
Student Number

Barker College

2021 YEAR 12

ASSESSMENT TASK 3

Mathematics Advanced

Staff Involved:

Wednesday 2nd June, 8:30am

- AYG* • LZM • LAK • DXC • PJR
- RJW • PDJ • DZP • ARP

135 copies

General Instructions

- Working time – 55 minutes
- Write using blue or black pen
- Write your Barker Student number in the spaces indicated
- Write your answers in the spaces provided
- Board approved calculators may be used
- The HSC Reference Sheet is provided.
- Attempt all questions
- ALL necessary working must be shown
- Marks may not be awarded for careless or badly arranged work
- Diagrams are NOT to scale

Topics Covered

- Exponential and Logarithmic Functions
- Trigonometric Functions

Section	Marks
Question 1	/ 12
Question 2	/ 15
Question 3	/ 12
Question 4	/ 12
Total Marks:	**/ 51**

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Question 1 (12 marks)

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(a) Differentiate with respect to x

(α) $y = \ln(7x - 2)$ **1**

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(β) $y = (e^x + x)^3$ **2**

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(γ) $y = \frac{\cos x}{x^2}$ **2**

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(b) Find the following indefinite integrals:

(α) $\int \cos 5x \, dx$ **1**

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(β) $\int \frac{1-2x^5}{x} dx$ **2**

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(c) Evaluate, writing your answer in simplest form.

(α) $\int_1^{e^3} \frac{5}{x} \, dx$ **2**

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(β) $\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$ **2**

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END OF QUESTION 1

Question 2 (15 marks)

Student number

(a) Find the equation of the normal to the curve $y = \ln(x - 1) + 3$ at the point $(2,3)$ on the curve. **3**

(b) Differentiate $y = \log_{10}(\sin x)$ **2**

(c) The function $f(x)$ has derivative $f'(x) = 8\sin(4x)$ and $f\left(\frac{\pi}{4}\right) = 1$. **3**

Find $f(x)$.

(d) (i) Find $\frac{d}{dx}(x^2 \ln x)$ **2**

(ii) Hence, or otherwise find $\int 3x(1 + \ln x^2)\ dx$ **2**

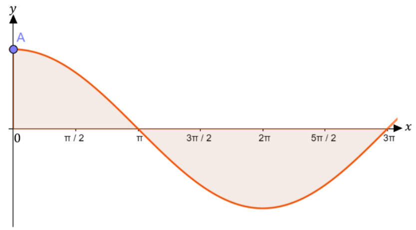
(e) Evaluate $\int_0^1 \frac{x-1}{x^2-2x+4}\ dx$ **3**

END OF QUESTION 2

Question 3 (12 marks)

Student number

(a) The shaded area between the curve $y = 2\cos\frac{x}{2}$ and the $x-$axis between $x = 0$ and $x = 3\pi$ is shown below. The sum of the shaded areas is equal to 12.

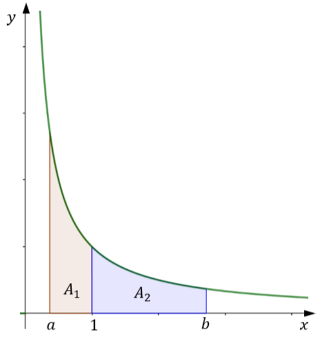


(i) What are the co-ordinates of A? **1**

(ii) What is the value of $\int_0^{3\pi} 2\cos\frac{x}{2}\,dx$? **1**

(b) Find $\int 2^x \ln 16\,dx$. Express your answer in simplest index form. **3**

(c) The diagram shows the curve $y = \frac{1}{x}$, for $x > 0$. **3**



The area under the curve between $x = a$ and $x = 1$ is A_1. The area under the curve between $x = 1$ and $x = b$ is A_2.

The areas A_1 and A_2 are each equal to 1 square unit.

Find the values of a and b.

(d) (i) Show that: $\log_x 2 = \frac{1}{\log_2 x}$ **1**

(ii) Hence, solve the equation $\log_2 x = 4\log_x 2$ **3**

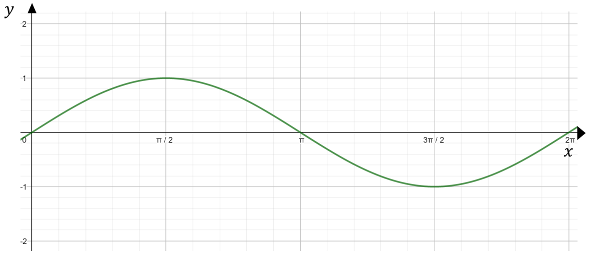
END OF QUESTION 3

Student number

Question 4 (12 marks)

(a) The curve $y = \sin x$ is shown.

(i) Sketch the curve $y = \sqrt{3}\cos x$ below on the same axes. **1**



(ii) Show that a point of intersection of $y = \sqrt{3}\cos x$ and $y = \sin x$

is $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$ **2**

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(iii) Hence, find the simplest exact area bounded by the curve $y = \sqrt{3}\cos x$,

$y = \sin x$ and the $x -$ axis between ordinates $x = 0$ and $x = \frac{\pi}{2}$. **3**

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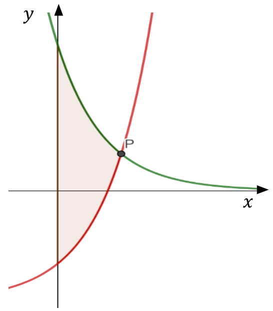
(b) The diagram shows the curves $y = 4e^{-x}$ and $y = e^{x} - 3$.



(i) The curves intersect at point P. Show the x co-ordinate of the point P is $x = \ln 4$ **3**

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(ii) Find the exact shaded area bounded by the two curves and the $y -$ axis, in simplest form **3**

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END OF ASSESSMENT

Student number

Extra writing space

If you use this space, clearly indicate which question you are answering.

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2021 Year 12 Mathematics Advanced Assessment Task 3 Student Solutions

Question 1.

a) α) $y = \ln(7x-2)$

$$\frac{dy}{dx} = \frac{7}{7x-2}$$

β) $y = (e^x + x)^3$

$$\frac{dy}{dx} = 3(e^x+x)^2 \times (e^x+1)$$
$$= 3(e^x+x)^2(e^x+1)$$

γ) $y = \dfrac{\cos x}{x^2}$ $\quad u = \cos x$, $u' = -\sin x$, $v = x^2$, $v' = 2x$

$$\therefore \frac{dy}{dx} = \frac{-\sin x \times x^2 - 2x \times \cos x}{x^4}$$
$$= \frac{-x(x\sin x + 2\cos x)}{x^4}$$
$$= \frac{-(x\sin x + 2\cos x)}{x^3}$$

b) α) $\displaystyle\int \cos 5x \, dx$

$$= \frac{1}{5}\sin 5x + C$$

b) β) $\displaystyle\int \frac{1-2x^5}{x} \, dx$

$$= \int \frac{1}{x} - 2x^4 \, dx$$
$$= \ln x - \frac{2x^5}{5} + C$$

c) α) $\displaystyle\int_1^{e^3} \frac{5}{x} \, dx$

$$= \Big[5\ln x\Big]_1^{e^3}$$
$$= 5\ln e^3 - 5\ln 1$$
$$= 5 \times 3 = 15$$

β) $\displaystyle\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$

$$= \frac{1}{2}\Big[\tan 2x\Big]_0^{\frac{\pi}{8}}$$
$$= \frac{1}{2}\left(\tan\frac{\pi}{4} - \tan 0\right)$$
$$= \frac{1}{2}(1-0) = \frac{1}{2}$$

Question 2.

a) Find the equation of the normal to $y = \ln(x-1) = 3$ at $(2,3)$

$$\frac{dy}{dx} = \frac{1}{x-1}$$

$\therefore$ gradient of tangent $= \dfrac{1}{2-1} = 1$

$\therefore$ gradient of normal $= -1$

$$\therefore y - 3 = -1(x-2)$$
$$y - 3 = -x + 2$$
$$\boxed{y = -x + 5}$$

b) $y = \log_{10}(\sin x)$

$$\frac{dy}{dx} = \frac{1}{\ln 10 \times \sin x} \times \cos x$$
$$= \frac{\cos x}{\ln 10 \times \sin x} = \frac{\cot x}{\ln 10}$$

c) $f'(x) = 8\sin(4x)$ & $f(\frac{\pi}{4}) = 1$

$$\therefore f(x) = \int f'(x)\,dx$$
$$= \int 8\sin(4x)\,dx$$
$$= -2\cos 4x + C$$

$f(\frac{\pi}{4}) = 1 \Rightarrow 1 = -2\cos\pi + C$

$$\therefore 1 = 2 + C$$
$$C = -1$$

$\therefore f(x) = -2\cos(4x) - 1.$

d) i) $\dfrac{d}{dx}(x^2 \ln x)$

$$= 2x\ln x + x^2 \times \frac{1}{x}$$
$$= 2x\ln x + x$$
$$= x(2\ln x + 1)$$

ii) $\displaystyle\int 3x(1 + \ln x^2)\,dx$

$$= 3\int x(1 + 2\ln x)\,dx$$
$$= 3x^2\ln x + C$$

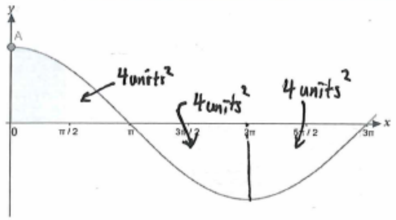
e) $\displaystyle\int_0^1 \frac{x-1}{x^2-2x+4}\,dx$

$$= \frac{1}{2}\int_0^1 \frac{2x-2}{x^2-2x+4}\,dx$$
$$= \frac{1}{2}\Big[\ln|x^2-2x+4|\Big]_0^1$$
$$= \frac{1}{2}\Big(\ln|1-2+4| - \ln 4\Big)$$
$$= \frac{1}{2}(\ln 3 - \ln 4)$$
$$= \frac{1}{2}\ln\left(\frac{3}{4}\right)$$

Question 3.

a) i) $A = (0, 2)$

ii)



$$\therefore \int_0^{3\pi} 2\cos\frac{x}{2}\,dx = 4 - 4 - 4$$
$$= -4$$

b) $\int 2^x \ln 16 \, dx$

$$= \frac{\ln 16 \times 2^x}{\ln 2} + C$$

$$= \frac{4\cancel{\ln 2} \times 2^x}{\cancel{\ln 2}} + C$$

$$= 4 \times 2^x + C$$

$$= 2^2 \times 2^x + C = 2^{x+2} + C$$

c) $\int_a^1 \frac{1}{x}\,dx = 1$

$$\therefore \left[\ln x\right]_a^1 = 1$$
$$\ln 1 - \ln a = 1$$
$$-\ln a = 1$$
$$\ln a = -1$$
$$\therefore a = e^{-1}$$

$$\int_1^b \frac{1}{x}\,dx = 1$$
$$\left[\ln x\right]_1^b = 1$$
$$\ln b - \ln 1 = 1$$
$$\ln b = 1$$
$$b = e^1 = e$$

d) i) $\log_x 2 = \frac{1}{\log_2 x}$

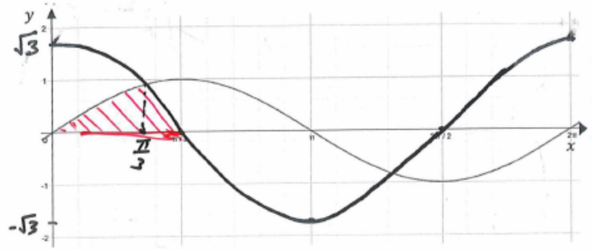
$$LHS = \log_x 2 = \frac{\log_2 2}{\log_2 x} = \frac{1}{\log_2 x} = RHS$$

ii) $\log_2 x = 4\log_x 2$

$$\therefore \log_2 x = \frac{4}{\log_2 x}$$
$$(\log_2 x)^2 = 4$$
$$\therefore \log_2 x = \pm 2$$
$$\therefore x = 2^2 = 4 \text{ or } x = 2^{-2} = \frac{1}{4}$$

Question 4.

a) i)



ii) $\sqrt{3}\cos x = \sin x$

$$\frac{\sin x}{\cos x} = \sqrt{3}$$
$$\tan x = \sqrt{3}$$
$$\therefore x = \frac{\pi}{3}$$

$$y = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$\therefore \left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$ is a point of intersection.

iii) $$\text{Area} = \int_0^{\frac{\pi}{3}} \sin x\,dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{3}\cos x\,dx$$

$$= \left[-\cos x\right]_0^{\frac{\pi}{3}} + \sqrt{3}\left[\sin x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \left(-\frac{1}{2} + 1\right) + \sqrt{3}\left(1 - \frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{2} + \sqrt{3} - \frac{3}{2} = (\sqrt{3} - 1)\text{ units}^2$$

b)

i) $4e^{-x} = e^x - 3$

$$\frac{4}{e^x} = e^x - 3$$
$$4 = e^{2x} - 3e^x$$
$$e^{2x} - 3e^x - 4 = 0$$
$$\therefore (e^x + 1)(e^x - 4) = 0$$

$\therefore e^x = -1$ or $e^x = 4$

No-solution. $x = \ln 4$

$\therefore$ Only one point of intersection at $x = \ln 4$.

ii) $$\text{Area} = \int_0^{\ln 4} 4e^{-x} - (e^x - 3)\,dx$$

$$= \int_0^{\ln 4} 4e^{-x} - e^x + 3\,dx$$

$$= \left[-4e^{-x} - e^x + 3x\right]_0^{\ln 4}$$

$$= (-4e^{-\ln 4} - e^{\ln 4} + 3\ln 4) - (-4e^0 - e^0 + 3 \times 0)$$

$$= -4 \times \frac{1}{4} - 4 + 3\ln 4 + 5$$

$$= -1 - 4 + 3\ln 4 + 5$$

$$= 3\ln 4 \text{ units}^2$$
$$= 6\ln 2 \text{ units}^2$$