Barker College

Student Number

2020 YEAR 12 ASSESSMENT BLOCK SEMESTER 1

Mathematics Advanced

Section I – Multiple Choice

Write your student number at the top of this page.

Remove this sheet.

Select the alternative A, B C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = A 2 B 6 C 8 D 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

correct ↓

A ● B ⊗ C ○ D ○

Start Here →

1.	A○	B○	C○	D○	6.	A○	B○	C○	D○
2.	A○	B○	C○	D○	7.	A○	B○	C○	D○
3.	A○	B○	C○	D○	8.	A○	B○	C○	D○
4.	A○	B○	C○	D○	9.	A○	B○	C○	D○
5.	A○	B○	C○	D○	10.	A○	B○	C○	D○

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Barker College

Student Number

AXD AYG*
ESP LMD
JZT RJW*
ALY RAS
ARP

Friday, 19th June 2020
12 PM
120 minutes

Year 12
MATHEMATICS ADVANCED
Assessment Task #2

Discrete Probability Distributions, Graphs & Equations,
Curve Sketching using the Derivative, Integration,
The Exponential & Logarithmic Functions,
The Trigonometric Functions

145 copies

General Instructions

- Write using blue or black pen
- NESA approved calculators may be used
- Answer each question in the space provided.
- A reference sheet is provided separately
- In Questions **11-15**, show relevant mathematical reasoning and/or calculations
- Diagrams are NOT to scale

	Section	Marks
I	Multiple Choice	/ 10
II	Question 11	/ 14
	Question 12	/ 14
	Question 13	/ 14
	Question 14	/ 15
	Question 15	/ 13
	Total Marks:	**/ 80**

Section I

10 Marks

Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

1 The domain of $f(x) = \ln(x-1)$ in interval notation is given by:

(A) $(-\infty, \infty)$

(B) $(-1, \infty)$

(C) $(0, \infty)$

(D) $(1, \infty)$

2 Using the log laws, fully simplify: $12\log_a(\sqrt{a})$

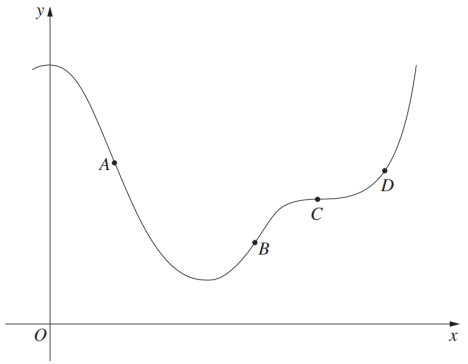
(A) 6

(B) 12

(C) $6\log_a(a)$

(D) $\sqrt{12}$

3 The diagram shows points A, B, C and D on the graph $y = f(x)$.



At which point is $f'(x) > 0$ and $f''(x) = 0$

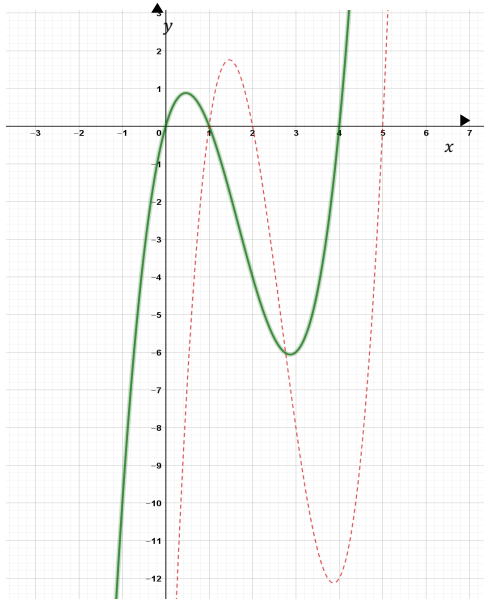
(A) A

(B) B

(C) C

(D) D

4 The graph of $y = f(x)$ is shown as a solid line.



Which of the following is a possible equation for the dotted line?

(A) $y = -2f(x)$

(B) $y = 2f(2x - 1)$

(C) $y = -2f(x - 1)$

(D) $y = 2f(x - 1)$

5 The gradient function of $y = x^3e^{2x}$ is given by:

(A) $3x^2e^{2x}$

(B) $6x^2e^{2x}$

(C) $3x^2e^{2x} + x^3e^{2x}$

(D) $x^2e^{2x}(2x + 3)$

6 The equation of the tangent to $y = \cos x$ at the point $(2\pi, 1)$ is given by:

(A) $y = -\sin(x) + 1$

(B) $y = x - 2\pi$

(C) $y = -\sin(x) - 2\pi$

(D) $y = 1$

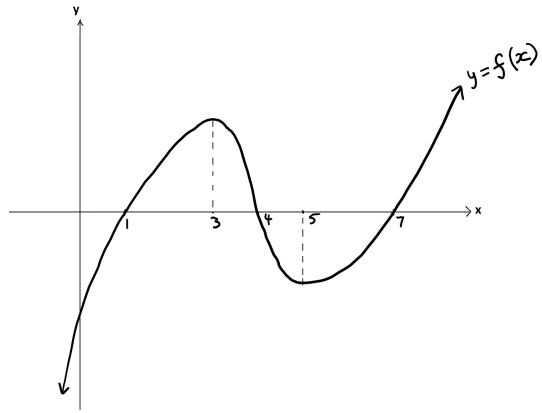
7 Using the sketch provided, determine when $f'(x) < 0$ in interval notation.

(A) $[1,4] \cup [7, \infty]$

(B) $(-\infty, 1) \cup (4,7)$

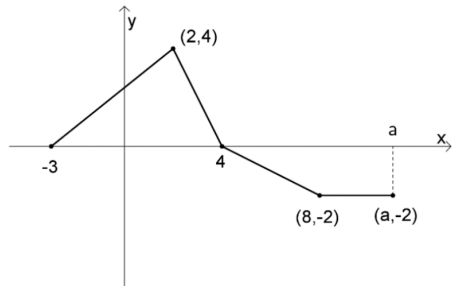
(C) $(3,5)$

(D) $(-\infty, 3) \cup (5, \infty)$



8 Using the graph of $y = f(x)$, determine the value of a which satisfies the condition:

$$\int_{-3}^{a} f(x)dx = 0$$



(A) 5

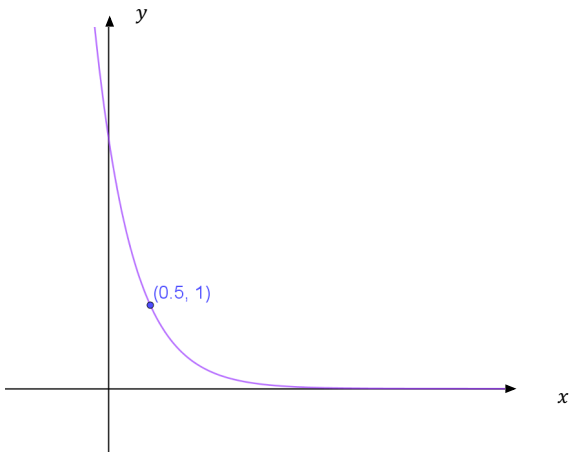
(B) 9.5

(C) 13

(D) 18

9 The graph below shows a translation and dilation of the curve $y = 3^{-x}$.

Which equation below matches the graph?



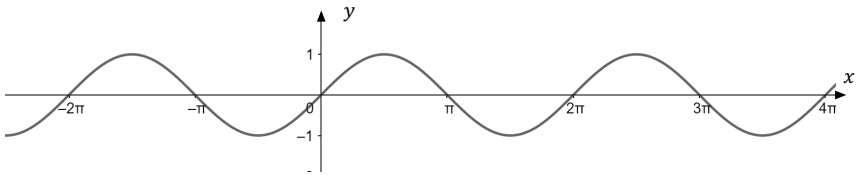
(A) $y = 3^{-2x+1}$

(B) $y = 3^{-2(x-1)}$

(C) $y = 3^{-2x-1}$

(D) $y = 3^{-2(x+1)}$

10 The graph of $y = \sin x$ is shown below.



Determine the maximum possible value of $\int_a^b \sin x \, dx$, where a and b are real numbers.

(A) 0

(B) 2

(C) ∞

(D) π/2

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Student Number

Section II

70 Marks
Attempt Questions 11-15
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate space provided.

In Questions 11-15, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (14 marks)

(a) Find these derivatives

(α) $\frac{d}{dx}(x^2-5)^6$ **1**

(β) $\frac{d}{dx}\frac{\ln(3x)}{e^x}$ **2**

(γ) $\frac{d}{dx}2^x e^x$ **2**

(b) Find the indefinite integrals

(α) $\int 5-x^3 dx$ **1**

Question 11 (continued)

(β) $\int (2x-1)^5 dx$ **2**

(γ) $\int \cos(3x-1)dx$ **2**

(c) Evaluate the following definite integrals

(α) **2**

$$\int_0^1 e^{5x}-1\,dx$$

(β) **2**

$$\int_{-2}^1 \frac{2}{x+3}dx$$

Student Number

Question 12 (14 marks)

(a) Consider the probability distribution below. **2**

(i) Show that $p = \frac{1}{12}$ and hence complete the discrete probability distribution table, assuming no other values of x are possible.

x	0	1	2	3	4	5	6	7
$P(X = x)$	p	$\frac{1}{12}$	$2p$	$2p$	$\frac{1}{6}$	$\frac{1}{6}$	p	$\frac{1}{12}$

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(ii) Find $E(X)$. **2**

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(iii) Calculate the $Var(X)$ **3**

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(iv) Calculate the standard deviation, to 2 decimal places. **1**

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Question 12 (continued)

(b)

(i) Sketch the curve $f(x) = -x(x-1)(x+3)$, showing x and y intercepts, in the space below. **2**

(ii) Hence or otherwise, solve $f(x) \leq 0$, giving your answer in interval notation. **1**

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(c) For a certain curve $y = f(x)$, $f''(x) = 6x - 1$. **3**

Find the equation of the curve given that it passes through the point $(1, -2)$ and has a gradient of -3 at that point.

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Student Number

Question 13 (14 marks)

(a) A surveyor measures the depth of a river at equal intervals across its width. **3**

The river is 18 m wide and the measurements are as follows

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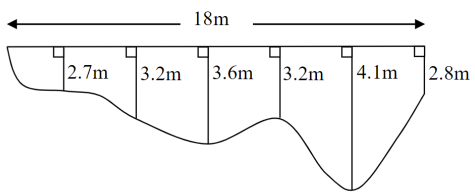

Using the trapezoidal rule with 7 function values, find the approximate cross – sectional area of the river.

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(b) Consider the function $f(x) = 2x^3 + x^2 + 5$. **2**

Find the values of x where the function is increasing.

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Question 13 (continued)

(c) Consider the function $y = x^3 - 3x^2 - 9x + 1$

(i) Find the stationary points and determine their nature. **4**

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(ii) Find the co-ordinates of the point of inflection. **2**

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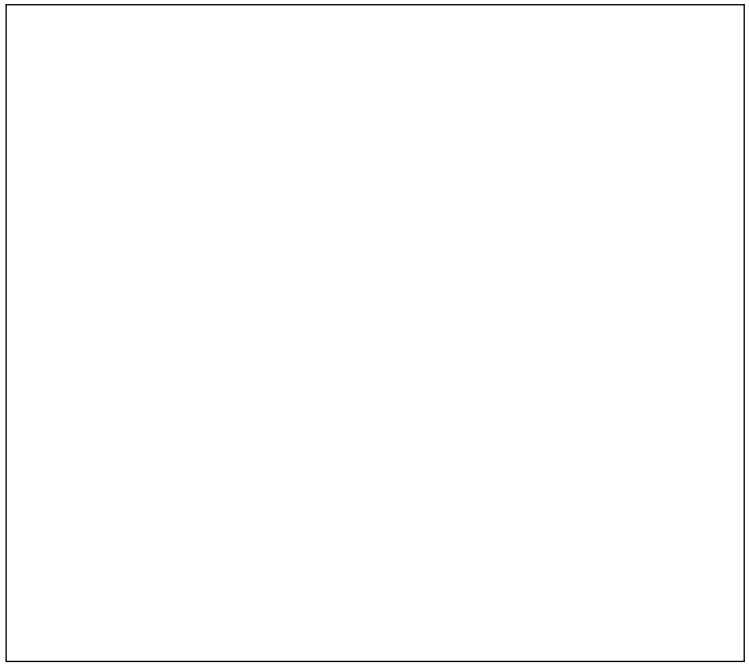
Question 13 (continued)

(iii) Sketch the graph $y = x^3 - 3x^2 - 9x + 1$ in the interval $[-3, 6]$ in the space below, showing all important features. (No need to find the x-intercepts) **2**



(iv) Find the maximum value of the function in the domain $[-3, 6]$. **1**

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Student Number

Question 14 (15 marks)

(a) The table below is a probability distribution table for a random variable X with $\mu = 23$.

x	10	a	30
$P(X = x)$	0.2	b	0.3

Find the values of a and b. **2**

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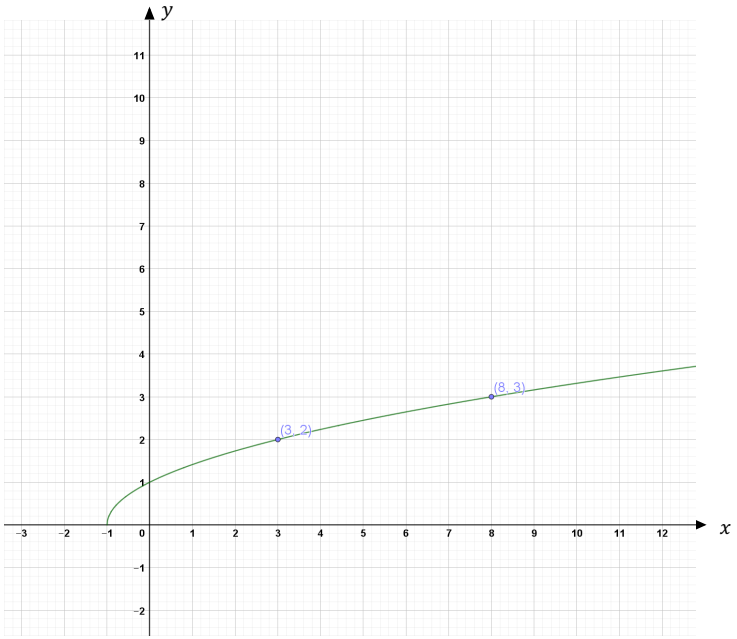
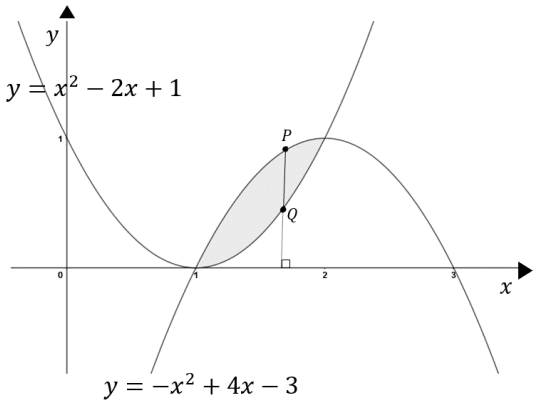
(b) The graph of $f(x) = \sqrt{x+1}$ is shown below. **3**

Use dilations to sketch $y = 3f(2x)$ on the same set of axes, labelling the image points of $(3, 2)$ and $(8, 3)$ and the y-intercept (points on the new graph which correspond to these points).



Question 14 (continued)

(c) The diagram shows the curves $y = x^2 - 2x + 1$ and $y = -x^2 + 4x - 3$



(i) Show that the curves intersect at the points where $x = 1$ and $x = 2$ **2**

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(ii) Hence, solve $x^2 - 2x + 1 < -x^2 + 4x - 3$. **1**

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(iii) If the length of PQ is l units, show that $l = -2x^2 + 6x - 4$. **1**

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Question 14 (continued)

(iv) Hence find the maximum length of PQ for $[1, 2]$. **3**

(v) Find the area of the shaded region enclosed between the two curves. **3**

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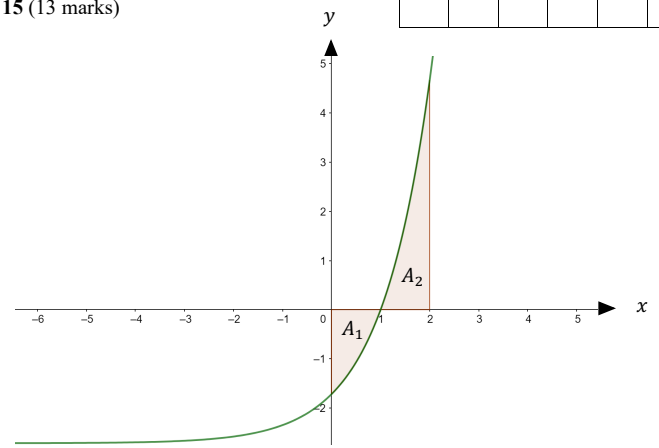
Student Number

Question 15 (13 marks)

(a)



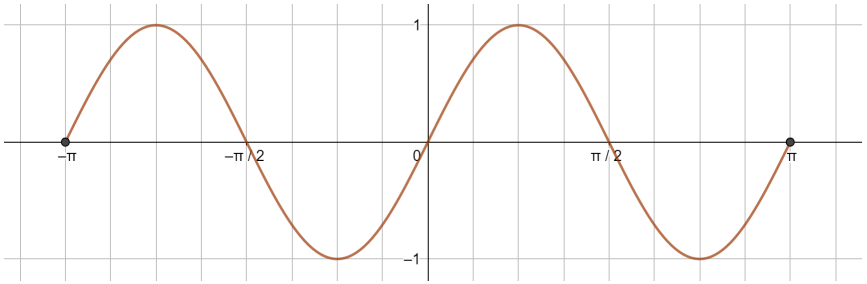
The graph of $y = e^x - e$ is shown above.

(i) Show that the x-intercept is at $(1, 0)$. **1**

(ii) Hence, or otherwise, find the total shaded area $(A_1 + A_2)$, in the diagram above. **3**
Give your answer in exact form.

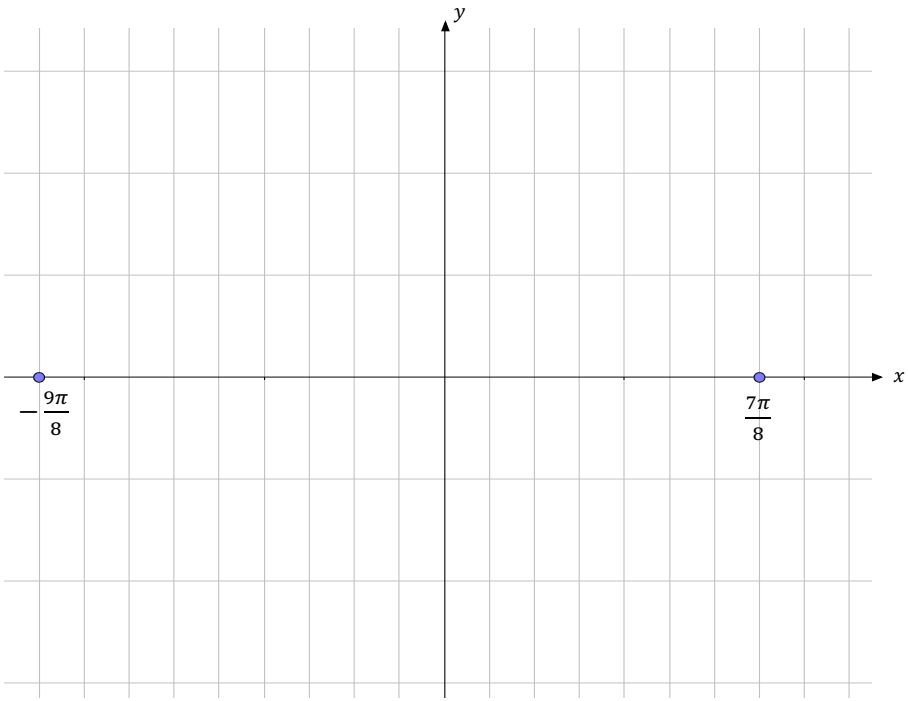
Question 15 (continued)

(b) The trigonometric graph below shows the function $y = \sin 2x$ in the interval $[-\pi, \pi]$



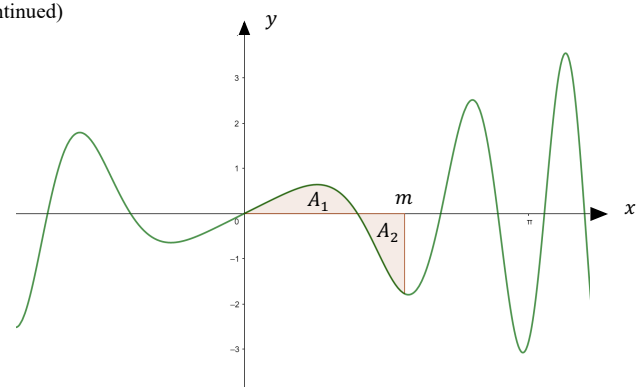
Hence, or otherwise sketch $y = 3\sin\left[2(x + \frac{\pi}{8})\right]$ in the interval $[-\frac{9\pi}{8}, \frac{7\pi}{8}]$ on the axes below.

Label all the intercepts and mark a scale on the y axis. **3**



Question 15 (continued)

(c)



The diagram above shows part of the curve $y = x\cos(x^2)$.

(i) Find $\frac{d}{dx}\sin(x^2)$. **1**

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(ii) Hence find $\int x\cos(x^2)dx$. **2**

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(iii) Given that $area\ A_1 = area\ A_2$, find the value of m marked on the diagram. **3**

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END OF ASSESSMENT

Year 2020 | Year 12 Mathematics Advanced Assessment Task 2 | Page 1

1. D. $\uparrow f(x)$ $(1,\infty)$
2. A. $12\log_a\sqrt{a} = 6$
3. B. Positive gradient and POI
4. D. $y = 2f(x-1)$
5. D. $u = x^3$ $v = e^{2x}$
   $u' = 3x^2$ $v' = 2e^{2x}$
   $y' = x^2e^{2x}(2x+3)$
6. D. $y' = -\sin x$
   $x = 2\pi$ $m = 0$
   $\therefore y = 1$
7. C. $(3,5)$
8. C. $A = 14 - 4 + 10$
   $\therefore a = 13$
9. A. $y = 3^{-2x+1}$
10. B. $\int_0^{\pi} \sin x\,dx = -\cos x\Big]_0^{\pi}$
    $= 2$
11. a)(a) $= 6(x^2-5)^5 \times 2x$
    $= 12x(x^2-5)^5$

    (b) $u = \ln 3x$ $v = e^x$
    $u' = \frac{1}{x}$ $v' = e^x$
    $y' = \frac{\frac{e^x}{x} - e^x\ln 3x}{e^{2x}}$
    $= \frac{1 - x\ln 3x}{xe^x}$

    (c) $u = 2^x$ $v = e^x$
    $u' = \ln 2\,2^x$ $v' = e^x$
    $y' = 2xe^x(\ln 2 + 1)$

    b)(a) $= 5x - \frac{x^4}{4} + C$

    (b) $= \frac{(2x-1)^6}{6\times 2} + C$
    $= \frac{(2x-1)^6}{12} + C$

    (c) $= \frac{e^{5x}}{5} - x\Big]_0^1$
    $= \frac{e^5}{5} - 1 - \frac{1}{5}$
    $= \frac{e^5}{5} - \frac{6}{5}$

    (b) $= 2\ln(x+3)\Big]_{-2}^{1}$
    $= 2(\ln 4 - \ln 1)$
    $= 2\ln 4$
    $= 4\ln 2$

12. a)(i) $6p + \frac{1}{2} = 1$
    $6p = \frac{1}{2}$
    $p = \frac{1}{12}$

    (ii) $E(X) = 0 + \frac{1}{12} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{8} + \frac{6}{12} + \frac{7}{12}$
    $= 3.5$

    (iii) $= E(X^2) - \mu^2$
    $= \frac{97}{6} - 3.5^2$
    $= 3.92$

    (iv) $= \sqrt{3.92}$
    $= 1.98$

    b)(i)

    (ii) $[-3,0] \cap [1,\infty)$

    c)

c) $f'(x) = 3x^2 - x + c$

$f'(1) = 2 + c = -3$

$c = -5$

$\therefore f'(x) = 3x^2 - x - 5$

$f(x) = x^3 - \frac{x^2}{2} - 5x + d$

$f(1) = 1 - \frac{1}{2} - 5 + d = -2$

$d = 2.5$

$\therefore f(x) = x^3 - \frac{x^2}{2} - 5x + \frac{5}{2}$

13. a) Area $\doteq \frac{3}{2}(0 + 2 \cdot 8 + 2(16 \cdot 8))$

$\doteq 54.6 m^2$

b) $f'(x) = 6x^2 + 2x > 0$

$2x(3x+1) > 0$

$x < -\frac{1}{3}, x > 0$

c)(i) $y' = 3x^2 - 6x - 9 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

$y = -26, 6$

$y'' = 6x - 6$

$y''(3) = 12 \therefore (3, -26)$ max

$y''(-1) = -12 \therefore (-1, 6)$ min

(ii) $y'' = 6x - 6 = 0$

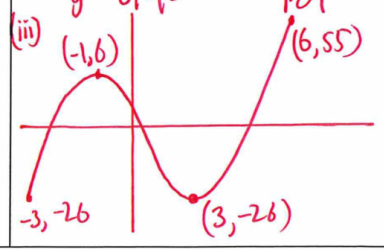
$x = 1$

$y = -10$

x	0	1	2
y''	-6	0	6

$\therefore (1, -10)$ is a POI

(iii)



(iv) $x = 6 \quad y = 55$

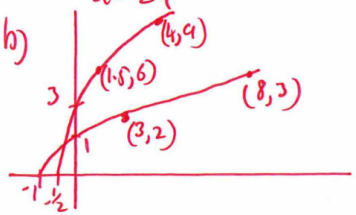
14. a) $0.2 + b + 0.3 = 1$

$b = 0.5$

$23 = 2 + \frac{a}{2} + 9$

$a = 24$

b)



c)(i) $x^2 - 2x + 1 = -x^2 + 4x - 3$

$(x-2)(x-1) = 0$

$\therefore x = 1, 2$

(ii) $1 < x < 2$

(iii) L = top − bottom

$= -2x^2 + 6x - 4$

(iv) $\frac{dL}{dx} = -4x + 6 = 0$

$x = \frac{3}{2}$

$\frac{d^2L}{dx^2} = -4 \therefore x = \frac{3}{2}$ max

Max $= -2\left(\frac{3}{2}\right)^2 + 6\left(\frac{3}{2}\right) - 4$

$= \frac{1}{2}$ unit

(v) $A = \int_1^2 \left(-2x^2 + 6x - 4\right) dx$

$= -\frac{2x^3}{3} + 3x^2 - 4x \Big]_1^2$

$= -\frac{16}{3} + 12 - 8 - \left(-\frac{2}{3} + 3 - 4\right)$

$= \frac{1}{3} u^2$

15 a) (i) $e^x - e = 0$

$x = 1$

$\therefore 1, 0$ is x int

(ii) $A_1 = \left| \int_0^1 e^x - e \, dx \right|$

$= \left| e^x - ex \right]_0^1 \Big|$

$= |e - e - 1|$

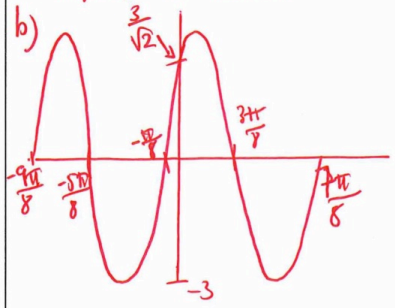
$= 1$

$A_2 = e^x - ex \Big]_1^2$

$= e^2 - 2e$

$A_1 + A_2 = e^2 - 2e + 1$

b)



c) (i) $2x \cos x^2$

(ii) $\int x \cos x^2 \, dx = \frac{1}{2} \int 2x \cos x^2 \, dx$

$= \frac{1}{2} \sin x^2 + C$

(iii) $\int_0^m x \cos x^2 \, dx = 0$

$\therefore \frac{\sin x^2}{2} \Big]_0^m = 0$

$\sin m^2 = 0$

$\therefore m^2 = 0, \pi, 2\pi \ldots$

$m = \sqrt{\pi}$

1st value