



NSW Education Standards Authority

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Centre Number

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Student Number

2024 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 2–8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9–39)

- Attempt Questions 11–31
- Allow about 2 hours and 45 minutes for this section

Section I

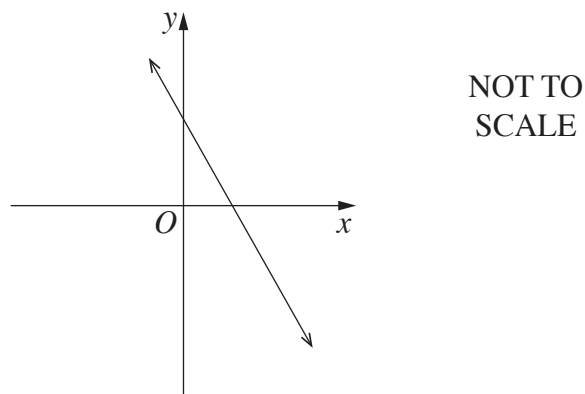
10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

- 1 Consider the function shown.



Which of the following could be the equation of this function?

- A. $y = 2x + 3$
 - B. $y = 2x - 3$
 - C. $y = -2x + 3$
 - D. $y = -2x - 3$
- 2 In a group of 60 students, 38 play basketball, 35 play hockey and 5 do not play either basketball or hockey.

How many students play both basketball and hockey?

- A. 55
- B. 18
- C. 13
- D. 8

- 3 Pia's marks in Year 10 assessments are shown. The scores for each subject were normally distributed.

	<i>Pia's mark</i>	<i>Year 10 mean</i>	<i>Year 10 standard deviation</i>
English	78	66	6
Mathematics	80	71	10
Science	77	70	15
History	85	72	9

In which subject did Pia perform best in comparison with the rest of Year 10?

- A. English
 - B. Mathematics
 - C. Science
 - D. History
- 4 The parabola $y = (x - 3)^2 - 2$ is reflected about the y -axis. This is then reflected about the x -axis.

What is the equation of the resulting parabola?

- A. $y = (x + 3)^2 + 2$
- B. $y = (x - 3)^2 + 2$
- C. $y = -(x + 3)^2 + 2$
- D. $y = -(x - 3)^2 + 2$

5 What is $\int (6x + 1)^3 dx$?

A. $\frac{1}{24}(6x + 1)^4 + C$

B. $\frac{1}{4}(6x + 1)^4 + C$

C. $\frac{2}{3}(6x + 1)^4 + C$

D. $\frac{3}{2}(6x + 1)^4 + C$

6 What is the domain of the function $f(x) = \frac{1}{\sqrt{x^2 - 1}}$?

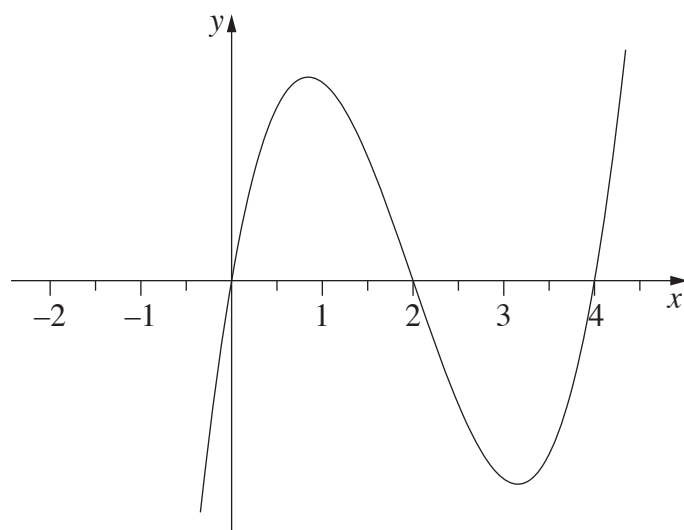
A. $[-1, 1]$

B. $(-\infty, -1] \cup [1, \infty)$

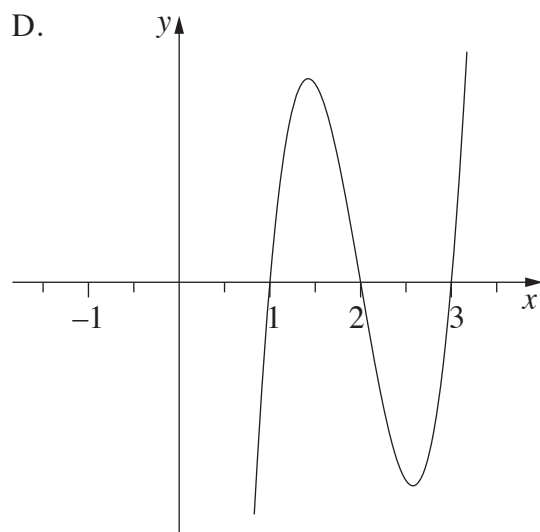
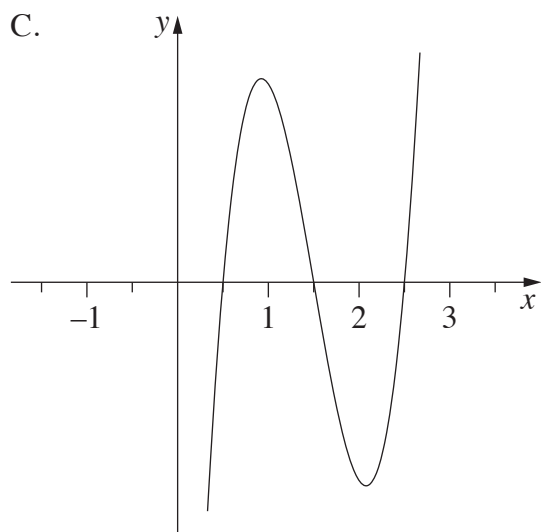
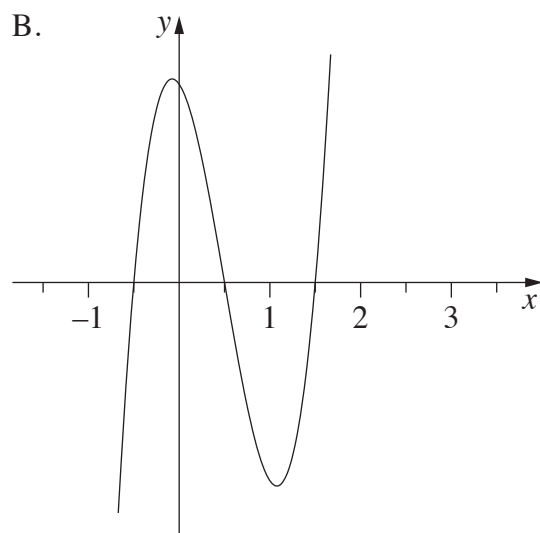
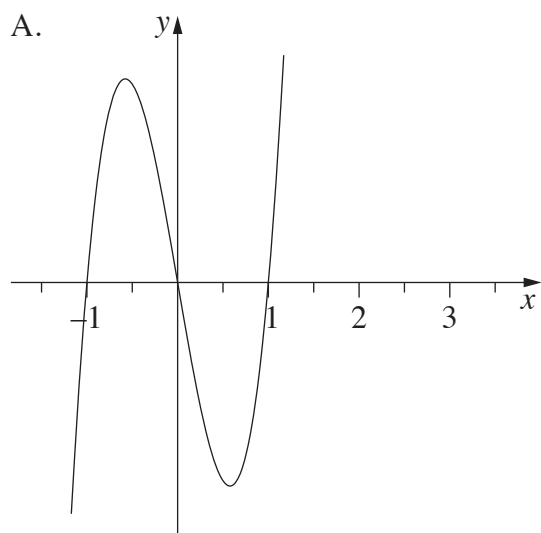
C. $(-1, 1)$

D. $(-\infty, -1) \cup (1, \infty)$

- 7 The diagram shows the graph $y = f(x)$.



Which of the following best represents the graph $y = f(2x - 1)$?

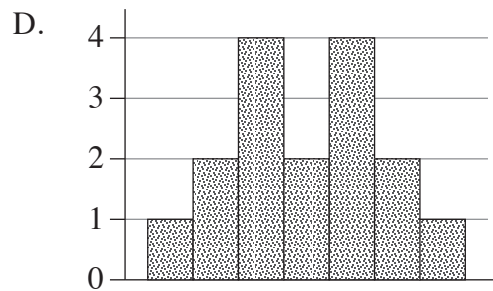
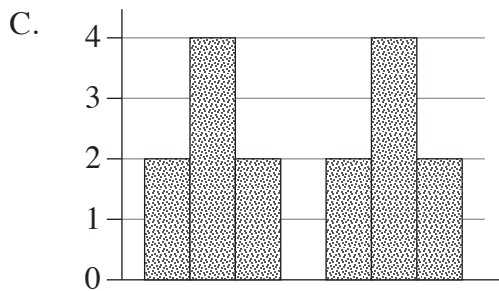
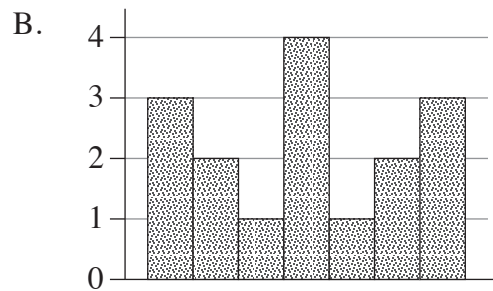
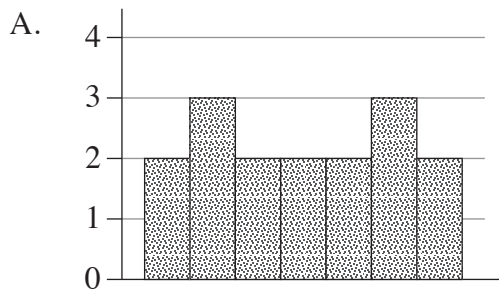


- 8 Some data are used to create a box plot shown.

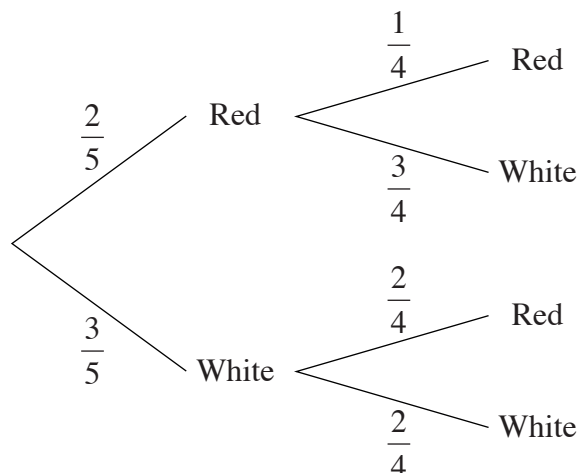


A histogram is created from the same set of data.

Which of these histograms is NOT possible for the given box plot?



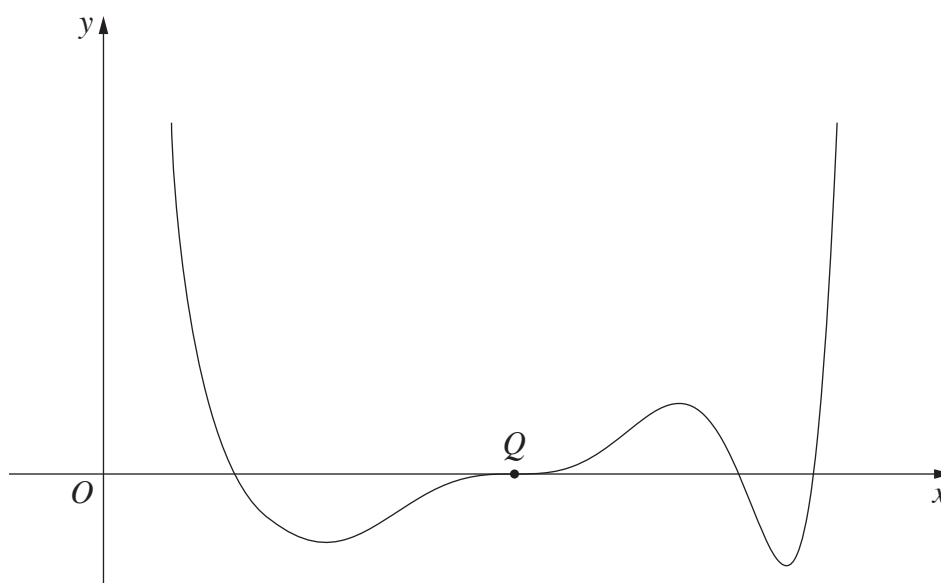
- 9 A bag contains 2 red and 3 white marbles. Jovan randomly selects two marbles at the same time from this bag. The probability tree diagram shows the probabilities for each of the outcomes.



Given that one of the marbles that Jovan has selected is red, what is the probability that the other marble that he has selected is also red?

- A. $\frac{1}{10}$
- B. $\frac{1}{7}$
- C. $\frac{1}{4}$
- D. $\frac{7}{10}$

- 10 The diagram shows the graph $y = f(x)$.



The point Q is a horizontal point of inflection.

Let $A(x) = \int_0^x f(t) dt$.

How many points of inflection does the graph $y = A(x)$ have?

- A. 2
- B. 3
- C. 4
- D. 5

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Centre Number

Mathematics Advanced

Section II Answer Booklet

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Student Number

90 marks

Attempt Questions 11–31

Allow about 2 hours and 45 minutes for this section

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

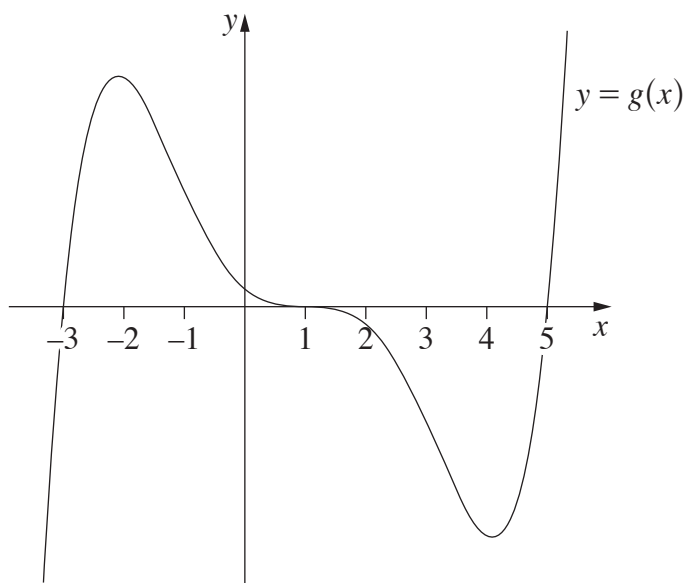
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Question 11 (3 marks)

The graph of the function $g(x)$ is shown.

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Using the graph, complete the table with the words *positive*, *zero* or *negative* as appropriate.

x -value	First derivative of $g(x)$ at x	Second derivative of $g(x)$ at x
$x = -3$		
$x = 1$		
$x = 5$		

Question 12 (3 marks)

Find the sum of the terms in the arithmetic series

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$$50 + 57 + 64 + \cdots + 2024.$$

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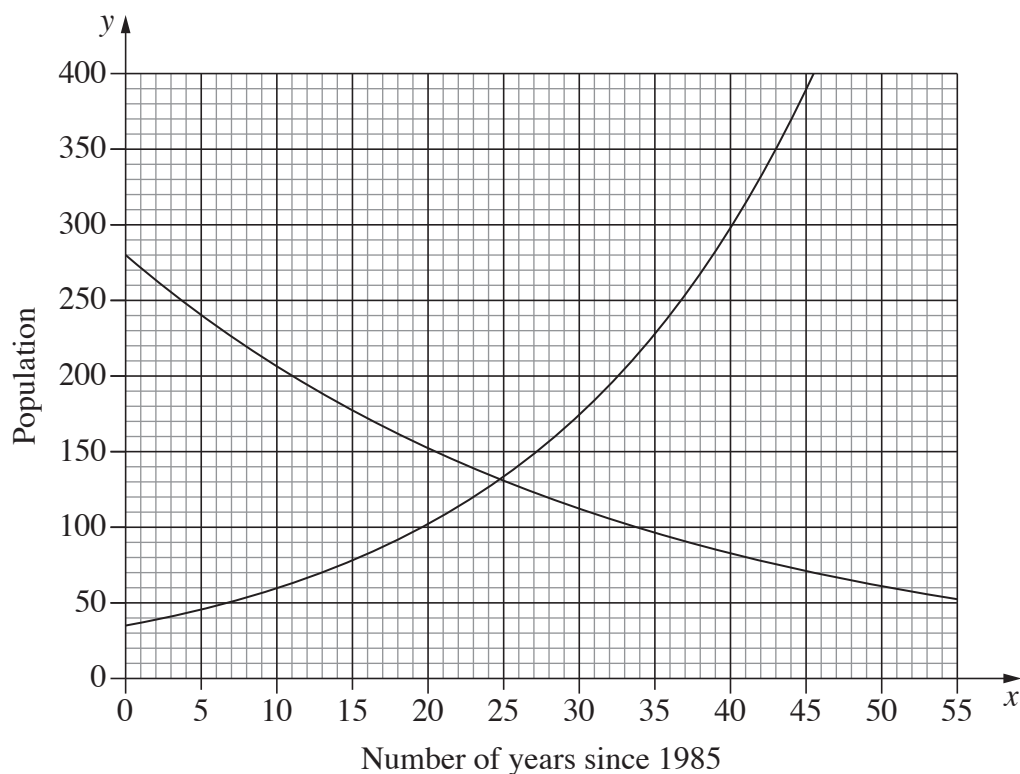
Question 13 (3 marks)

The graph shows the populations of two different animals, W and K , in a conservation park over time. The y -axis is the size of the population and the x -axis is the number of years since 1985.

3

Population W is modelled by the equation $y = A \times (1.055)^x$.

Population K is modelled by the equation $y = B \times (0.97)^x$.

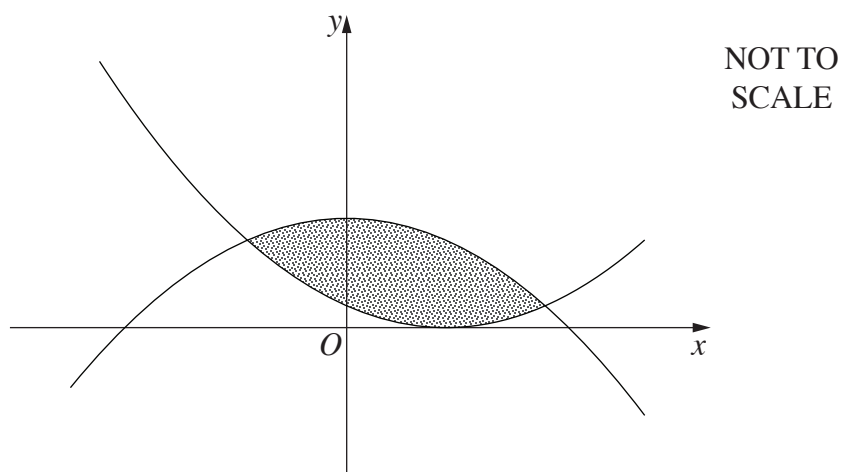


Complete the table using the information provided.

	<i>Population W</i>	<i>Population K</i>
Population in 1985	$A = 34$	$B =$
Percentage yearly change in the population		
Predicted population when $x = 50$		61

Question 14 (4 marks)

The curves $y = (x - 1)^2$ and $y = 5 - x^2$ intersect at two points, as shown in the diagram.



- (a) Find the x -coordinates of the points of intersection of the two curves.

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- (b) Find the area enclosed by the two curves.

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Question 15 (3 marks)

Initially there are 350 litres of water in a tank. Water starts flowing into the tank.

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The rate of increase of the volume V of water in litres is given by $\frac{dV}{dt} = 300 - 7.5t$, where t is the time in hours.

Find the volume of water in the tank when $\frac{dV}{dt} = 0$.

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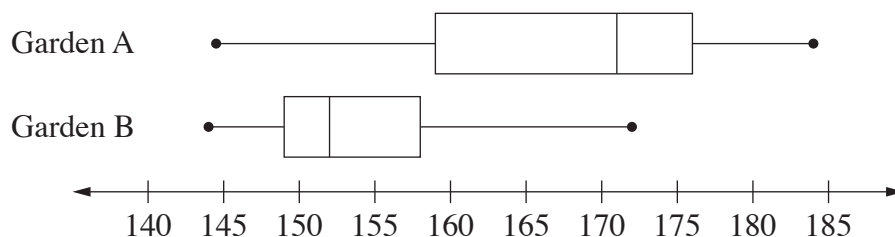
Question 16 (3 marks)

Flowers were planted in two gardens (Garden A and Garden B).

3

On a particular day, 25 flowers were randomly selected from each garden and their heights measured in millimetres.

The data are represented in parallel box-plots.



Compare the two datasets by examining the skewness of the distributions, and the measures of central tendency and spread.

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Question 17 (6 marks)

In a particular electrical circuit, the voltage V (volts) across a capacitor is given by

$$V(t) = 6.5(1 - e^{-kt}),$$

where k is a positive constant and t is the number of seconds after the circuit is switched on.

- (a) Draw a sketch of the graph of $V(t)$, showing its behaviour as t increases.

2

- (b) When $t = 1$, the voltage across the capacitor is 2.6 volts.

2

Find the value of k , correct to 3 decimal places.

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- (c) Find the rate at which the voltage is increasing when $t = 2$, correct to 3 decimal places.

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Question 18 (3 marks)

In a game, the probability that a particular player scores a goal at each attempt is 0.15.

- (a) What is the probability that this player does NOT score a goal in the first two attempts? **1**

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- (b) Determine the least number of attempts that this player must make so that the probability of scoring at least one goal is greater than 0.8. **2**

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Sketch the curve $y = x^4 - 2x^3 + 2$ by first finding all stationary points, checking their nature, and finding the points of inflection.

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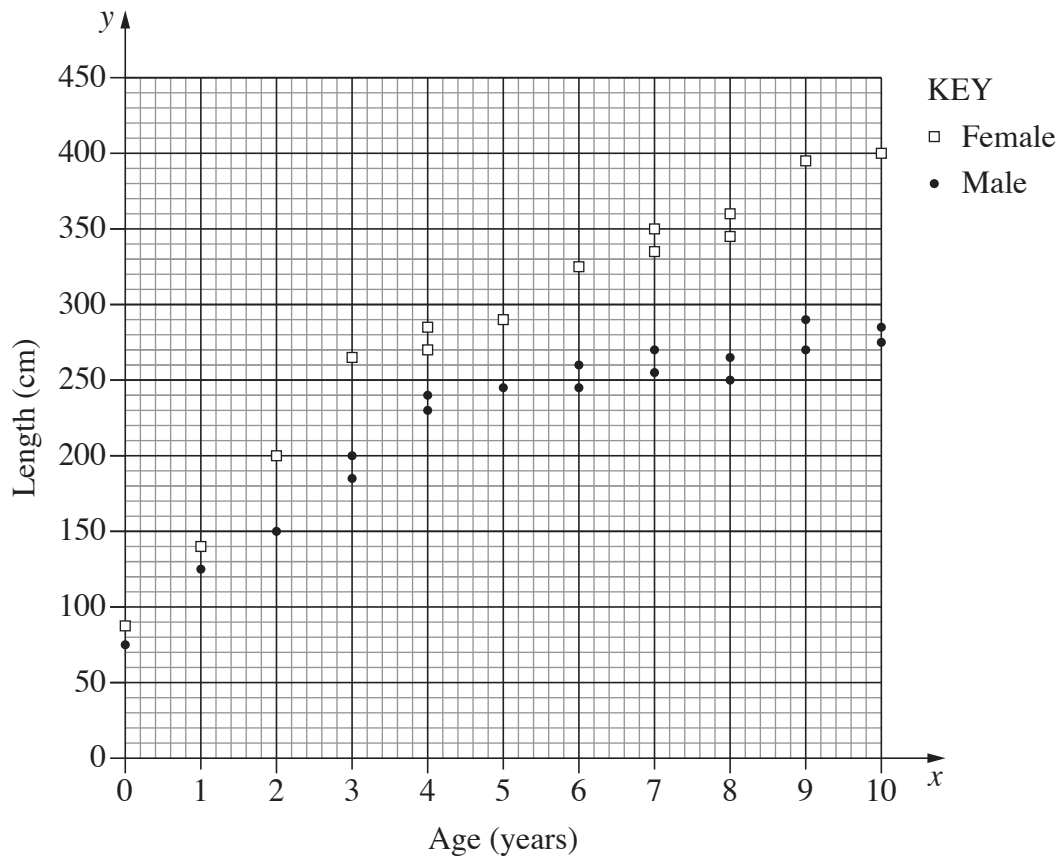
Question 21 (3 marks)

A researcher is studying anacondas (a type of snake).

3

A dataset recording the age (in years) and length (in cm) of female and male anacondas is displayed on the graph.

Anacondas reach maturity at about 4 years of age.



Write THREE observations about anacondas that may be made from the scatterplot. (Note: No calculations are required.)

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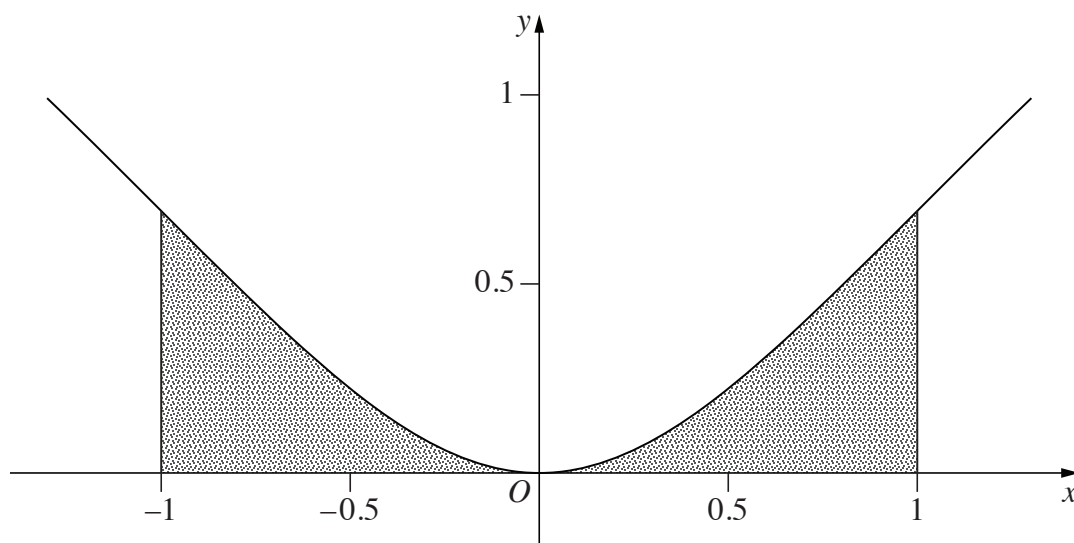
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Question 22 (6 marks)

The graph of the function $f(x) = \ln(1 + x^2)$ is shown.



- (a) Prove that $f(x)$ is concave up for $-1 < x < 1$.

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Question 22 continues on page 23

Question 22 (continued)

- (b) A table of function values, correct to 4 decimal places, for some x values is provided. 2

x	0	0.25	0.5	0.75	1
$\ln(1 + x^2)$	0	0.0606	0.2231	0.4463	0.6931

Using the function values provided and the trapezoidal rule, estimate the shaded area in the diagram.

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- (c) Is the answer to part (b) an overestimate or underestimate? Give a reason for your answer. 1

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End of Question 22

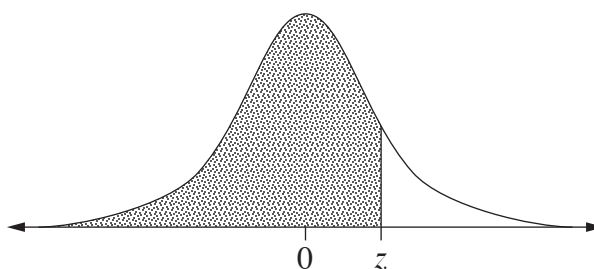
Questions 11–22 are worth 46 marks in total

Question 23 (5 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table gives the probability that this random variable is less than z .

z	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
<i>Probability</i>	0.7257	0.7580	0.7881	0.8159	0.8413	0.8643	0.8849	0.9032	0.9192

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



The scores in a university examination with a large number of candidates are normally distributed with mean 58 and standard deviation 15.

- (a) By calculating a z -score, find the percentage of scores that are between 58 and 70. 2

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- (b) Explain why the percentage of scores between 46 and 70 is twice your answer to part (a). 1

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- (c) By using the values in the table above, find an approximate minimum score that a candidate would need to be placed in the top 10% of the candidates. 2

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Question 24 (4 marks)

Jack intends to deposit \$80 into a savings account on the first day of each month for 24 months. The interest rate during this time is 6% per annum, compounded monthly.

- (a) Calculate how much money Jack will have in his account at the end of the 24 months.

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- (b) A table of future value interest factors could have been used to calculate how much money Jack would have in his account at the end of the 24 months.

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Part of the table is shown.

Table of future value interest factors for an annuity of \$1

<i>Number of periods</i>	<i>Interest rate per period</i>		
		0.5%	
24		A	

What is the value of A in the table? Give your answer correct to 3 decimal places.

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Question 25 (6 marks)

A function $f(x)$ is defined as

$$f(x) = \begin{cases} 0, & \text{for } x < 0 \\ 1 - \frac{x}{h}, & \text{for } 0 \leq x \leq h, \\ 0, & \text{for } x > h \end{cases}$$

where h is a constant.

- (a) Find the value of h such that $f(x)$ is a probability density function.

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- (b) By first finding a formula for the cumulative distribution function, sketch its graph.

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Question 25 continues on page 27

Question 25 (continued)

- (c) Find the value of the median of the probability density function $f(x)$. Give your answer correct to 3 decimal places.

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End of Question 25

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Question 26 (4 marks)

Twenty-five years ago, Phoenix deposited a single sum of money into a new bank account, earning 2.4% interest per annum compounding monthly.

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Present value interest factors for an annuity of \$1 for various interest rates (r) and numbers of periods (n) are given in the table.

<i>Rate (r)</i> <i>Period (n)</i>	<i>0.001</i>	<i>0.002</i>	<i>0.003</i>	<i>0.004</i>
60	58.207	56.487	54.835	53.249
120	113.026	106.592	100.649	95.156
180	164.655	151.036	138.927	128.137
240	213.278	190.460	170.908	154.093
300	259.071	225.430	197.627	174.521

Phoenix made the following withdrawals from this account.

- \$2000 at the end of each month for the first 15 years, starting at the end of the first month.
- \$1200 at the end of each month for the next 10 years, starting at the end of the 181st month after the account was opened.

Calculate the minimum sum that Phoenix could have deposited in order to make these withdrawals.

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Question 27 (5 marks)

- (a) Find the derivative of $x^2 \tan x$.

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- (b) Hence, find $\int (x \tan x + 1)^2 dx$.

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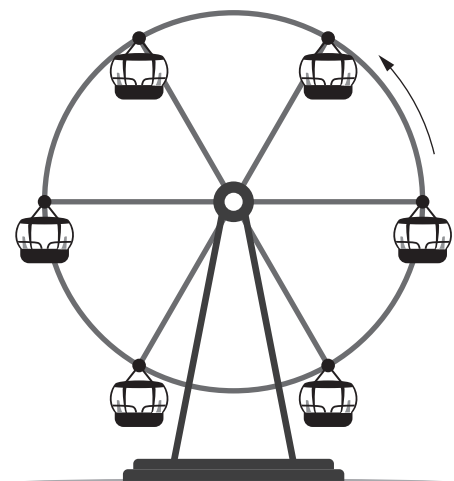
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Question 28 (7 marks)

Anna is sitting in a carriage of a Ferris wheel which is revolving. The height, $A(t)$, in metres above the ground of the top of her carriage is given by

$$A(t) = c - k \cos\left(\frac{\pi t}{24}\right),$$

where t is the time in seconds after Anna's carriage first reaches the bottom of its revolution and c and k are constants.



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The top of each carriage reaches a greatest height of 39 metres and a smallest height of 3 metres.

- (a) Find the value of c and k .

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- (b) How many seconds does it take for one complete revolution of the Ferris wheel?

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Question 28 continues on page 31

Question 28 (continued)

- (c) Billie is in another carriage. The height, $B(t)$, in metres above the ground of the top of her carriage is given by

4

$$B(t) = c - k \cos\left(\frac{\pi}{24}(t - 6)\right),$$

where c and k are as found in part (a).

During each revolution, there are two occasions when Anna's and Billie's carriages are at the same heights. At what two heights does this occur? Give your answer correct to 2 decimal places.

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End of Question 28

Question 29 (4 marks)

Consider the curve $y = ax^2 + bx + c$, where $a \neq 0$.

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At a particular point, the tangent and normal to the curve are given by $t(x) = 2x + 3$ and $n(x) = -\frac{1}{2}x - 2$ respectively.

The curve has a minimum turning point at $x = -4$.

Find the values of a, b and c .

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Question 30 (3 marks)

Suppose the geometric series $x + x^2 + x^3 + \dots$ has a limiting sum, S .

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By considering the graph $y = -1 - \frac{1}{x-1}$, or otherwise, find the range of possible values of S .

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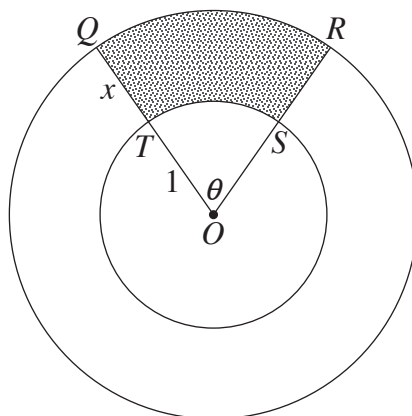
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Question 31 (6 marks)

Two circles have the same centre O . The smaller circle has radius 1 cm, while the larger circle has radius $(1 + x)$ cm. The circles enclose a region $QRST$, which is subtended by an angle θ at O , as shaded.

The area of $QRST$ is $A \text{ cm}^2$, where A is a constant and $A > 0$.



Let P cm be the perimeter of $QRST$.

- (a) By finding expressions for the area and perimeter of $QRST$, show that
- $$P(x) = 2x + \frac{2A}{x}.$$

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Question 31 continues on page 35

Question 31 (continued)

- (b) Show that if the perimeter, $P(x)$, is minimised, then θ must be less than 2.

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Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

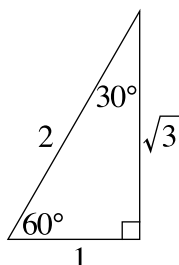
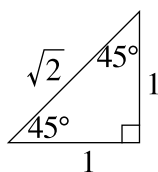
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1 + t^2}$$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2} (1 - \cos 2nx)$$

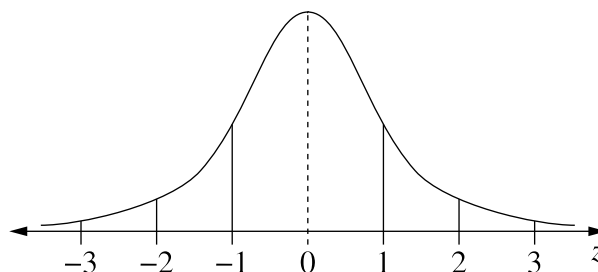
$$\cos^2 nx = \frac{1}{2} (1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z -scores between -1 and 1
- approximately 95% of scores have z -scores between -2 and 2
- approximately 99.7% of scores have z -scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function

Derivative

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

Integral Calculus

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \left\{ f(a) + f(b) + 2[f(x_1) + \dots + f(x_{n-1})] \right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \dots + \binom{n}{r}x^{n-r}a^r + \dots + a^n$$

Vectors

$$|\underline{u}| = |x\underline{i} + y\underline{j}| = \sqrt{x^2 + y^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1x_2 + y_1y_2,$$

$$\text{where } \underline{u} = x_1\underline{i} + y_1\underline{j}$$

$$\text{and } \underline{v} = x_2\underline{i} + y_2\underline{j}$$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

Complex Numbers

$$\begin{aligned} z &= a + ib = r(\cos \theta + i \sin \theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos \theta + i \sin \theta)]^n &= r^n(\cos n\theta + i \sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2024 HSC Mathematics Advanced Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	C
2	B
3	A
4	C
5	A
6	D
7	C
8	D
9	B
10	B

Section II

Question 11

Criteria	Marks
• Provides 6 correct entries	3
• Provides 4 correct entries	2
• Provides 2 correct entries	1

Sample answer:

x -value	First derivative at x	Second derivative at x
$x = -3$	Positive	Negative
$x = 1$	Zero	Zero
$x = 5$	Positive	Positive

Question 12

Criteria	Marks
• Provides correct solution	3
• Finds the value of n , or equivalent merit	2
• Finds the correct values of a or d , or equivalent merit	1

Sample answer:

$$T_1 = a = 50 \quad d = 7$$

$$T_n = a + (n - 1)d = 2024$$

$$50 + (n - 1)7 = 2024$$

$$(n - 1)7 = 1974$$

$$n - 1 = 282$$

$$n = 283$$

$$S_{283} = \frac{n}{2}(a + l)$$

$$= \frac{283}{2}(50 + 2024)$$

$$= 293\,471$$

Question 13

Criteria	Marks
• Completes the table correctly	3
• Provides two correct entries	2
• Provides one correct entry	1

Sample answer:

	<i>Population W</i>	<i>Population K</i>
Population in 1985	$A = 34$	$B = \mathbf{280}$
Percentage yearly change in the population	5.5%	-3%
Population when $x = 50$	494	61

$$\begin{aligned}
 &\uparrow \\
 &34(1.055)^{50} \\
 &= 494.426\dots \\
 &= 494
 \end{aligned}$$

Question 14 (a)

Criteria	Marks
• Finds x-coordinates of the points of intersection	1

Sample answer:

Points of intersection:

$$\begin{aligned}
 (x-1)^2 &= 5-x^2 \\
 x^2-2x+1 &= 5-x^2 \\
 2x^2-2x-4 &= 0 \\
 2(x-2)(x+1) &= 0 \\
 x &= 2, -1
 \end{aligned}$$

Question 14 (b)

Criteria	Marks
• Provides correct solution	3
• Finds the correct antiderivative	2
• Finds the correct integrand, or equivalent merit	1

Sample answer:

$$\begin{aligned}
 \text{Area} &= \int_{-1}^2 5-x^2-(x-1)^2 dx \\
 &= \int_{-1}^2 4-2x^2+2x dx \\
 &= \left[4x - \frac{2x^3}{3} + x^2 \right]_{-1}^2 \\
 &= \left(8 - \frac{16}{3} + 4 \right) - \left(-4 + \frac{2}{3} + 1 \right) \\
 &= 9
 \end{aligned}$$

Question 15

Criteria	Marks
• Provides correct solution	3
• Uses the initial volume to provide a correct expression for V	2
• Provides the correct anti-derivative, or equivalent merit	1

Sample answer:

$$\frac{dV}{dt} = 300 - 7.5t$$

$$V = 300t - \frac{7.5t^2}{2} + C$$

When $t = 0$, $V = 350$

$$350 = 0 - 0 + C$$

$$\therefore V = 300t - 3.75t^2 + 350$$

When $\frac{dV}{dt} = 0$

$$300 - 7.5t = 0$$

$$7.5t = 300$$

$$t = 40$$

When $t = 40$

$$V = 300 \times 40 - 3.75 \times 40^2 + 350$$

$$V = 6350 \text{ L}$$

Question 16

Criteria	Marks
• Compares the two datasets on skewness, central tendency and spread	3
• Compares two of the measures of centre and spread and skewness for the two gardens, or equivalent merit	2
• Compares skewness, measures of centre or spread for the two gardens, or equivalent merit	1

Sample answer:

The dataset for Garden A is negatively skewed while the dataset for Garden B is positively skewed.

The median for Garden A is higher than the median for Garden B.

The *IQR* of Garden A is larger than the *IQR* of Garden B.

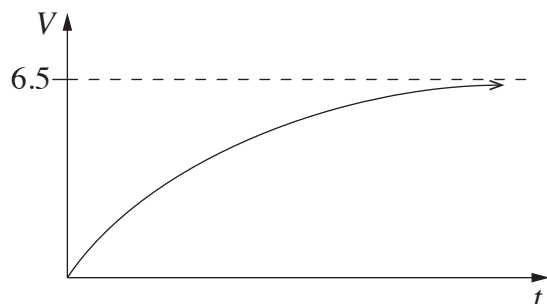
Answers could include:

The range of Garden A is larger than the range of Garden B.

Question 17 (a)

Criteria	Marks
• Provides correct graph, including labelled horizontal asymptote	2
• Provides a graph with correct shape, or equivalent merit	1

Sample answer:



Question 17 (b)

Criteria	Marks
• Provides correct solution	2
• Provides an expression with correct substitutions, or equivalent merit	1

Sample answer:

$$\begin{aligned}
 \text{When } t = 1, \quad 2.6 &= 6.5(1 - e^{-k}) \\
 1 - e^{-k} &= \frac{2.6}{6.5} \\
 1 - e^{-k} &= 0.4 \\
 e^{-k} &= 0.6 \\
 k &= -\ln(0.6) \\
 &= 0.511 \quad (3 \text{ decimal places})
 \end{aligned}$$

Question 17 (c)

Criteria	Marks
• Provides correct solution	2
• Finds the correct derivative, or equivalent merit	1

Sample answer:

$$\frac{dV}{dt} = 6.5ke^{-kt}$$

$$\begin{aligned}
 \text{When } t = 2, \quad \frac{dV}{dt} &= 6.5ke^{-2k} \\
 &= 1.195 \text{ volts/second}
 \end{aligned}$$

Question 18 (a)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Probability (player misses in 1 attempt)

$$= 1 - 0.15 = 0.85$$

Probability (player misses in both attempts)

$$= (0.85)^2 = 0.7225$$

Question 18 (b)

Criteria	Marks
• Provides correct solution	2
• Provides the correct inequality, $(0.85)^n < 0.2$, or equivalent merit	1

Sample answer:

Probability (player scores at least one goal)

$$= 1 - \text{Prob}(\text{player misses in } n \text{ attempts})$$

$$= 1 - (0.85)^n$$

When $1 - (0.85)^n > 0.8$

$$(0.85)^n < 0.2$$

Taking logs, $n \ln(0.85) < \ln(0.2)$

$$n > \frac{\ln(0.2)}{\ln(0.85)}$$

$$\div 9.90$$

The player needs 10 attempts.

Question 19

Criteria	Marks
• Provides correct graph	5
• Correctly identifies all points of inflection and stationary points	4
• Finds the nature of the stationary points	3
• Finds the x values of both stationary points, or equivalent merit	2
• Finds the first derivative, or equivalent merit	1

Sample answer:

$$y = x^4 - 2x^3 + 2$$

$$y' = 4x^3 - 6x^2$$

$$y'' = 12x^2 - 12x$$

Stationary points: $y' = 0$

$$4x^3 - 6x^2 = 0$$

$$2x^2(2x - 3) = 0$$

$$x = 0 \text{ or } x = \frac{3}{2}$$

When $x = 0$, $y = 2$ and $y'' = 0$

Check concavity:

x	-1	0	$\frac{1}{2}$
y''	24	0	-3

Concavity changes sign $\therefore$ Horizontal point of inflection at $(0, 2)$.

When $x = \frac{3}{2}$, $y = \frac{5}{16}$ and $y'' = 9$

$\therefore$ Local minimum at $\left(\frac{3}{2}, \frac{5}{16}\right)$

Points of inflection: $y'' = 0$

$$12x^2 - 12x = 0$$

$$12x(x - 1) = 0$$

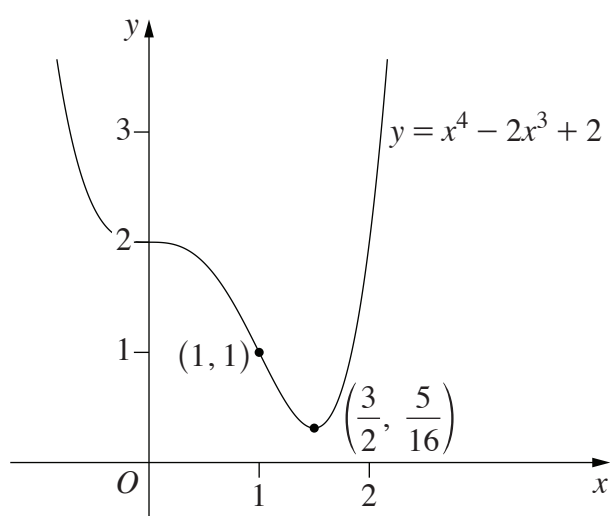
$$x = 0 \text{ or } x = 1$$

Already found $(0, 2)$

Check concavity:

x	$\frac{1}{2}$	1	2
y''	-3	0	24

Concavity changes sign $\therefore$ Point of inflection at $(1, 1)$.



Question 20 (a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\tan 35^\circ = \frac{40}{AC}$$

$$\therefore AC = \frac{40}{\tan 35^\circ}$$

$$= 57.1259\dots$$

$$= 57.13 \text{ metres (2 decimal places)}$$

Question 20 (b)

Criteria	Marks
• Provides correct solution	3
• Finds the angle at C, or equivalent merit	2
• Finds the length of BC, or equivalent merit	1

Sample answer:

$$\tan 30^\circ = \frac{40}{BC}$$

$$\therefore BC = \frac{40}{\tan 30^\circ}$$

$$= 69.28 \text{ metres (2 decimal places)}$$

Using cosine rule for $\angle ACB$:

$$\cos \angle ACB = \frac{57.13^2 + 69.28^2 - 100^2}{2 \times (57.13) \times (69.28)}$$

$$= -0.24462$$

$$\therefore \angle ACB = 104^\circ \text{ (nearest degree)}$$

$\therefore$ Bearing of B from C

$$= 104^\circ + 90^\circ$$

$$= 194^\circ$$

Question 21

Criteria	Marks
• Provides three correct observations	3
• Provides two correct observations	2
• Provides one correct observation	1

Sample answer:

- Female anacondas grow at a faster rate than males.
- Both females and males continue to grow after 4 years.
- Females are longer than males.

Question 22 (a)

Criteria	Marks
• Provides correct solution	3
• Finds the correct second derivative, or equivalent merit	2
• Finds the correct first derivative, or equivalent merit	1

Sample answer:

$$f(x) = \ln(1 + x^2)$$

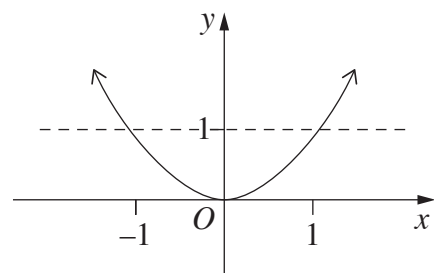
$$f'(x) = \frac{2x}{1 + x^2}$$

$$f''(x) = \frac{2(1 + x^2) - 2x(2x)}{(1 + x^2)^2}$$

$$= \frac{2 - 2x^2}{(1 + x^2)^2} > 0$$

When $2 - 2x^2 > 0$ (since denominator always positive)

ie when $x^2 < 1$



$\therefore$ when $-1 < x < 1$

Question 22 (b)

Criteria	Marks
• Provides correct solution	2
• Uses the values from the table correctly in the trapezoidal rule, or equivalent merit	1

Sample answer:

$$\begin{aligned}
 \int_{-1}^1 f(x) dx &= 2 \int_0^1 f(x) dx \\
 &\doteq 2 \times \frac{1-0}{2 \times 4} [0 + 0.6931 + 2(0.0606 + 0.2231 + 0.4463)] \\
 &= 0.538275
 \end{aligned}$$

Question 22 (c)

Criteria	Marks
• Provides correct answer with a reason	1

Sample answer:

From part (a) $f(x)$ is concave up when $-1 < x < 1$, so the straight line segments will lie above the curve.

So this is an overestimate.

Question 23 (a)

Criteria	Marks
• Provides correct solution, including a correct z-score	2
• Finds z-score corresponding to 70, or equivalent merit	1

Sample answer:

$$z = \frac{70 - 58}{15}$$

$$= 0.8$$

P from table = 0.7881

P required = $0.7881 - 0.5 = 0.2881$

$\therefore 28.81\%$

Question 23 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

The z -score for 46 is -0.8 .

By symmetry the area between 46 and 58 is equal to the area between 58 and 70.

So, the percentage of scores between 46 and 70 is twice 28.81%.

Question 23 (c)

Criteria	Marks
• Provides correct solution, with evidence of using the table	2
• Finds the z-score corresponding to 90th percentile, or equivalent merit	1

Sample answer:

$$1.3 = \frac{x - 58}{15}$$

$$x = 77.5$$

$\therefore$ Approximate minimum score = 77

Answers could include:

78

Question 24 (a)

Criteria	Marks
• Provides correct solution	3
• Sets up an appropriate series, or equivalent merit	2
• Finds a correct term in the series, or equivalent merit	1

Sample answer:

The first deposit accrues interest for 24 months

∴ Its future value is $80(1.005)^{24}$

The next deposit accrues interest for 23 months and so on

∴ Total amount after 24 months is

$$\begin{aligned}
 & 80(1.005)^{24} + 80(1.005)^{23} + \cdots + 80(1.005)^1 \\
 &= 80[(1.005)^1 + (1.005)^2 + \cdots + (1.005)^{24}] \\
 &= 80 \times (1.005) \left[\frac{(1.005)^{24} - 1}{1.005 - 1} \right] \\
 &= 2044.73
 \end{aligned}$$

Question 24 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$80A = 2044.73$$

$$\begin{aligned}
 A &= \frac{2044.73}{80} \\
 &= 25.559
 \end{aligned}$$

Question 25 (a)

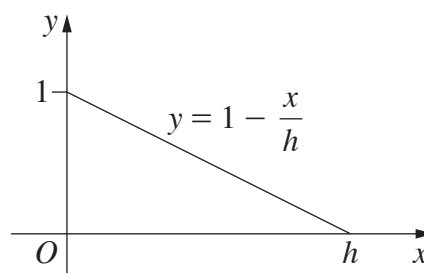
Criteria	Marks
• Provides correct solution	2
• Identifies that the area under the curve = 1, or equivalent merit	1

Sample answer:

We want the area of the triangle = 1

$$\therefore \frac{1}{2} \times 1 \times h = 1$$

$$h = 2$$



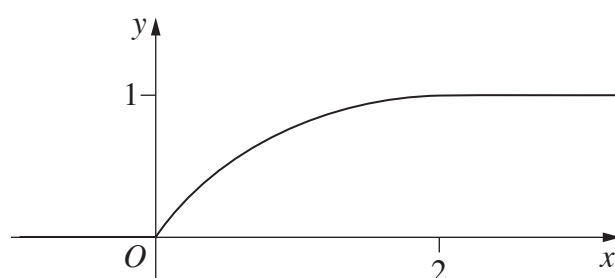
Question 25 (b)

Criteria	Marks
• Provides correct formula and sketch	2
• Finds the formula for the CDF, or equivalent merit	1

Sample answer:

$$\int_0^x 1 - \frac{t}{2} dx = \left[t - \frac{t^2}{4} \right]_0^x$$

$$= \begin{cases} 0, & \text{for } x < 0 \\ x - \frac{x^2}{4}, & \text{for } 0 \leq x \leq 2, \\ 1, & \text{for } x > 2 \end{cases}$$



Question 25 (c)

Criteria	Marks
• Provides correct solution	2
• Finds the correct quadratic equation, or equivalent merit	1

Sample answer:

$$t - \frac{t^2}{4} = \frac{1}{2}$$

$$\therefore t^2 - 4t + 2 = 0$$

$$t = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 2}}{2 \times 1}$$

$$t = \frac{4 \pm \sqrt{8}}{2}$$

$$t = 2 \pm \sqrt{2}$$

We need $t < 2$

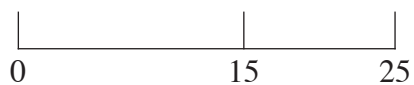
$$\begin{aligned}\therefore t &= 2 - \sqrt{2} \\ &\approx 0.586\end{aligned}$$

Question 26

Criteria	Marks
• Provides correct solution	4
• Makes substantial progress towards a solution	3
• Demonstrates in calculations understanding of importance of 15 years and 25 years	2
• Identifies the required interest rate, or equivalent merit	1

Sample answer:

$$r = \frac{2.4\%}{12} = 0.002$$



For 25 years need $1200 \times 225.430 = 270\,516$

For 15 years need extra $800 \times 151.036 = 120\,828.80$

Total = \$391 344.80

Question 27 (a)

Criteria	Marks
• Provides correct solution	2
• Applies the product rule, or equivalent merit	1

Sample answer:

$$y = x^2 \tan x$$

$$y' = x^2 \sec^2 x + 2x \tan x$$

Question 27 (b)

Criteria	Marks
• Provides correct solution	3
• Correctly uses the identity, $\tan^2 x = \sec^2 x - 1$	2
• Expands the integrand, or equivalent merit	1

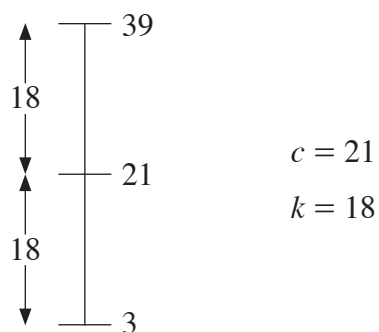
Sample answer:

$$\begin{aligned}
 \int (x \tan x + 1)^2 dx &= \int x^2 \tan^2 x + 2x \tan x + 1 dx \\
 &= \int x^2 (\sec^2 x - 1) + 2x \tan x + 1 dx \\
 &= \int x^2 \sec^2 x + 2x \tan x - x^2 + 1 dx \\
 &= x^2 \tan x - \frac{x^3}{3} + x + C
 \end{aligned}$$

Question 28 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the value of c or k , or equivalent merit	1

Sample answer:



Question 28 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\text{Period} = \frac{2\pi}{\frac{\pi}{24}} = 48$$

It takes 48 seconds.

Question 28 (c)

Criteria	Marks
• Provides a correct solution	4
• Finds $t = 3$, or equivalent merit	3
• Makes correct use of the periodic nature of cosine function	2
• Equates the appropriate trigonometric ratios, or equivalent merit	1

Sample answer:

We want $c - k \cos\left(\frac{\pi}{24}t\right) = c - k \cos\left(\frac{\pi}{24}(t - 6)\right)$

$$\therefore \cos\left(\frac{\pi}{24}t\right) = \cos\left(\frac{\pi}{24}(t - 6)\right)$$

$$\therefore \frac{\pi}{24}t = \frac{\pi}{24}(t - 6) \text{ which is impossible.}$$

or $-\frac{\pi}{24}t = \frac{\pi}{24}(t - 6)$

$$\therefore -\frac{\pi}{24}t = \frac{\pi}{24}t - \frac{\pi}{4}$$

$$\therefore \frac{\pi}{12}t = \frac{\pi}{4}$$

$$\therefore t = 3$$

or $2\pi - \frac{\pi}{24}t = \frac{\pi}{24}(t - 6)$

$$-\frac{2\pi}{24}t = -2\pi - \frac{\pi}{4}$$

$$\frac{\pi t}{12} = \frac{9\pi}{4}$$

$$\therefore t = 27$$

When $t = 3$ height is $21 - 18 \cos \frac{3\pi}{24} = 4.37 \text{ m}$

When $t = 27$ height is $21 - 18 \cos \frac{27\pi}{24} = 37.63 \text{ m}$

Question 29

Criteria	Marks
• Provides correct solution	4
• Finds the values of a and b	3
• Finds a relevant equation linking a and b , or equivalent merit	2
• Finds the x-ordinate of the point of intersection of the tangent and normal, or equivalent merit	1

Sample answer:

The tangent and normal meet where

$$2x + 3 = -\frac{1}{2}x - 2$$

$$\frac{5x}{2} = -5$$

$$x = -2$$

$$y = 2(-2) + 3$$

$$y = -1$$

$\therefore$ They meet at $(-2, -1)$.

If $y = ax^2 + bx + c$, then $y' = 2ax + b$

When $x = -4$ $y' = 0$

$$\therefore -8a + b = 0$$

$$\therefore b = 8a$$

When $x = -2$ $y' = 2$

$$-4a + b = 2$$

$$\therefore -4a + 8a = 2$$

$$\therefore a = \frac{1}{2}$$

$$\therefore b = 4$$

When $x = -2$ $y = -1$

$$\therefore a(-2)^2 + b(-2) + c = -1$$

$$\frac{1}{2} \times 4 + 4 \times -2 + c = -1$$

$$-6 + c = -1$$

$$\therefore c = 5$$

Question 30

Criteria	Marks
• Provides the correct solution	3
• Shows that the graph given is equal to S_{∞} , or equivalent merit	2
• Finds the limiting sum S , or equivalent merit	1

Sample answer:

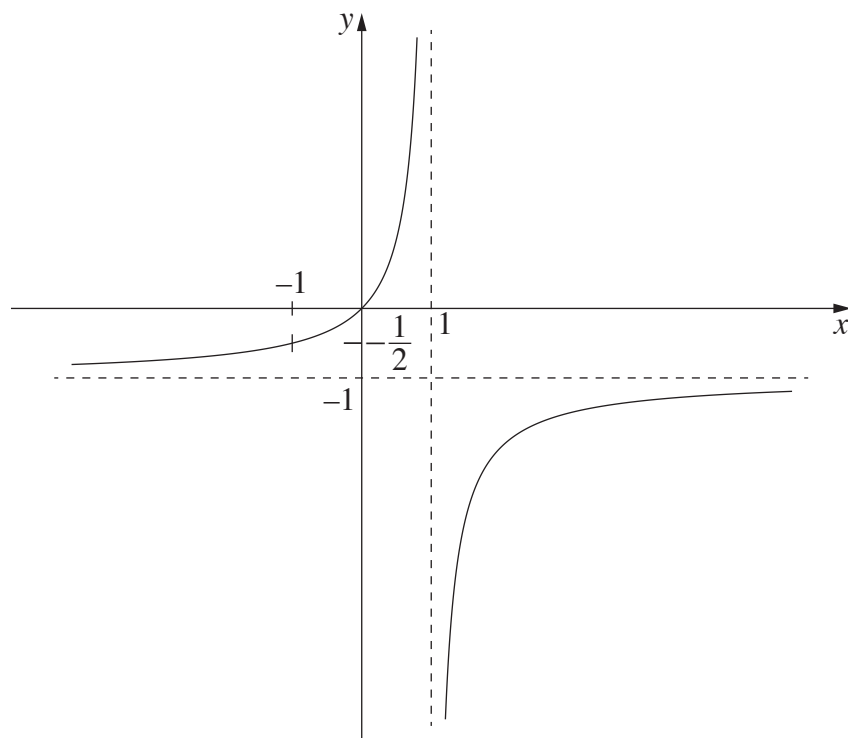
$$S = x + x^2 + x^3 + \dots$$

$$= \frac{x}{1-x} \quad \text{where } -1 < x < 1$$

$$y = -1 - \frac{1}{x-1} = \frac{-(x-1)-1}{x-1}$$

$$= \frac{-x}{x-1} = \frac{x}{1-x}$$

So, the graph of $y = \frac{x}{1-x}$ is as shown.



When $x = -1$ $y = -\frac{1}{2}$

From the graph, when $-1 < x < 1$, $y > -\frac{1}{2}$

Hence, $S > -\frac{1}{2}$

Question 31 (a)

Criteria	Marks
• Provides correct solution	3
• Finds correct expressions for the perimeter and area, or equivalent merit	2
• Finds the correct expression for the perimeter or area, or equivalent merit	1

Sample answer:

$$\begin{aligned}
 P &= 1\theta + (1+x)\theta + 2x \\
 &= (2+x)\theta + 2x \\
 A &= \frac{1}{2}(1+x)^2\theta - \frac{1}{2} \cdot 1^2\theta \\
 &= \frac{1}{2}\theta[1+2x+x^2-1] \\
 &= \frac{1}{2}\theta(2x+x^2) \\
 &= \frac{1}{2}\theta x(2+x) \quad \text{①}
 \end{aligned}$$

Since $P = (2+x)\theta + 2x$ and $(2+x)\theta = \frac{2A}{x}$

$$\therefore P(x) = 2x + \frac{2A}{x}$$

Question 31 (b)

Criteria	Marks
• Provides correct solution	3
• Expresses θ in terms of A , or equivalent merit	2
• Shows $x = \sqrt{A}$ is a minimum, or equivalent merit	1

Sample answer:

$$\begin{aligned}
 P' &= 2 - \frac{2A}{x^2} = 0 \text{ for minimum} \\
 \therefore x^2 &= A \quad \therefore x = \sqrt{A} \quad (x > 0) \\
 P'' &= \frac{4A}{x^3} > 0 \text{ when } x = \sqrt{A} \\
 \therefore &\text{ This gives a minimum value.}
 \end{aligned}$$

$$\text{Using ①} \quad A = \frac{1}{2}\theta\sqrt{A}(2+\sqrt{A})$$

$$\begin{aligned}
 \therefore \theta &= \frac{2A}{\sqrt{A}(2+\sqrt{A})} = \frac{2\sqrt{A}}{2+\sqrt{A}} < \frac{2\sqrt{A}}{\sqrt{A}} \\
 &< 2
 \end{aligned}$$

2024 HSC Mathematics Advanced Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	MA-F1 Working with Functions	MA11-1
2	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
3	1	MA-S3 Random Variables	MA12-8
4	1	MA-F2 Graphing Techniques	MA12-1
5	1	MA-C4 Integral Calculus	MA12-3
6	1	MA-F1 Working with Functions	MA11-2
7	1	MA-F2 Graphing Techniques	MA12-10
8	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
9	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
10	1	MA-C3 Applications of Differentiation	MA12-7

Section II

Question	Marks	Content	Syllabus outcomes
11	3	MA-C3 Applications of Differentiation	MA12-6
12	3	MA-M1 Modelling Financial Situations	MA12-4
13	3	MA-E1 Logarithms and Exponentials	MA11-6
14 (a)	1	MA-F1 Linear, quadratic and cubic functions	MA11-1
14 (b)	3	MA-C4 Integral Calculus	MA12-3
15	3	MA-C4 Integral Calculus	MA12-3
16	3	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-10
17 (a)	2	MA-F2 Graphing Techniques	MA12-1
17 (b)	2	MA-C3 Applications of Differentiation	MA12-3
17 (c)	2	MA-C3 Applications of Differentiation	MA12-3
18 (a)	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
18 (b)	2	MA-E1 Logarithms and Exponentials	MA11-6
19	5	MA-C3 Applications of Differentiation	MA12-6
20 (a)	1	MA-T1 Trigonometry and Measure of Angles	MA11-3
20 (b)	3	MA-T1 Trigonometry and Measure of Angles	MA11-3
21	3	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-10
22 (a)	3	MA-C2 Differential Calculus MA-C3 Applications of Differentiation	MA12-6
22 (b)	2	MA-C4 Integral Calculus	MA12-7
22 (c)	1	MA-C4 Integral Calculus	MA12-10
23 (a)	2	MA-S3 Random Variables	MA12-8
23 (b)	1	MA-S3 Random Variables	MA12-8
23 (c)	2	MA-S3 Random Variables	MA12-8

Question	Marks	Content	Syllabus outcomes
24 (a)	3	MA-M1 Modelling Financial Situations	MA12-4
24 (b)	1	MA-M1 Modelling Financial Situations	MA12-4
25 (a)	2	MA-S3 Random Variables	MA12-8
25 (b)	2	MA-S3 Random Variables	MA12-8
25 (c)	2	MA-S3 Random Variables	MA12-8
26	4	MA-M1 Modelling Financial Situations	MA12-9
27 (a)	2	MA-C2 Differential Calculus	MA12-6
27 (b)	3	MA-C4 Integral Calculus	MA12-7
28 (a)	2	MA-T3 Trigonometric Functions and Graphs	MA12-5
28 (b)	1	MA-T3 Trigonometric Functions and Graphs	MA12-5
28 (c)	4	MA-T3 Trigonometric Functions and Graphs	MA12-5
29	4	MA-C3 Applications of Differentiation	MA12-6
30	3	MA-F2 Graphing Techniques MA-M1 Modelling Financial Situations	MA12-9, MA12-10
31 (a)	3	MA-T1 Trigonometry and Measure of Angles	MA11-9
31 (b)	3	MA-C3 Applications of Differentiation	MA12-10