Period} = \frac{2\pi}{\left(\frac{4\pi}{25}\right)} \text{ hours}$$

$$= 12.5 \text{ hours}$$

Question 23 (c)

Criteria	Marks
• Provides correct solution	3
• Finds one value of t for $d = 1$, or equivalent merit	2
• Attempts to solve the given equation when $d = 1$	1

Sample answer:

$$d = 1.3 - 0.6\cos\left(\frac{4\pi}{25}t\right)$$

$$1 = 1.3 - 0.6\cos\left(\frac{4\pi}{25}t\right), \qquad \text{when } d = 1$$

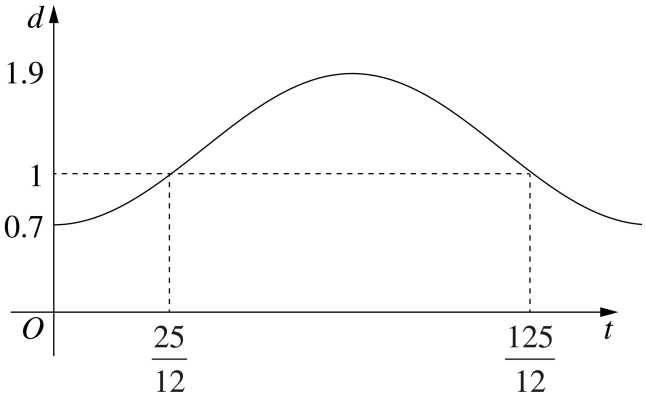
$$-0.3 = -0.6\cos\left(\frac{4\pi}{25}t\right)$$

$$\frac{1}{2} = \cos\left(\frac{4\pi}{25}t\right)$$

$$\frac{4\pi}{25}t = \frac{\pi}{3},\ \frac{5\pi}{3}$$

$$t = \frac{\pi}{3}\times\frac{25}{4\pi},\ \frac{5\pi}{3}\times\frac{25}{4\pi}$$

$$= \frac{25}{12},\ \frac{125}{12}$$



$$\text{Time interval} = \frac{125}{12} - \frac{25}{12}$$

$$= 8\frac{1}{3} \text{ hours}$$

Question 24

Criteria	Marks
• Provides a comprehensive description and interpretation of the data and other information, in the given context	4
• Provides a sound description and interpretation of the data and other information, in the given context	3
• Provides some description and interpretation of the data and/or other information	2
• Provides some relevant information	1

Answers could include:

The data show that:

- The relationship is linear and positive/increasing. The correlation of 0.4564 is weaker/moderate.
- As character age increases by 1 year, the actor age increases by almost 2 years.
- The actors playing teenagers do not need to be teenagers themselves. The ages of characters range from 14 to 17 but the ages of actors playing these characters range from 14 to 30. This means that there is no need for just young actors but older people can play teenage characters.
- There are only a few characters of age 14 and they are played by actors ranging in age between 14–23.
- Characters of age 15 is the largest group and they are played by actors with the widest age range (14–27).
- Characters older than 15 are all played by older actors. The youngest actors in the dataset are playing characters close to their own age.

Question 25

Criteria	Marks
• Provides correct solution	3
• Finds the values of $x,\ 0 < x < \pi$, for each derivative equation	2
• Attempts to find a value for x that solves $f'(x)$ or $f''(x)$, or equivalent merit	1

Sample answer:

$f(x) = \sin 2x$

$f'(x) = 2\cos 2x$

$f''(x) = -4\sin 2x$

$f'(x) = -\sqrt{3} \quad \Rightarrow \quad 2\cos 2x = -\sqrt{3}$

$$\cos 2x = \frac{-\sqrt{3}}{2} \qquad 0 < 2x < 2\pi$$

$$2x = \frac{5\pi}{6},\ \frac{7\pi}{6}$$

$$x = \frac{5\pi}{12},\ \frac{7\pi}{12}$$

$f''(x) = 2 \quad \Rightarrow \quad -4\sin 2x = 2$

$$\sin 2x = \frac{-1}{2} \qquad 0 < 2x < 2\pi$$

$$2x = \frac{7\pi}{6},\ \frac{11\pi}{6}$$

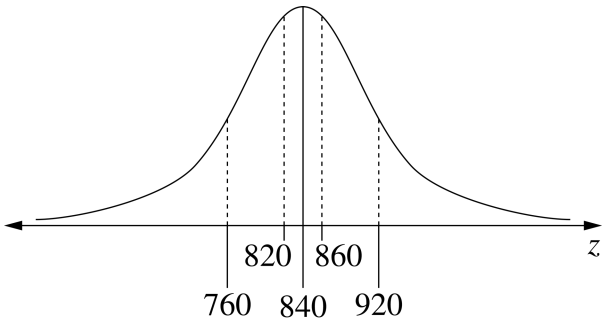
$$x = \frac{7\pi}{12},\ \frac{11\pi}{12}$$

$\therefore\ x = \dfrac{7\pi}{12}$ for $0 < x < \pi$

Question 26

Criteria	Marks
• Provides correct solution	3
• Recognises that 34% have a life span between 840 and 920 AND attempts to use the information given regarding the statistical tables	2
• Recognises that 68% have a life span between 760 and 920, or equivalent merit	1

Sample answer:



60% have a life span < 860.

∴ 10% have a life span between 840 and 860.

∴ 10% have a life span between 820 and 840.

68% have a life span between 760 and 920.

∴ 34% have a life span between 840 and 920.

∴ percentage with life span between 820 and 920 hours is 10% + 34% = 44%.

Question 27 (a)

Criteria	Marks
• Provides correct solution	2
• Attempts to use the product rule	1

Sample answer:

$$f'(x) = e^{-2x} - 2xe^{-2x}$$

$$\begin{aligned} f''(x) &= -2e^{-2x} - 2\left(x \times -2e^{-2x} + e^{-2x}\right) \\ &= -2e^{-2x} + 4xe^{-2x} - 2e^{-2x} \\ &= (4x-4)e^{-2x} \\ &= 4(x-1)e^{-2x} \end{aligned}$$

Question 27 (b)

Criteria	Marks
• Provides correct solution	2
• Finds the x value of the stationary point	1

Sample answer:

Stationary points where $f'(x) = 0$

$$(1-2x)e^{-2x} = 0$$

$$x = \frac{1}{2}, \qquad \text{since } e^{-2x} \neq 0, \qquad f\left(\frac{1}{2}\right) = \frac{1}{2}e^{-1} = \frac{1}{2e}$$

$$f''\left(\frac{1}{2}\right) = 4 \times -\frac{1}{2} \times e^{-1} = -2e^{-1} < 0$$

$\therefore \left(\frac{1}{2}, \frac{1}{2e}\right)$ is a local maximum.

Question 27 (c)

Criteria	Marks
• Provides correct graph	3
• Sketches the curve showing most of the main features	2
• Provides a sketch showing the stationary point and 1 other feature	1

Sample answer:

Possible point of inflection where $f''(x) = 0$

$4(x-1)e^{-2x} = 0$

$\therefore x = 1, \quad$ since $e^{-2x} \neq 0$

x	0	1	2
$f''(x)$	-4	0	$4e^{-4}$

Changes concavity at $x = 1$

$\therefore$ Inflection point at $x = 1$

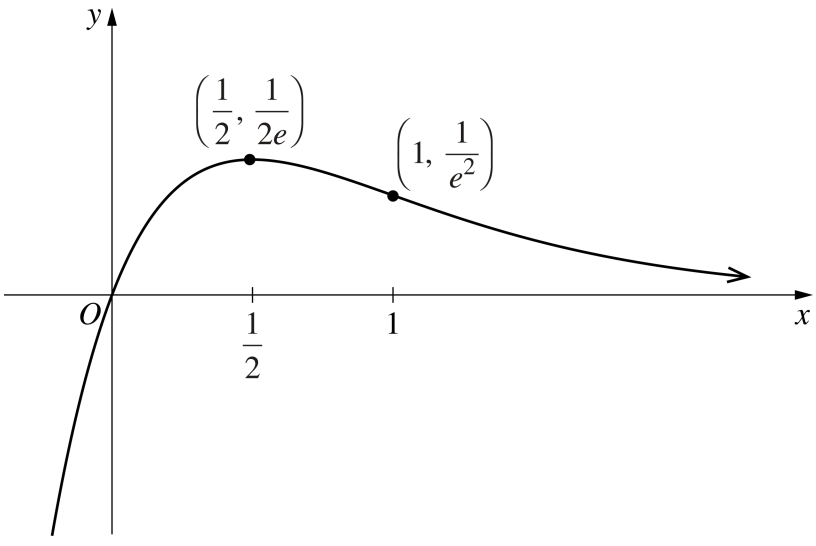
$f(1) = 1e^{-2\times 1}$

$= e^{-2}$

$\left(1, \frac{1}{e^2}\right)$ — Inflection point

$f(0) = 0 \quad \therefore$ graph passes through $(0, 0)$

As $x \to \infty, \quad f(x) \to 0$

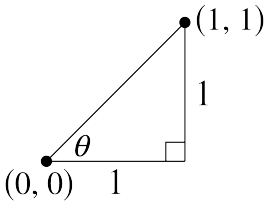


Question 28 (a)

Criteria	Marks
• Provides correct solution	2
• Finds area of the sector	1

Sample answer:

Area = Area sector – Area triangle



$\theta = \frac{\pi}{4}$

$$\text{Area} = \frac{1}{2}r^2\theta - \frac{1}{2}\times 1\times 1$$

$$= \frac{1}{2}\left(\sqrt{2}\right)^2 \times \frac{\pi}{4} - \frac{1}{2}$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

Question 28 (b)

Criteria	Marks
• Provides correct solution	2
• Shows that $a = b$	1

Sample answer:

Substitute $x = y = 0$

$$0 = \frac{a}{b} - 1 \qquad \therefore \frac{a}{b} = 1$$

$$\therefore\ a = b$$

Substitute $x = y = 1$

$$1 = \frac{a}{b-1} - 1$$

$$1 = \frac{a}{a-1} - 1$$

$$\frac{a}{a-1} = 2$$

$$a = 2a - 2$$

$$\therefore\ a = 2 \qquad \therefore b = 2$$

Question 28 (c)

Criteria	Marks
• Provides correct solution	3
• Correctly evaluates integral	2
• Gives the area as a sum of an integral and the area from part (a), or equivalent merit	1

Sample answer:

$$\text{Area under curve (hyperbola)} = \int_0^1 \left(\frac{2}{2-x} - 1\right) dx$$

$$= \left[-2\ln(2-x) - x\right]_0^1$$

$$= -2\ln 1 - 1 + 2\ln 2 - 0$$

$$= 2\ln 2 - 1$$

$$\text{Total area} = 2\ln 2 - 1 + \frac{\pi}{4} - \frac{1}{2} \text{ (units}^2\text{)}$$

$$= 2\ln 2 + \frac{\pi}{4} - \frac{3}{2} \text{ (units}^2\text{)}$$

Question 29 (a)

Criteria	Marks
• Shows the limiting sum	1

Sample answer:

Limiting sum $= 2^0 + 2^{-1} + 2^{-2} + \cdots$

$$S = \frac{a}{1-r}, \quad \text{where } a = 1 \text{ and } r = \frac{1}{2}$$

$$= \frac{1}{1-\frac{1}{2}}$$

$$= 2$$

Question 29 (b)

Criteria	Marks
• Provides correct solution	2
• Substitutes correctly into anti-derivative, or equivalent merit	1

Sample answer:

$$\int_0^4 2^{-x}\,dx = \left[\frac{-1}{\ln 2}2^{-x}\right]_0^4 \qquad \text{(using the reference sheet)}$$

$$= \frac{-1}{\ln 2}\left[2^{-4} - 2^0\right]$$

$$= \left(1 - \frac{1}{16}\right)\frac{1}{\ln 2} = \frac{15}{16\ln 2}$$

Question 29 (c)

Criteria	Marks
• Provides correct solution	2
• Sets up correct inequality, or equivalent merit	1

Sample answer:

Limiting sum of rectangles > area under curve between 0 and 4

$\therefore \quad \frac{15}{16\ln 2} < 2$ area under the curve between 0 and 4 < limiting sum of rectangles

$\therefore \quad 15 < 32\ln 2$

$\therefore \quad 15 < \ln\left(2^{32}\right)$

$\therefore \quad e^{15} < 2^{32}$

Question 30 (a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

We need $F\left(e^3\right)=1$, since $F(x)$ is a CDF

$$F\left(e^3\right)=\frac{1}{k}\ln e^3$$

$$=\frac{3}{k}$$

So $\frac{3}{k}=1$

$\therefore \quad k=3$

Question 30 (b)

Criteria	Marks
• Provides correct solution	2
• Recognises that $P(X>c)=1-P(X<c)$, or equivalent merit	1

Sample answer:

$$P(X>c)=1-P(X<c)$$

$$P(X<c)=2\left[1-P(X<c)\right]$$

$$3P(X<c)=2$$

$$P(X<c)=\frac{2}{3}$$

$\therefore \quad F(c)=\frac{2}{3}$

$$\frac{1}{3}\ln c=\frac{2}{3}$$

$$\ln c=2$$

$$c=e^2$$

Question 31 (a)

Criteria	Marks
• Provides correct solution	2
• Equates the two expressions for the gradients, or equivalent merit	1

Sample answer:

gradient XP = gradient PY

$$\frac{0-2}{x-1} = \frac{y-2}{0-1}$$

$$(x-1)(y-2) = 2$$

$$y-2 = \frac{2}{x-1}$$

$$y = \frac{2}{x-1} + 2$$

$$= \frac{2+2x-2}{x-1}$$

$$= \frac{2x}{x-1}$$

Question 31 (b)

Criteria	Marks
• Provides correct solution	4
• Solves $\frac{dA}{dx} = 0$ and determines the nature of stationary point at $x = 2$	3
• Finds $\frac{dA}{dx}$, or equivalent merit	2
• Attempts to find the area in terms of x	1

Sample answer:

$$A = \frac{x^2}{x-1} \qquad \left[\text{from } A = \frac{1}{2}xy \text{ and } y = \frac{2x}{x-1}\right]$$

$$\frac{dA}{dx} = \frac{(x-1)2x - x^2(1)}{(x-1)^2}$$

$$= \frac{2x^2 - 2x - x^2}{(x-1)^2}$$

$$= \frac{x^2 - 2x}{(x-1)^2}$$

$$= \frac{x(x-2)}{(x-1)^2}$$

$\frac{dA}{dx} = 0$ when $x = 0$ or $x = 2$

discard $x = 0$ since $x > 1$. (from question)

x	1.5	2	3
$\frac{dA}{dx}$	-3	0	$\frac{3}{4}$

Slope changes from negative to positive so

$x = 2$ is a minimum.

$$A = \frac{4}{2-1}$$

$\therefore$ Minimum area = 4 units^2

Question 32 (a)

Criteria	Marks
• Provides correct solution	2
• Uses the sum for a geometric progression	1

Sample answer:

$A_{180} = 0$

$$\therefore\ 200\,000(1.0025)^{180} = M\left[1 + (1.0025)^1 + \cdots + (1.0025)^{179}\right]$$

$$200\,000(1.0025)^{180} = M\left[\frac{(1.0025)^{180} - 1}{1.0025 - 1}\right]$$

$$M = \frac{200000(1.0025)^{180} \times 0.0025}{(1.0025)^{180} - 1}$$

$$= 1381.163281$$

$$= 1381.16 \text{ when rounded down}$$

Question 32 (b)

Criteria	Marks
• Provides correct solution	3
• Writes the equation in simplest exponential form	2
• Writes the equation in terms of n using the new interest rate and the new principal	1

Sample answer:

$$100\,032(1.0035)^n = 1381.16\left[\frac{(1.0035)^n - 1}{1.0035 - 1}\right]$$

$$100\,032 \times 0.0035(1.0035)^n = 1381.16(1.0035)^n - 1381.16$$

$$1031.048(1.0035)^n = 1381.16$$

$$(1.0035)^n = 1.33956906$$

$$n = \frac{\log(1.33956906)}{\log(1.0035)}$$

$$= 83.67407889$$

$$\therefore 83 \text{ full payments}$$

Question 32 (c)

Criteria	Marks
• Provides correct solution	2
• Substitutes $n = 83$ into amount owing, or equivalent merit	1

Sample answer:

After 83 payments

$$\text{Amount owing} = 100\,032(1.0035)^{83} - 1381.16\left[\frac{(1.0035)^{83}-1}{1.0035-1}\right]$$
$$= 928.29$$

$$\text{Final payment (84th month)} = 928.29 \times 1.0035 \text{ (1-month interest)}$$
$$= 931.54$$

2022 HSC Mathematics Advanced Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	MA-F1 Working with Functions	MA11-1
2	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
3	1	MA-T1 Trigonometry and Measure of Angles	MA11-3
4	1	MA-F1 Working with Functions	MA11-1
5	1	MA-C1 Introduction to Differentiation	MA11-5
6	1	MA-C4 Integral Calculus	MA12-7
7	1	MA-S3 Random Variables	MA12-8
8	1	MA-C4 Integral Calculus	MA12-7
9	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
10	1	MA-F2 Graphing Techniques	MA12-1

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
11 (b)	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
12 (a)	2	MA-F1 Working with Functions	MA11-1
12 (b)	2	MA-F1 Working with Functions	MA11-2
13	2	MA-C4 Integral Calculus	MA12-7
14	2	MA-T3 Trigonometric Functions and Graphs	MA12-5
15 (a)	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
15 (b)	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
16	3	MA-C4 Integral Calculus	MA12-7
17 (a)	2	MA-M1 Modelling Financial Situations	MA12-4
17 (b)	3	MA-M1 Modelling Financial Situations	MA12-4
18 (a)	2	MA-C2 Differential Calculus	MA12-6
18 (b)	1	MA-C4 Integral Calculus	MA12-7
19	3	MA-F2 Graphing Techniques	MA12-1

Question	Marks	Content	Syllabus outcomes
20 (a)	1	MA-E1 Logarithms and Exponentials	MA11-6
20 (b)	1	MA-E1 Logarithms and Exponentials	MA11-6
20 (c)	2	MA-C3 Applications of Differentiation	MA12-6
21 (a)	2	MA-M1 Modelling Financial Situations	MA12-2
21 (b)	2	MA-M1 Modelling Financial Situations	MA12-2
22	4	MA-C3 Applications of Differentiation	MA12-6
23 (a)	2	MA-T3 Trigonometric Functions and Graphs	MA12-5
23 (b)	1	MA-T3 Trigonometric Functions and Graphs	MA12-5
23 (c)	3	MA-T3 Trigonometric Functions and Graphs	MA12-5
24	4	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-10
25	3	MA-C2 Differential Calculus	MA12-6
26	3	MA-S3 Random Variables	MA12-8
27 (a)	2	MA-C2 Differential Calculus	MA12-6
27 (b)	2	MA-C3 Applications of Differentiation	MA12-6
27 (c)	3	MA-C3 Applications of Differentiation	MA12-6
28 (a)	2	MA-T1 Trigonometry and Measure of Angles	MA11-3
28 (b)	2	MA-F1 Working with Functions	MA11-1
28 (c)	3	MA-C4 Integral Calculus	MA12-7
29 (a)	1	MA-M1 Modelling Financial Situations	MA12-4
29 (b)	2	MA-C4 Integral Calculus	MA12-7
29 (c)	2	MA-C4 Integral Calculus	MA12-10
30 (a)	1	MA-S3 Random Variables	MA12-8
30 (b)	2	MA-S3 Random Variables	MA12-8
31 (a)	2	MA-F1 Working with Functions	MA11-1
31 (b)	4	MA-C3 Applications of Differentiation	MA12-6
32 (a)	2	MA-M1 Modelling Financial Situations	MA12-4
32 (b)	3	MA-M1 Modelling Financial Situations	MA12-4
32 (c)	2	MA-M1 Modelling Financial Situations	MA12-2