NSW GOVERNMENT

NSW Education Standards Authority

Centre Number

Student Number

2023 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions	• Reading time – 10 minutes • Working time – 3 hours • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • For questions in Section II, show relevant mathematical reasoning and/or calculations • Write your Centre Number and Student Number at the top of this page
Total marks: 100	**Section I – 10 marks** (pages 2–8) • Attempt Questions 1–10 • Allow about 15 minutes for this section **Section II – 90 marks** (pages 9–40) • Attempt Questions 11–32 • Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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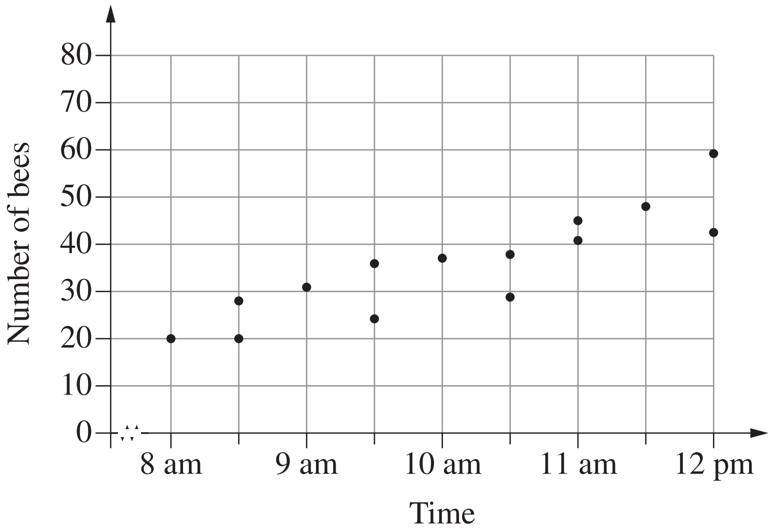
1 The number of bees leaving a hive was observed and recorded over 14 days at different times of the day.



Which Pearson’s correlation coefficient best describes the observations?

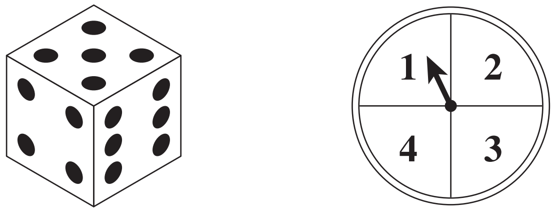
A. –0.8

B. –0.2

C. 0.2

D. 0.8

2 A game involves throwing a die and spinning a spinner.



The sum of the two numbers obtained is the score.

The table of scores below is partially completed.

		SPINNER			
		1	2	3	4
DIE	1	2	3	4	
	2	3	4	5	
	3		5	6	
	4			7	
	5				
	6				

What is the probability of getting a score of 7 or more?

A. $\frac{1}{6}$

B. $\frac{1}{4}$

C. $\frac{5}{18}$

D. $\frac{5}{12}$

3 What is the domain of $f(x) = \dfrac{1}{\sqrt{1-x}}$?

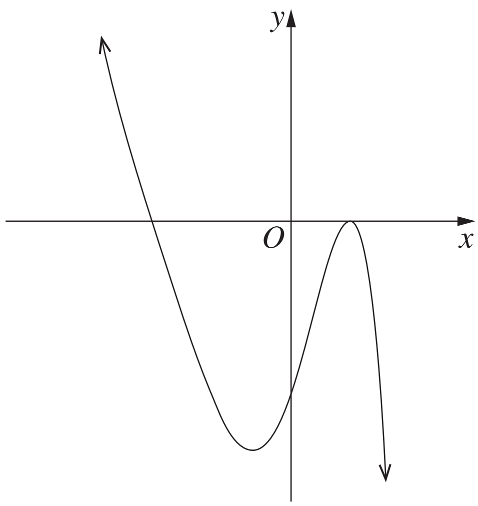
A. $x < 1$

B. $x \leq 1$

C. $x > 1$

D. $x \geq 1$

4 The graph of a polynomial is shown.



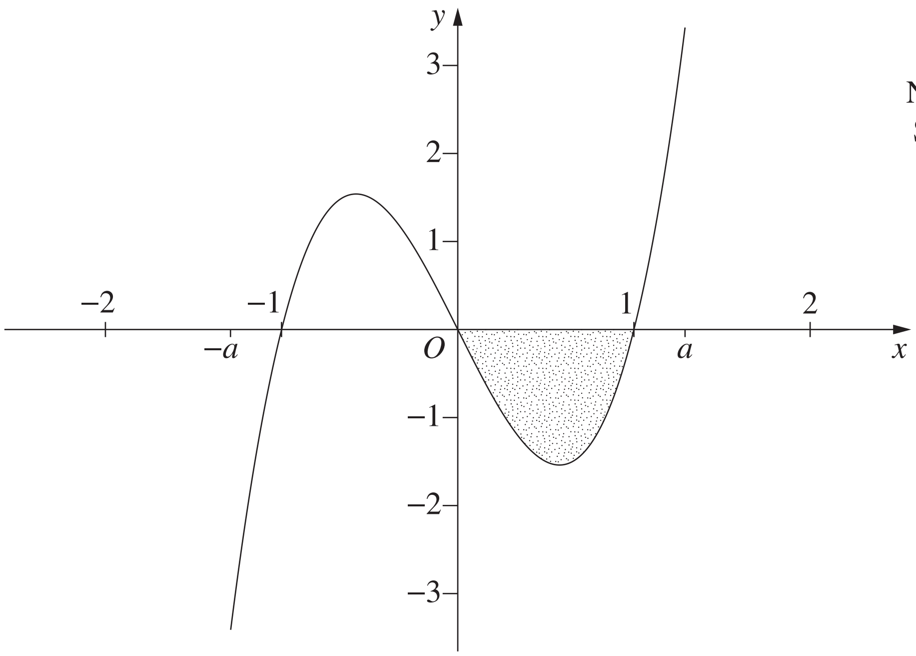
Which row of the table is correct for this polynomial?

	Equation	*Value of b*	*Value of c*
A.	$y = -(x-b)(x-c)^2$	$b > 0$	$c < 0$
B.	$y = -(x-b)(x-c)^2$	$b < 0$	$c > 0$
C.	$y = -x(x-b)(x-c)$	$b > 0$	$c < 0$
D.	$y = -x(x-b)(x-c)$	$b < 0$	$c > 0$

5 The diagram shows the graph $y = f(x)$, where $f(x)$ is an odd function.

The shaded area is 1 square unit.

The number a, where $a > 1$, is chosen so that $\int_0^a f(x)\,dx = 0$.



NOT TO SCALE

What is the value of $\int_{-a}^{1} f(x)\,dx$?

A. -1

B. 0

C. 1

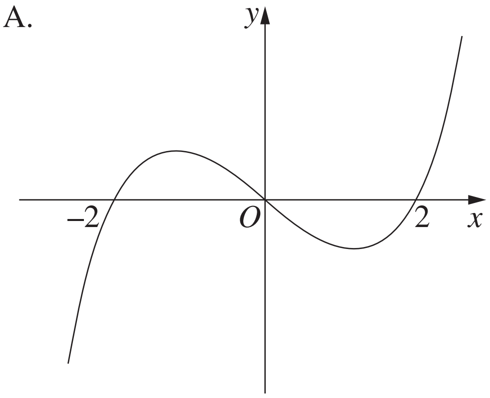
D. 3

6 The following table gives the signs of the first and second derivatives of a function $y = f(x)$ for different values of x.

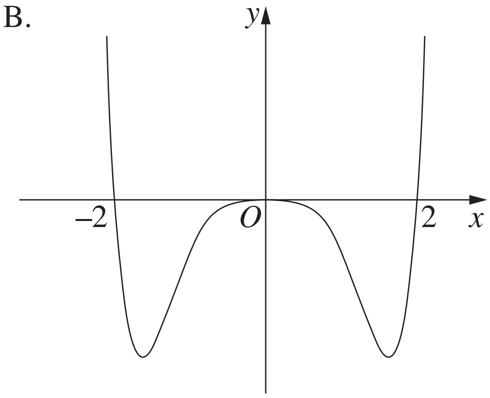
x	-2	0	2
$f'(x)$	$+$	0	$+$
$f''(x)$	$-$	0	$+$

Which of the following is a possible sketch of $y = f(x)$?

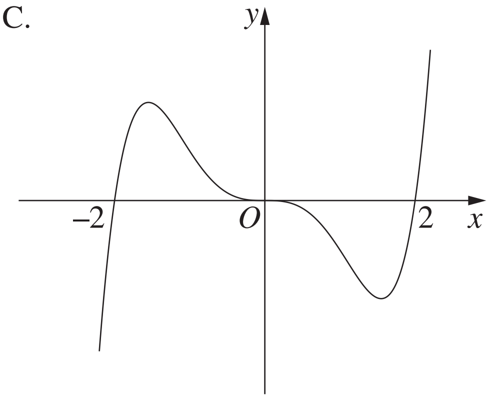
A.



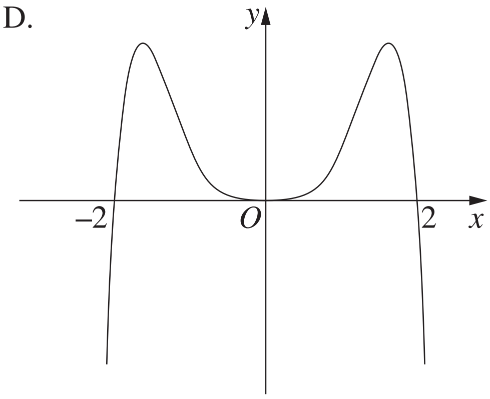
B.



C.



D.



7 It is given that $y = f(g(x))$, where $f(1) = 3$, $f'(1) = -4$, $g(5) = 1$ and $g'(5) = 2$.

What is the value of y' at $x = 5$?

A. -8

B. -4

C. 3

D. 6

8 What is the solution of the equation $\log_a x^3 = b$, where a and b are positive constants?

A. $x = b^{\frac{a}{3}}$

B. $x = a^{\frac{b}{3}}$

C. $x = \dfrac{b^a}{3}$

D. $x = \dfrac{a^b}{3}$

9 Let $f(x)$ be any function with domain all real numbers.

Which of the following is an even function, regardless of the choice of $f(x)$?

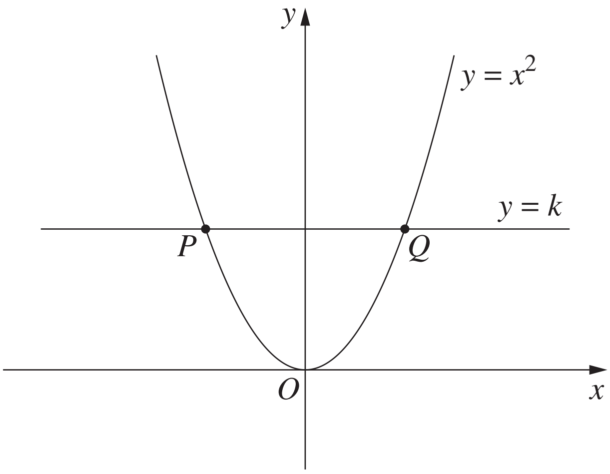
A. $2f(x)$

B. $f(f(x))$

C. $(f(-x))^2$

D. $f(x)f(-x)$

10 The graph $y = x^2$ meets the line $y = k$ (where $k > 0$) at points P and Q as shown in the diagram. The length of the interval PQ is L.



Let a be a positive number. The graph $y = \dfrac{x^2}{a^2}$ meets the line $y = k$ at points S and T.

What is the length of ST?

A. $\dfrac{L}{a}$

B. $\dfrac{L}{a^2}$

C. aL

D. a^2L

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2023 HIGHER SCHOOL CERTIFICATE EXAMINATION

Centre Number

Student Number

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

The first three terms of an arithmetic sequence are 3, 7 and 11. **2**

Find the 15th term.

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Question 12 (3 marks)

The table shows the probability distribution of a discrete random variable.

x	0	1	2	3	4
$P(X = x)$	0	0.3	0.5	0.1	0.1

(a) Show that the expected value $E(X) = 2$. **1**

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(b) Calculate the standard deviation, correct to one decimal place. **2**

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Question 13 (2 marks)

Let $P(t)$ be a function such that $\dfrac{dP}{dt} = 3000e^{2t}$. **2**

When $t = 0$, $P = 4000$.

Find an expression for $P(t)$.

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Question 14 (3 marks)

Find the equation of the tangent to the curve $y = (2x + 1)^3$ at the point $(0, 1)$. **3**

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Question 15 (5 marks)

A table of future value interest factors for an annuity of $1 is shown.

Period \ *Rate*	*1.5%*	*3%*	*4.5%*	*6%*
5	5.152	5.309	5.471	5.637
10	10.703	11.464	12.288	13.181
20	23.124	26.870	31.371	36.786
40	54.268	75.401	107.030	154.762

(a) Micky wants to save $450 000 over the next 10 years. **2**

If the interest rate is 6% per annum compounding annually, how much should Micky contribute each year? Give your answer to the nearest dollar.

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(b) Instead, Micky decides to contribute $8535 every three months for 10 years to an annuity paying 6% per annum, compounding quarterly. **3**

How much will Micky have at the end of 10 years?

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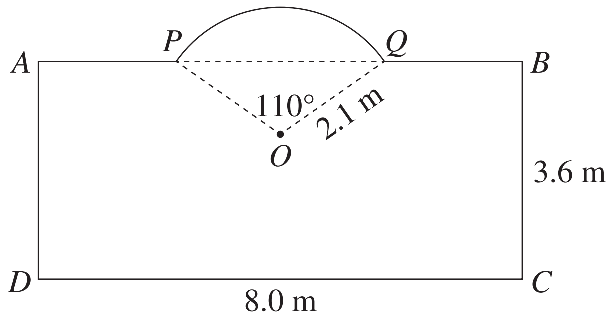
Question 16 (4 marks)

The diagram shows a shape *APQBCD*. The shape consists of a rectangle *ABCD* with an arc *PQ* on side *AB* and with side lengths $BC = 3.6$ m and $CD = 8.0$ m. **4**

The arc *PQ* is an arc of a circle with centre *O* and radius 2.1 m and $\angle POQ = 110°$.



NOT TO SCALE

What is the perimeter of the shape *APQBCD*? Give your answer correct to one decimal place.

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Question 17 (2 marks)

Find $\int x\sqrt{x^2+1}\,dx$. **2**

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Please turn over

Question 18 (6 marks)

A university uses gas to heat its buildings. Over a period of 10 weekdays during winter, the gas used each day was measured in megawatts (MW) and the average outside temperature each day was recorded in degrees Celsius (°C).

Using x as the average daily outside temperature and y as the total daily gas usage, the equation of the least-squares regression line was found.

The equation of the regression line predicts that when the temperature is 0°C, the daily gas usage is 236 MW.

The ten temperatures measured were: 0°, 0°, 0°, 2°, 5°, 7°, 8°, 9°, 9°, 10°.

The total gas usage for the ten weekdays was 1840 MW.

In any bivariate dataset, the least-squares regression line passes through the point $(\bar{x}, \bar{y})$, where $\bar{x}$ is the sample mean of the x-values and $\bar{y}$ is the sample mean of the y-values.

(a) Using the information provided, plot the point $(\bar{x}, \bar{y})$ and the y-intercept of the least-squares regression line on the grid. **3**

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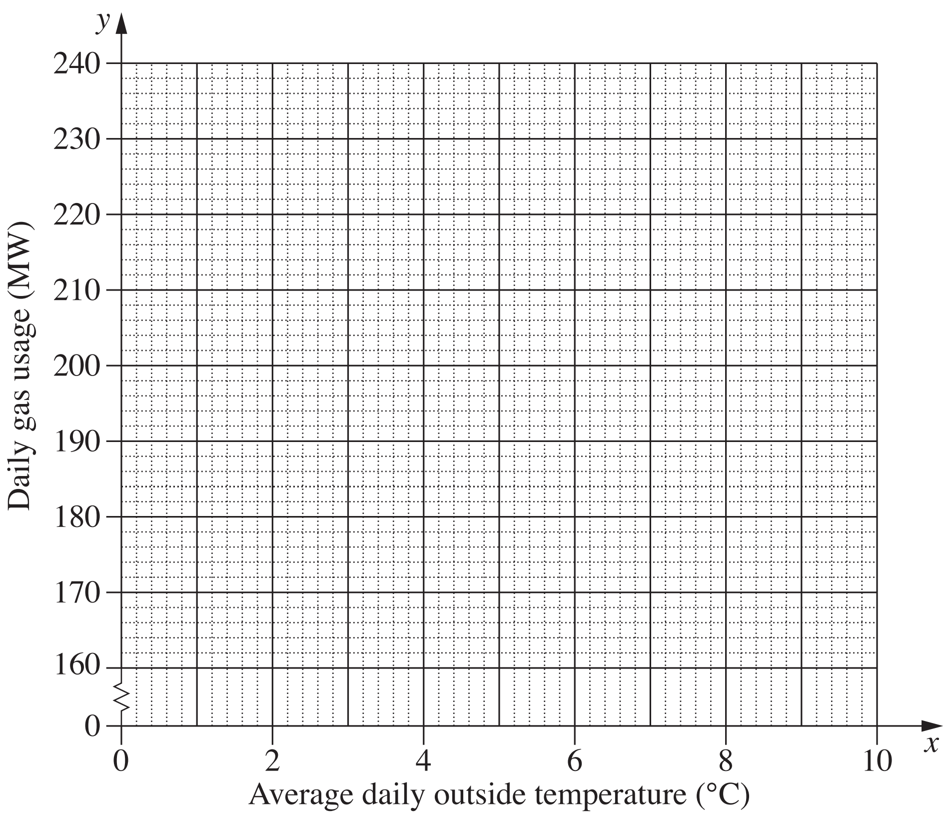

Question 18 continues on page 17

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Question 18 (continued)

(b) What is the equation of the regression line? **2**

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(c) In the context of the dataset, identify ONE problem with using the regression line to predict gas usage when the average outside temperature is 23°C. **1**

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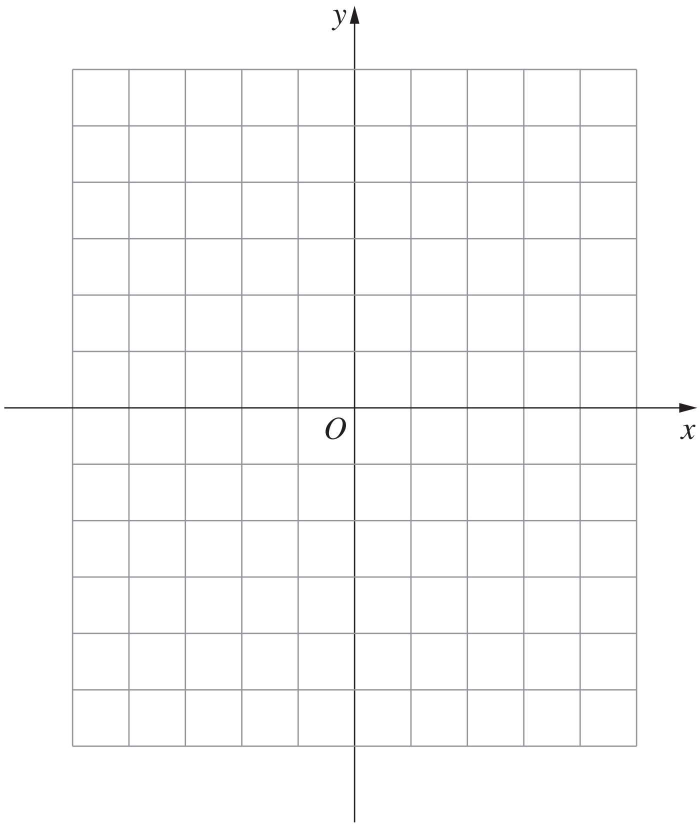
End of Question 18

Please turn over

Question 19 (4 marks)

(a) Sketch the graphs of the functions $f(x) = x - 1$ and $g(x) = (1 - x)(3 + x)$ showing the x-intercepts. **2**



(b) Hence, or otherwise, solve the inequality $x - 1 < (1 - x)(3 + x)$. **2**

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Question 20 (3 marks)

Find all the values of θ, where $0° \leq \theta \leq 360°$, such that **3**

$$\sin(\theta - 60°) = -\frac{\sqrt{3}}{2}.$$

Please turn over

Question 21 (3 marks)

The fourth term of a geometric sequence is 48. **3**

The eighth term of the same sequence is $\frac{3}{16}$.

Find the possible value(s) of the common ratio and the corresponding first term(s).

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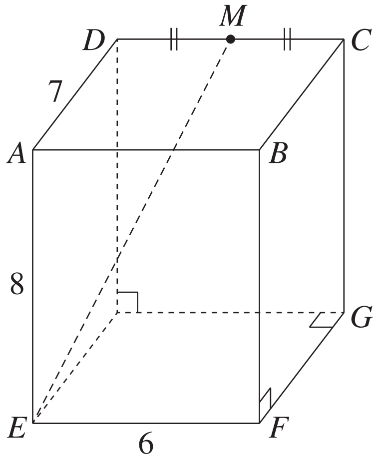
Question 22 (3 marks)

In the rectangular prism shown, $AD = 7$ cm, $AE = 8$ cm, $EF = 6$ cm. Point M is the midpoint of CD. **3**



NOT TO SCALE

Find $\angle AEM$, to the nearest degree.

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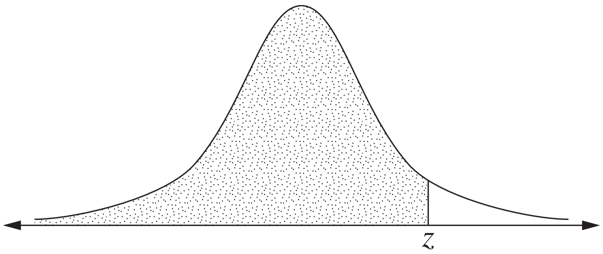
Question 23 (4 marks)

A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table gives the probability that this random variable lies below z for some positive values of z. **4**

z	1.30	1.31	1.32	1.33	1.34	1.35	1.36	1.37	1.38	1.39
Probability	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

The probability values given in the table are represented by the shaded area in the following diagram.



The weights of adult male koalas form a normal distribution with mean $\mu = 10.40$ kg, and standard deviation $\sigma = 1.15$ kg.

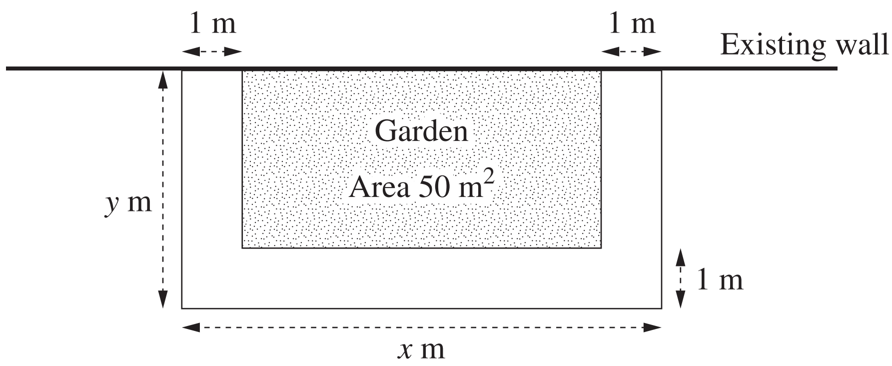
In a group of 400 adult male koalas, how many would be expected to weigh more than 11.93 kg?

Questions 11–23 are worth 44 marks in total

Please turn over

Question 24 (5 marks)

A gardener wants to build a rectangular garden of area 50 m^2 against an existing wall as shown in the diagram. A concrete path of width 1 metre is to be built around the other three sides of the garden.



Let x and y be the dimensions, in metres, of the outer rectangle as shown.

(a) Show that $y = \dfrac{50}{x-2} + 1$. **1**

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Question 24 continues on page 25

Question 24 (continued)

(b) Find the value of x such that the area of the concrete path is a minimum. Show that your answer gives a minimum area. **4**

End of Question 24

Question 25 (6 marks)

On the first day of November, Jia deposits \$10 000 into a new account which earns 0.4% interest per month, compounded monthly. At the end of each month, after the interest is added to the account, Jia intends to withdraw \$$M$ from the account.

Let A_n be the amount (in dollars) in Jia's account at the end of n months.

(a) Show that $A_2 = 10\,000(1.004)^2 - M(1.004) - M$. **1**

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(b) Show that $A_n = (10\,000 - 250M)(1.004)^n + 250M$. **3**

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Question 25 continues on page 27

Question 25 (continued)

(c) Jia wants to be able to make at least 100 withdrawals. **2**

What is the largest value of M that will enable Jia to do this?

End of Question 25

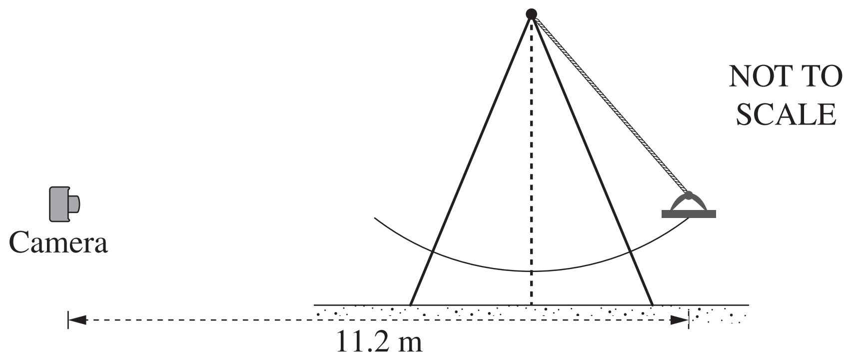
Please turn over

Question 26 (4 marks)

A camera films the motion of a swing in a park.

Let $x(t)$ be the horizontal distance, in metres, from the camera to the seat of the swing at t seconds.

The seat is released from rest at a horizontal distance of 11.2 m from the camera.



(a) The rate of change of x can be modelled by the equation **2**

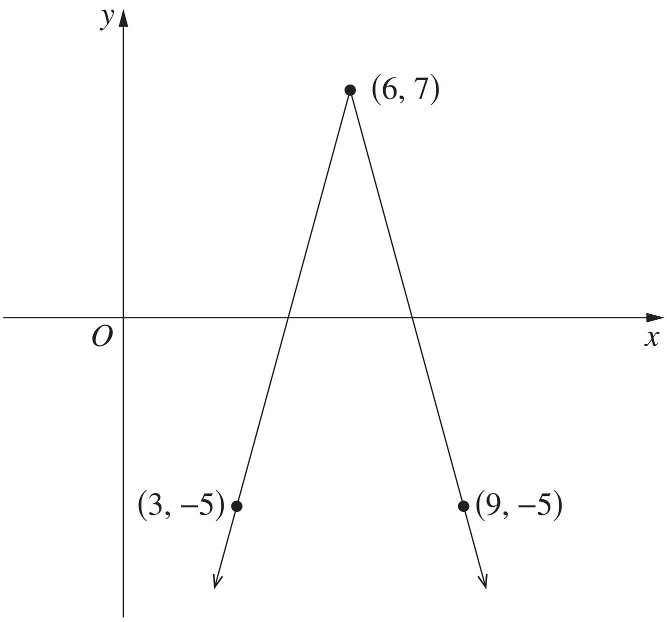
$$\frac{dx}{dt} = -1.5\pi \sin\left(\frac{5\pi}{4}t\right).$$

Find an expression for $x(t)$.

(b) How many times does the swing reach the closest point to the camera during the first 10 seconds? **2**

Question 27 (5 marks)

The graph of $y = f(x)$, where $f(x) = a|x - b| + c$, passes through the points $(3, -5)$, $(6, 7)$ and $(9, -5)$ as shown in the diagram.



(a) Find the values of a, b and c. **3**

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(b) The line $y = mx$ cuts the graph of $y = f(x)$ in two distinct places. **2**

Find all possible values of m.

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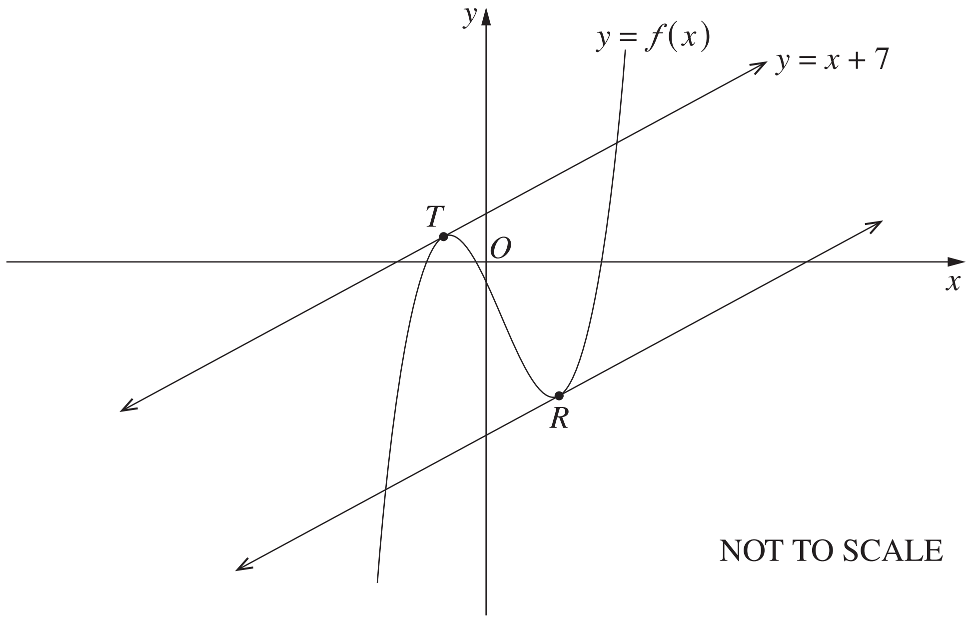
Question 28 (4 marks)

The curve $y = f(x)$ is shown on the diagram. The equation of the tangent to the curve at point $T\ (-1, 6)$ is $y = x + 7$. At a point R, another tangent parallel to the tangent at T is drawn. **4**



The gradient function of the curve is given by $\dfrac{dy}{dx} = 3x^2 - 6x - 8$.

Find the coordinates of R.

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Question 29 (6 marks)

A continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} 12x^2(1-x), & \text{for } 0 \le x \le 1 \\ 0, & \text{for all other values of } x \end{cases}.$$

(a) Find the mode of X. **2**

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(b) Find the cumulative distribution function for the given probability density function. **2**

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(c) Without calculating the median, show that the mode is greater than the median. **2**

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Question 30 (5 marks)

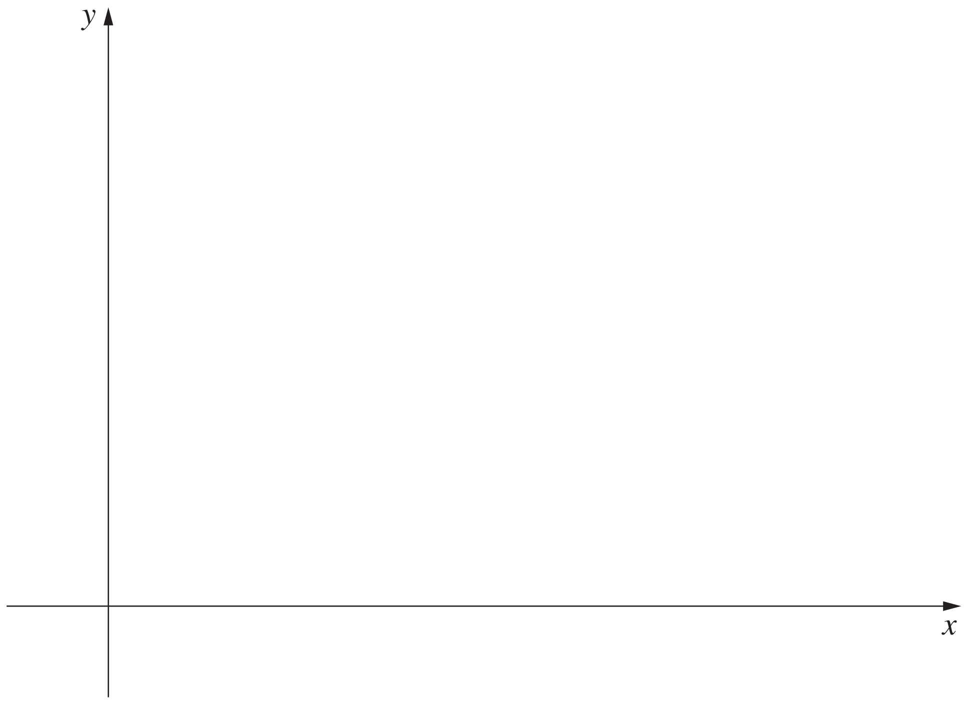
Let $f(x) = e^{-x} \sin x$.

(a) Find the coordinates of the stationary points of $f(x)$ for $0 \le x \le 2\pi$. You do NOT need to check the nature of the stationary points. **3**

Question 30 continues on page 33

Question 30 (continued)

(b) Without using any further calculus, sketch the graph of $y = f(x)$, for $0 \le x \le 2\pi$, showing stationary points and intercepts. **2**



End of Question 30

Please turn over

Question 31 (5 marks)

Four Year 12 students want to organise a graduation party. All four students have the same probability, $P(F)$, of being available next Friday. All four students have the same probability, $P(S)$, of being available next Saturday.

It is given that $P(F) = \frac{3}{10}$, $P(S|F) = \frac{1}{3}$, and $P(F|S) = \frac{1}{8}$.

Kim is one of the four students.

(a) Is Kim's availability next Friday independent from his availability next Saturday? Justify your answer. **1**

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(b) Show that the probability that Kim is available next Saturday is $\frac{4}{5}$. **2**

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Question 31 continues on page 35

Question 31 (continued)

(c) What is the probability that at least one of the four students is NOT available next Saturday? **2**

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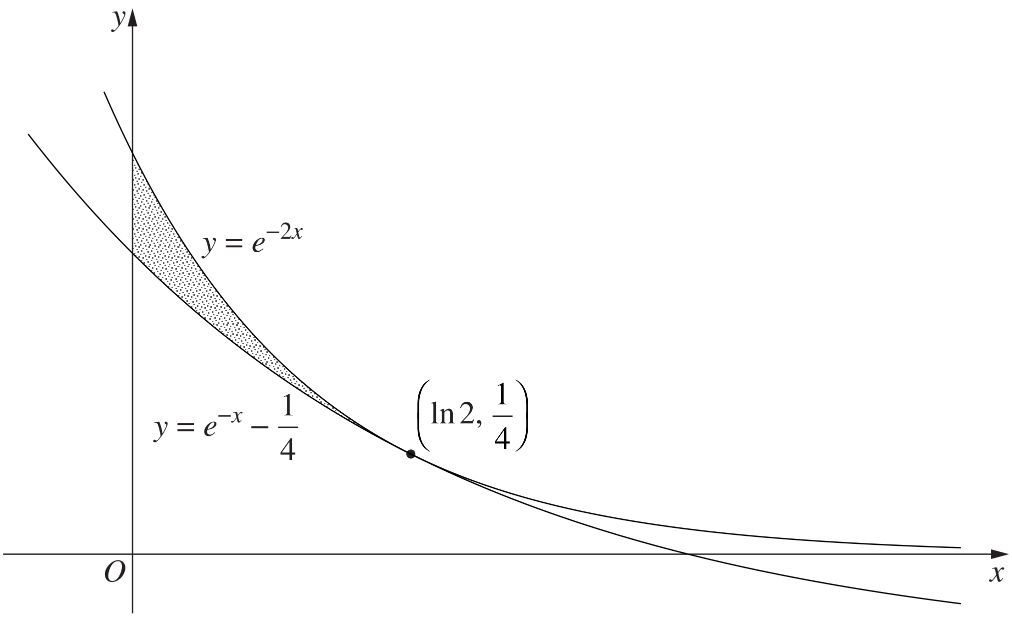
End of Question 31

Please turn over

Question 32 (6 marks)

The curves $y = e^{-2x}$ and $y = e^{-x} - \frac{1}{4}$ intersect at exactly one point as shown in the diagram. The point of intersection has coordinates $\left(\ln 2, \frac{1}{4}\right)$. (Do NOT prove this.)



(a) Show that the area bounded by the two curves and the y-axis, as shaded in the diagram, is $\frac{1}{4}\ln 2 - \frac{1}{8}$. **3**

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Question 32 continues on page 37

Question 32 (continued)

(b) Find the values of k such that the curves $y = e^{-2x}$ and $y = e^{-x} + k$ intersect at two points. **3**

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NSW Education Standards Authority

2023 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

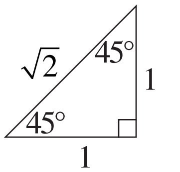
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

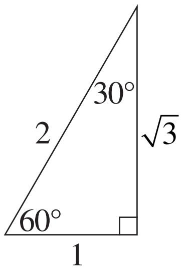


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

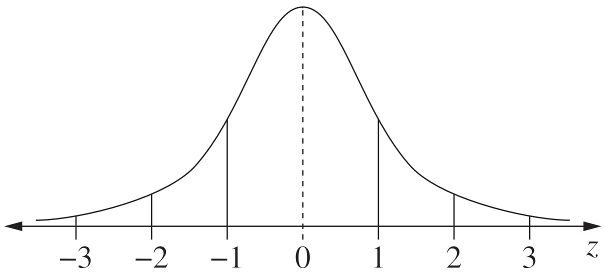
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n\,dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2\,,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2023 HSC Mathematics Advanced Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	D
2	D
3	A
4	B
5	A
6	C
7	A
8	B
9	D
10	C

Section II

Question 11

Criteria	Marks
• Provides correct solution	2
• Finds the common difference, or equivalent merit	1

Sample answer:

$d = 7 - 3 = 4$

$d = 4$
$a = 3$

$$\begin{aligned} t_{15} = a + (15 - 1)d &= 3 + 14 \times 4 \\ &= 59 \end{aligned}$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$
\begin{aligned}
E(X) &= \sum x\,P(x) \\
&= 0\times 0 + 1\times 0.3 + 2\times 0.5 + 3\times 0.1 + 4\times 0.1 \\
&= 2
\end{aligned}
$$

Question 12 (b)

Criteria	Marks
• Provides correct solution	2
• Attempts to find Var(X), or equivalent merit	1

Sample answer:

$$
\begin{aligned}
\mathrm{Var}(X) &= E\left(X^2\right) - \mu^2 \\
&= E\left(X^2\right) - \left[E(X)\right]^2 \\
&= \sum x^2\,P(x) - 4 \\
&= 0\times 0 + 1^2\times 0.3 + 2^2\times 0.5 + 3^2\times 0.1 + 4^2\times 0.1 - 4 \\
&= 0.8
\end{aligned}
$$

$$
\begin{aligned}
\text{Standard deviation} &= \sqrt{0.8} \\
&= 0.8944\ldots \\
&= 0.9 \qquad \text{(to 1 decimal place)}
\end{aligned}
$$

Question 13

Criteria	Marks
• Provides correct solution	2
• Attempts to find antiderivative, or equivalent merit	1

Sample answer:

$$\frac{dP}{dt} = 3000e^{2t}$$

$$\therefore \quad P = \frac{3000}{2}e^{2t} + C$$

$$= 1500e^{2t} + C$$

When $t = 0$, $P = 4000$

$$\therefore \quad 4000 = 1500e^{2\times 0} + C$$

$$\therefore \quad C = 2500$$

So $P(t) = 1500e^{2t} + 2500$

Question 14

Criteria	Marks
• Provides correct solution	3
• Correctly finds slope of tangent, or equivalent merit	2
• Attempts to find the correct derivative, or equivalent merit	1

Sample answer:

$$y = (2x+1)^3$$

$$y' = 3 \times (2x+1)^2 \times 2$$

$$= 6(2x+1)^2$$

When $x = 0$ $y' = 6$

Equation of tangent given by:

$$y - 1 = 6(x - 0)$$
$$y - 1 = 6x$$
$$y = 6x + 1$$

Question 15 (a)

Criteria	Marks
• Provides correct solution	2
• Identifies the correct factor from the table	1

Sample answer:

$$\text{Amount} = \frac{\$450\,000}{13.181}$$
$$= \$34\,140 \qquad \text{(to the nearest dollar)}$$

Question 15 (b)

Criteria	Marks
• Provides correct solution	3
• Provides the correct interest rate and the correct number of periods, or equivalent merit	2
• Multiplies a factor from the table by $8535, or equivalent merit	1

Sample answer:

$$r = \frac{6}{4}\%$$
$$= 1.5\%$$

$$n = 10 \times 4$$
$$= 40$$

$$\text{Amount} = \$8535 \times 54.268$$
$$= \$463\,177.38$$

Question 16

Criteria	Marks
• Provides correct solution	4
• Calculates the arc length AND the length of line segment *PQ*, or equivalent merit	3
• Calculates the arc length OR the length of line segment *PQ*, or equivalent merit	2
• Attempts to calculate the perimeter of the shape by adding some appropriate portions, or equivalent merit	1

Sample answer:

$$\text{Arc length } PQ = \frac{110}{360} \times 2 \times \pi \times 2.1$$
$$= 4.03171\ldots$$

$$\text{Length } PQ = \sqrt{2.1^2 + 2.1^2 - 2 \times 2.1 \times 2.1 \times \cos 110^\circ}$$
$$= 3.4404\ldots$$

$$\text{Total perimeter} = (3.6 \times 2) + 8.0 + (8.0 - 3.4404) + 4.0317$$
$$= 23.7913$$
$$= 23.8 \text{ m}$$

Question 17

Criteria	Marks
• Provides correct solution	2
• Recognises the integral is of the form $k\int f'(x)\left[f(x)\right]^n dx$, or equivalent merit	1

Sample answer:

$$\int x\left(x^2+1\right)^{\frac{1}{2}}dx$$

$$= \frac{1}{2}\int 2x\left(x^2+1\right)^{\frac{1}{2}}dx$$

$$= \frac{1}{2}\left[\frac{\left(x^2+1\right)^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$$

$$= \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}} + C$$

Question 18 (a)

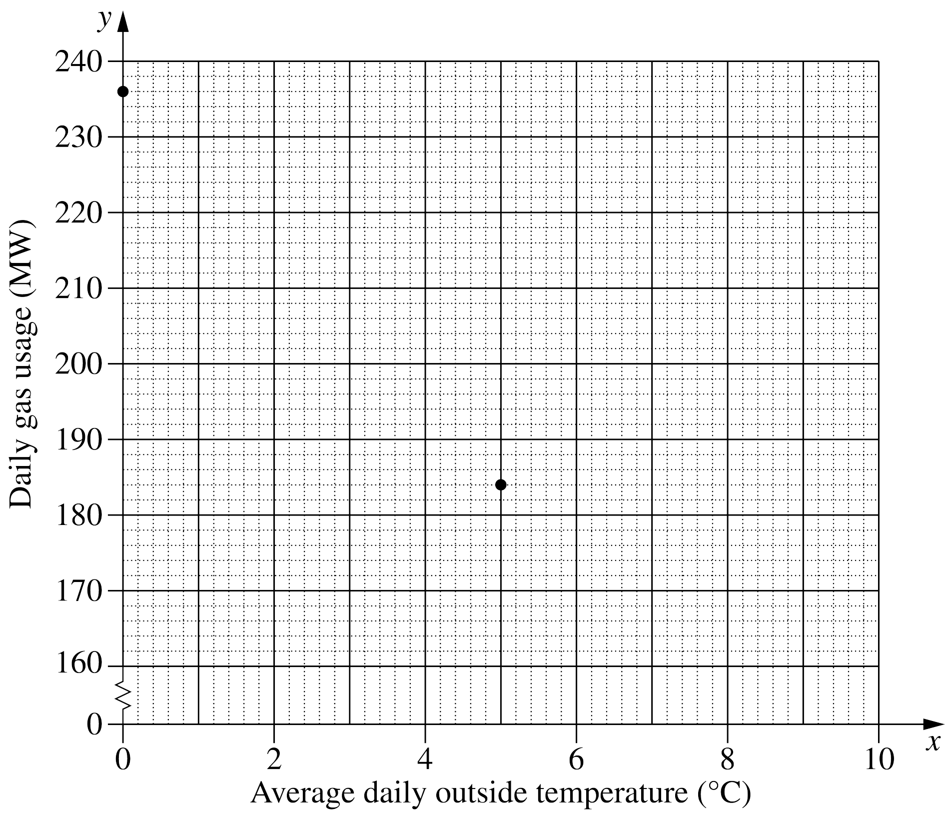
Criteria	Marks
• Correctly plots both points on the graph	3
• Calculates $\bar{x}$ and $\bar{y}$, and plots this point on the grid, or equivalent merit	2
• Calculates $\bar{x}$ or $\bar{y}$, or equivalent merit	1

Sample answer:

$$\bar{x} = \frac{0+0+0+2+5+7+8+9+9+10}{10}$$
$$= 5$$

$$\bar{y} = \frac{1840}{10}$$
$$= 184$$

$$\therefore\ (\bar{x}, \bar{y}) = (5, 184)$$



Question 18 (b)

Criteria	Marks
• Provides correct solution	2
• Finds the slope of the regression line, or equivalent merit	1

Sample answer:

$$\text{Slope of regression line} = \frac{184 - 236}{5}$$

$$= -10.4$$

Gas usage = 236 − 10.4 × temperature

ie $y = 236 - 10.4x$

Question 18 (c)

Criteria	Marks
• Identifies one problem with predicting using the regression line	1

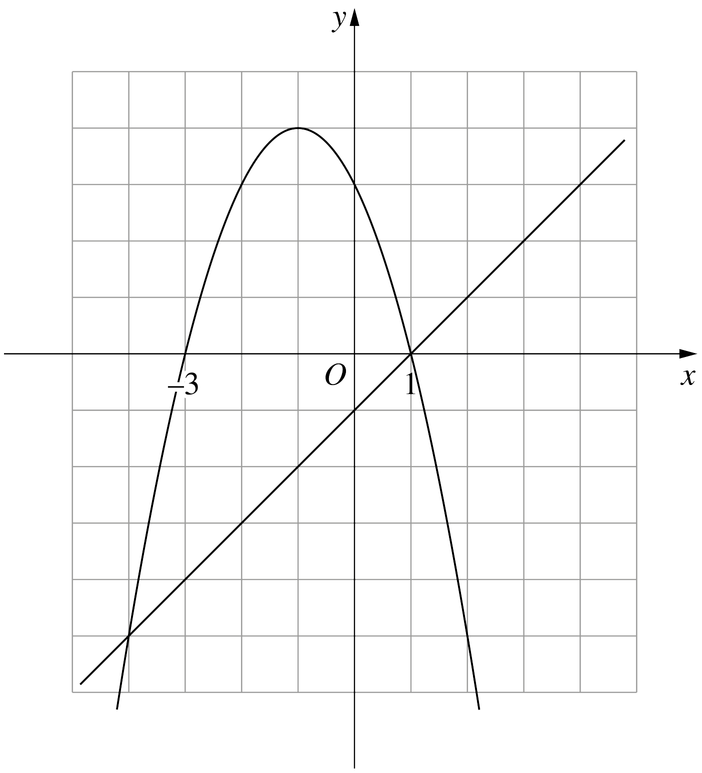
Sample answer:

When temperature is 23°C, the regression equation provides a negative answer, which is not physically possible (negative gas usage).

Question 19 (a)

Criteria	Marks
• Provides correct graphs	2
• Provides a sketch of $f(x)$, or equivalent merit	1

Sample answer:



Question 19 (b)

Criteria	Marks
• Provides correct solution	2
• Finds that the graphs intersect at $x = -4$, or equivalent merit	1

Sample answer:

The graphs meet when
$x - 1 = (1 - x)(3 + x)$

$\therefore \quad x = 1 \quad \text{or} \quad 3 + x = -1$
ie $\quad x = 1 \quad \text{or} \quad x = -4$

From part (a), $-4 < x < 1$.

Question 20

Criteria	Marks
• Provides correct solution	3
• Provides $\theta - 60° = 240°, 300°$, or equivalent merit	2
• Recognises that $\sin 60° = \frac{\sqrt{3}}{2}$	1

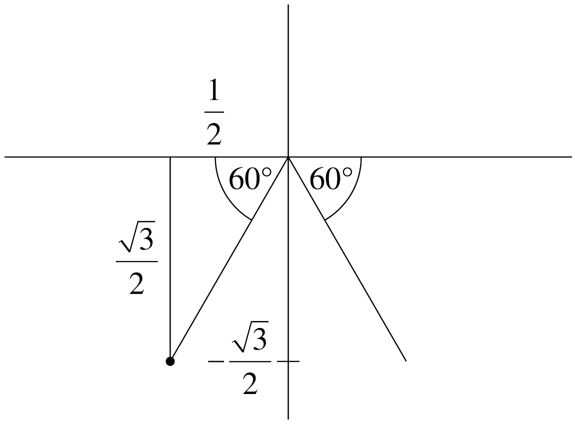
Sample answer:

$$0° \le \theta \le 360°$$
$$-60° \le \theta - 60° \le 300°$$

$$\sin(\theta - 60°) = -\frac{\sqrt{3}}{2}$$

$$\theta - 60° = 240° \text{ or } 300° \text{ or } -60°$$

$$\theta = 300°, 360°, 0°$$



Question 21

Criteria	Marks
• Provides correct solution	3
• Finds r^4, or equivalent merit	2
• Writes $ar^3 = 48$, or equivalent merit	1

Sample answer:

Let a = first term and r = common ratio

Then $ar^3 = 48$ ———— (1)

and $ar^7 = \frac{3}{16}$ ———— (2)

Dividing (2) by (1),

$$\frac{ar^7}{ar^3} = \frac{\frac{3}{16}}{48}$$

$$\therefore r^4 = \frac{1}{256}$$

$$\therefore r = \pm\frac{1}{4}$$

If $r = \frac{1}{4}$ $\quad a\left(\frac{1}{4}\right)^3 = 48 \quad \therefore a = 3072$

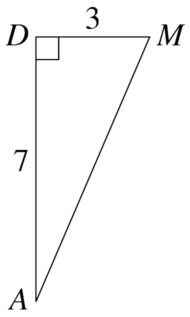
If $r = -\frac{1}{4}$ $\quad a\left(-\frac{1}{4}\right)^3 = 48 \quad \therefore a = -3072$

Question 22

Criteria	Marks
• Provides correct solution	3
• Finds the length of *AM*, or equivalent merit	2
• Indicates that triangle *ADM* is useful, or equivalent merit	1

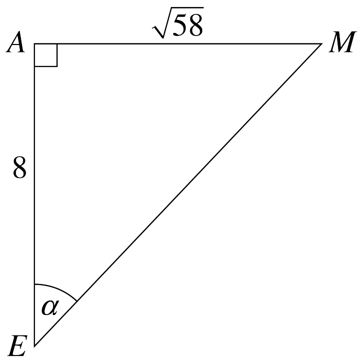
Sample answer:

Find AM



$$\begin{aligned} AM^2 &= 3^2 + 7^2 \\ &= 9 + 49 \\ &= 58 \\ AM &= \sqrt{58} \end{aligned}$$

Triangle AME,



$$\text{So} \quad \tan\alpha = \frac{\sqrt{58}}{8}$$

$$\alpha = 43.59°$$

$$\text{so} \quad \alpha = 44° \qquad \text{(to the nearest degree)}$$

Question 23

Criteria	Marks
• Provides correct solution	4
• Finds the correct proportion of the group of koalas, or equivalent merit	3
• Finds the correct probability from the table, or equivalent merit	2
• Calculates the correct z value, or equivalent merit	1

Sample answer:

$$
\begin{aligned}
z &= \frac{x-\mu}{\sigma} \\
&= \frac{11.93-10.40}{1.15} \\
&= 1.33 \qquad \text{(2 decimal places)}
\end{aligned}
$$

$\therefore$ Probability from table = 0.9082

$$
\begin{aligned}
P(\text{more than } 11.93) &= 1-0.9082 \\
&= 0.0918
\end{aligned}
$$

$$
\begin{aligned}
\text{Number of koalas} &= 0.0918 \times 400 \\
&= 36.72 \\
&= 36 \qquad \text{(accept 37 as well)}
\end{aligned}
$$

Question 24 (a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\begin{aligned} 50 &= (x-2)(y-1) \\ \frac{50}{x-2} &= y-1 \end{aligned}$$

So $\quad y = \dfrac{50}{x-2} + 1 \quad$ as required.

Question 24 (b)

Criteria	Marks
• Provides correct solution	4
• Finds $x = 12$, or equivalent merit	3
• Finds A', or equivalent merit	2
• Finds an expression for the Area in terms of x, or equivalent merit	1

Sample answer:

Area of concrete path is $2y + x - 2$

$$A = 2\left(\frac{50}{x-2}+1\right) + x - 2$$

$$A = 2\left(\frac{50}{x-2}\right) + 2 + x - 2$$

$$A = \frac{100}{x-2} + x$$

$$= 100(x-2)^{-1} + x$$

$$A' = 100\left(-1(x-2)^{-2}\right)+1$$

$$= \frac{-100}{(x-2)^2} + 1$$

$$A' = 0 \quad \text{when} \quad \frac{-100}{(x-2)^2} = -1$$

$$100 = (x-2)^2$$

$$\pm 10 = x - 2$$

$$x = 12 \text{ or } -8$$

Since x is a distance, discard -8.

Stationary point at $x = 12$.

x	11	12	13
A'	$\frac{-100}{81}+1$	0	$\frac{-100}{121}+1$
	$\diagdown$	$-$	$\diagup$

So there is a minimum turning point at $x = 12$.

The minimum area of the path is when $x = 12$.

Question 25 (a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$A_1 = 10\,000\,(1.004) - M$$

$$A_2 = \left(10\,000\,(1.004) - M\right)(1.004) - M$$

$$= 10\,000\,(1.004)^2 - M(1.004) - M \qquad \text{as required.}$$

Question 25 (b)

Criteria	Marks
• Provides correct solution	3
• Provides an expression for A_n involving the sum of a geometric series, or equivalent merit	2
• Finds an expression for A_n using part (a), or equivalent merit	1

Sample answer:

$$A_n = 10\,000\,(1.004)^n - M\left(1 + 1.004 + \cdots + 1.004^{n-1}\right)$$

$$= 10\,000\,(1.004)^n - \frac{M\left(1.004^n - 1\right)}{0.004}$$

$$= 10\,000\,(1.004)^n - \frac{M}{0.004} \times 1.004^n + \frac{M}{0.004}$$

$$= 10\,000\,(1.004)^n - 250M \times 1.004^n + 250M$$

$$A_n = \left(10\,000 - 250M\right)(1.004)^n + 250M$$

Question 25 (c)

Criteria	Marks
• Provides correct solution	2
• Identifies $A_n > 0$ and $n = 100$, or equivalent merit	1

Sample answer:

$A_{100} > 0$

$$(10\,000 - 250M)(1.004)^{100} + 250M > 0$$

$$10\,000 \times 1.004^{100} - 250M \times 1.004^{100} + 250M > 0$$

$$14\,906.34886 - 250M\left(1.004^{100} - 1\right) > 0$$

$$14\,906.34886 - 250M \times 0.4\,9063 > 0$$

$$14\,906.34886 > 122.6587\ldots M$$

$$\frac{14\,906.34886}{122.6587\ldots} > M$$

$$121.527 > M$$

The largest amount Jia could withdraw is \$121.52.

Question 26 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the antiderivative, or equivalent merit	1

Sample answer:

$$\frac{dx(t)}{dt} = -1.5\pi \sin\left(\frac{5\pi}{4}t\right)$$

$$x(t) = \int -1.5\pi \sin\left(\frac{5\pi}{4}t\right) dt$$

$$= \frac{-1.5\pi}{\frac{5\pi}{4}} \times -\cos\left(\frac{5\pi}{4}t\right) + k$$

When $t = 0$ $x = 11.2$

So $11.2 = 1.2\cos(0) + k$

$11.2 = 1.2 + k$

$k = 10$

$$x(t) = 1.2\cos\left(\frac{5\pi}{4}t\right) + 10$$

Question 26 (b)

Criteria	Marks
• Provides correct solution	2
• Finds the period, or equivalent merit	1

Sample answer:

$$\text{Period} = \frac{2\pi}{\frac{5\pi}{4}} = 1.6$$

$10 \div 1.6 = 6.25$

$\therefore$ Number of complete periods in 10 seconds is 6.

$\therefore$ Reaches closest point to camera 6 times.

Question 27 (a)

Criteria	Marks
• Provides correct solution	3
• Finds the value of *b* and *c*, or equivalent merit	2
• Finds value of *c*, or equivalent merit	1

Sample answer:

$c = 7$ Since the absolute value graph has been shifted by 7 vertically

$b = 6$ Shifted by 6 to the right

Let $x = 3$, $y = -5$

$$f(3) = a|3-6|+7 = -5$$
$$3a + 7 = -5$$
$$3a = -12$$
$$a = -4$$

$\therefore\ a = -4,\quad b = 6,\quad c = 7$

Question 27 (b)

Criteria	Marks
• Provides correct solution	2
• Finds that $m < \frac{7}{6}$, or equivalent merit	1

Sample answer:

Line joining (6, 7) with (0, 0) has slope $\frac{7}{6}$

m must be less than $\frac{7}{6}$ to cut the graph in two places.

Slope of right side of graph is -4

m must be greater than -4 or it will only cut graph once

Hence $-4 < m < \frac{7}{6}$.

Question 28

Criteria	Marks
• Provides correct solution	4
• Finds the x-coordinate of R AND the antiderivative for y	3
• Finds the x-coordinate of R OR the antiderivative for y	2
• Attempts to solve $\frac{dy}{dx} = 1$, or equivalent merit	1

Sample answer:

$$\frac{dy}{dx} = 3x^2 - 6x - 8$$

Tangent at $(-1, 6)$ is $y = x + 7$

Slope of tangent is 1.

Solve $\frac{dy}{dx} = 1$ ie

$$\begin{aligned} 3x^2 - 6x - 8 &= 1 \\ 3x^2 - 6x - 9 &= 0 \\ 3(x^2 - 2x - 3) &= 0 \\ 3(x - 3)(x + 1) &= 0 \end{aligned}$$

So x coordinate of R is 3.

When $\frac{dy}{dx} = 3x^2 - 6x - 8$

$y = x^3 - 3x^2 - 8x + k$ and when $x = -1$ $y = 6$.

So

$$\begin{aligned} 6 &= (-1)^3 - 3(-1)^2 - 8(-1) + k \\ 6 &= -1 - 3 + 8 + k \\ 6 - 4 &= k \\ k &= 2 \end{aligned}$$

When $x = 3$

$$\begin{aligned} y &= x^3 - 3x^2 - 8x + 2 \\ &= 27 - 27 - 24 + 2 \\ &= -22 \end{aligned}$$

$\therefore$ Coordinates of R are $(3, -22)$.

Question 29 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the derivative of $f(x)$, or equivalent merit	1

Sample answer:

Mode of X will be when $f(x)$ has a maximum.

$$f(x) = 12x^2 - 12x^3, \qquad 0 \le x \le 1$$

$$\begin{aligned} f'(x) &= 24x - 36x^2 \\ &= 12x(2 - 3x) \end{aligned}$$

$f'(x) = 0$ when $x = 0$ and when $2 - 3x = 0$

$$x = \frac{2}{3}$$

Discard $x = 0$ since $f(0) = 0$ so

the mode of X is $\dfrac{2}{3}$.

Question 29 (b)

Criteria	Marks
• Provides correct solution	2
• Expresses $F(x)$ as an integral of $f(x)$, or equivalent merit	1

Sample answer:

$$F(x) = \int_0^x 12t^2(1-t)\,dt$$

$$= \int_0^x 12t^2 - 12t^3\,dt$$

$$= \left[4t^3 - 3t^4\right]_0^x$$

$$= 4x^3 - 3x^4$$

Question 29 (c)

Criteria	Marks
• Provides correct solution	2
• Substitutes the mode from part (a) into their cumulative distribution function, or equivalent merit	1

Sample answer:

When $x = \frac{2}{3}$ $\quad 4x^3 - 3x^4 = 4\times\left(\frac{8}{27}\right) - 3\times\left(\frac{16}{81}\right)$

$$= 0.59$$

The probability of the variable being less than $\frac{2}{3}$ is greater than 0.5, therefore the mode is greater than the median.

Question 30 (a)

Criteria	Marks
• Provides correct solution	3
• Finds the *x* values of the stationary points, or equivalent merit	2
• Finds correct derivative, or equivalent merit	1

Sample answer:

$$f(x) = e^{-x}\sin x$$

$$\begin{aligned} f'(x) &= e^{-x}\cos x + -e^{-x}\sin x \\ &= e^{-x}(\cos x - \sin x) \end{aligned}$$

$$f'(x) = 0 \quad \text{when} \quad \cos x = \sin x$$

$$x = \frac{\pi}{4} \text{ or } \frac{5\pi}{4}$$

$$\text{so} \quad f(x) = e^{-\frac{\pi}{4}}\sin\frac{\pi}{4} \quad \text{or} \quad e^{-\frac{5\pi}{4}}\sin\frac{5\pi}{4}$$

The two stationary points are

$$\left(\frac{\pi}{4}, \frac{e^{-\frac{\pi}{4}}}{\sqrt{2}}\right) \quad \text{and} \quad \left(\frac{5\pi}{4}, \frac{-e^{-\frac{5\pi}{4}}}{\sqrt{2}}\right)$$

approx. approx.

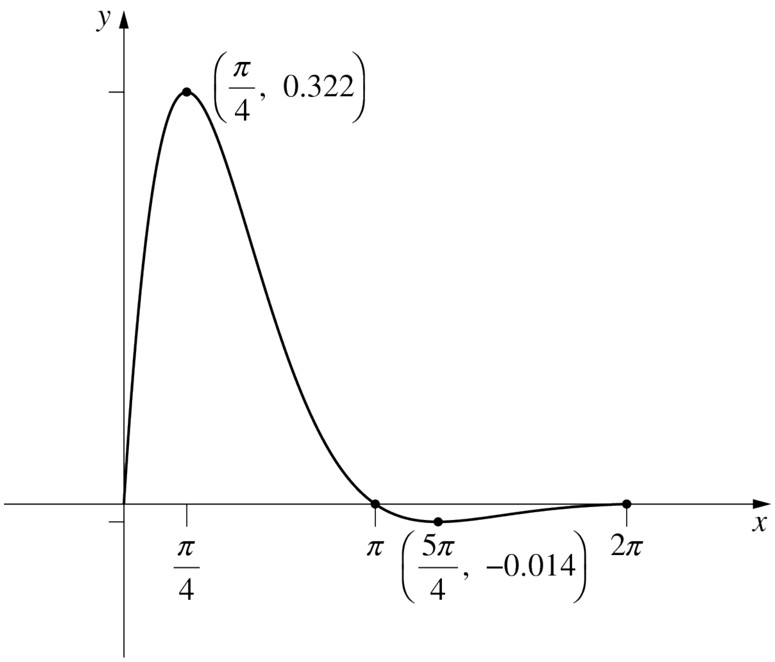
$$\left(\frac{\pi}{4}, 0.322\right) \quad \text{and} \quad \left(\frac{5\pi}{4}, -0.014\right)$$

Question 30 (b)

Criteria	Marks
• Provides correct graph	2
• Provides a graph with some correct features, or equivalent merit	1

Sample answer:

$f(x) = 0$ when $\sin x = 0$ $\quad x = 0,\ \pi,\ 2\pi$



Question 31 (a)

Criteria	Marks
• Provides correct reason	1

Sample answer:

No, since $P(F|S) \neq P(F)$

Question 31 (b)

Criteria	Marks
• Provides correct solution	2
• Attempts to use conditional probability formula, or equivalent merit	1

Sample answer:

$$P(S|F) = \frac{P(S \cap F)}{P(F)}$$

$$\frac{1}{3} = \frac{P(S \cap F)}{\frac{3}{10}}$$

$$P(S \cap F) = \frac{1}{10}$$

$$P(F|S) = \frac{P(S \cap F)}{P(S)}$$

$$\frac{1}{8} = \frac{\frac{1}{10}}{P(S)} \qquad (\text{since } P(S \cap F) = P(F \cap S))$$

$$P(S) = \frac{8}{10}$$

$$= \frac{4}{5}$$

Question 31 (c)

Criteria	Marks
• Provides correct answer	2
• Uses expression for complementary events, or equivalent merit	1

Sample answer:

$$1-\left(\frac{4}{5}\right)^4 = 1-\frac{256}{625} = \frac{369}{625} = 0.5904$$

Question 32 (a)

Criteria	Marks
• Provides correct solution	3
• Provides an antiderivative, or equivalent merit	2
• Provides an integral expression for the area, or equivalent merit	1

Sample answer:

$$\text{Shaded area } = \int_0^{\ln 2} e^{-2x} - \left(e^{-x} - \frac{1}{4}\right) dx$$

$$= \int_0^{\ln 2} e^{-2x} - e^{-x} + \frac{1}{4}\, dx$$

$$= \left[-\frac{1}{2}e^{-2x} + e^{-x} + \frac{1}{4}x\right]_0^{\ln 2}$$

$$= \left(-\frac{1}{2}e^{-2\ln 2} + e^{-\ln 2} + \frac{1}{4}\ln 2\right) - \left(-\frac{1}{2} + 1 + 0\right)$$

$$= -\frac{1}{2}e^{\ln\left(2^{-2}\right)} + e^{\ln\left(2^{-1}\right)} + \frac{1}{4}\ln 2 - \frac{1}{2}$$

$$= -\frac{1}{2}\times\frac{1}{4} + \frac{1}{2} + \frac{1}{4}\ln 2 - \frac{1}{2}$$

$$= \frac{1}{4}\ln 2 - \frac{1}{8}$$

Question 32 (b)

Criteria	Marks
• Provides correct solution	3
• Uses the discriminant to find $k > -\frac{1}{4}$, or equivalent merit	2
• Attempts to form a quadratic equation, or equivalent merit	1

Sample answer:

We want the equation $e^{-2x} = e^{-x} + k$ to have 2 solutions.

ie $\quad e^{-2x} - e^{-x} - k = 0 \qquad$ has 2 solutions

ie $\quad \left(e^{-x}\right)^2 - \left(e^{-x}\right) - k = 0 \qquad$ has 2 solutions

Using the quadratic formula,

$$e^{-x} = \frac{1 \pm \sqrt{1+4k}}{2}$$

For two real solutions we want $1 + 4k > 0$, ie $k > -\frac{1}{4}$

But for two solutions for e^{-x}, both the real solutions to the quadratic must be positive.

$\therefore \quad \sqrt{1+4k} < 1$

$\therefore \quad 1 + 4k < 1$

$\therefore \quad k < 0$

Hence $\quad -\frac{1}{4} < k < 0$.

2023 HSC Mathematics Advanced Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
2	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
3	1	MA-F1 Working with Functions	MA11-1
4	1	MA-F1 Working with Functions	MA11-1
5	1	MA-C4 Integral Calculus	MA12-7
6	1	MA-C3 Applications of Differentiation	MA12-6
7	1	MA-C2 Differential Calculus	MA12-6
8	1	MA-E1 Logarithms and Exponentials	MA11-6
9	1	MA-F1 Working with Functions	MA11-2
10	1	MA-F2 Graphing Techniques	MA12-1

Section II

Question	Marks	Content	Syllabus outcomes
11	2	MA- M1 Modelling Financial Situations	MA12-6
12 (a)	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
12 (b)	2	MA-S1 Probability and Discrete Probability Distributions	MA11-7
13	2	MA-C4 Integral Calculus	MA12-7
14	3	MA-C2 Differential Calculus	MA12-6
15 (a)	2	MA-M1 Modelling Financial Situations	MA12-2
15 (b)	3	MA-M1 Modelling Financial Situations	MA12-2
16	4	MA-T1 Trigonometry and Measure of Angles	MA11-3
17	2	MA-C4 Integral Calculus	MA12-7
18 (a)	3	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
18 (b)	2	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
18 (c)	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-10
19 (a)	2	MA-F2 Graphing Techniques	MA12-1
19 (b)	2	MA-F2 Graphing Techniques	MA12-1, MA12-10
20	3	MA-T2 Trigonometric Functions and Identities	MA11-1
21	3	MA-M1 Modelling Financial Situations	MA12-4
22	3	MA- T1 Trigonometry and Measure of Angles	MA11-3
23	4	MA-S3 Random Variables	MA12-8
24 (a)	1	MA-F1 Working with Functions	MA11-2
24 (b)	4	MA-C3 Applications of Differentiation	MA12-3, MA12-10
25 (a)	1	MA-M1 Modelling Financial Situations	MA12-10
25 (b)	3	MA-M1 Modelling Financial Situations	MA12-4
25 (c)	2	MA-M1 Modelling Financial Situations	MA12-2

Question	Marks	Content	Syllabus outcomes
26 (a)	2	MA-C4 Integral Calculus	MA12-3
26 (b)	2	MA-T3 Trigonometric Functions and Graphs	MA12-5
27 (a)	3	MA-F2 Graphing Techniques	MA12-1
27 (b)	2	MA-F2 Graphing Techniques	MA12-1
28	4	MA-C1 Introduction to Differentiation, MA-C4 Integral Calculus	MA12-3
29 (a)	2	MA-S3 Random Variables	MA12-8
29 (b)	2	MA-S3 Random Variables	MA12-8
29 (c)	2	MA-S3 Random Variables	MA12-8
30 (a)	3	MA-C3 Applications of Differentiation	MA12-6
30 (b)	2	MA-C3 Applications of Differentiation	MA12-3
31 (a)	1	MA-S1 Probability and Discrete Probability Distributions	MA11-9
31 (b)	2	MA-S1 Probability and Discrete Probability Distributions	MA11-7
31 (c)	2	MA-S1 Probability and Discrete Probability Distributions	MA11-7
32 (a)	3	MA-C4 Integral Calculus	MA12-7
32 (b)	3	MA-F1 Working with Functions	MA11-1