Student Number

Barker College

2021 YEAR 12 ASSESSMENT BLOCK

Mathematics Advanced

Staff Involved: FRIDAY 26th MARCH, NOON

- AYG • LZM • LAK • DXC • PJR
- RJW* • PDJ • DZP • ARP

140 copies

TOPICS COVERED:	
Year 11 Content	Sequences & Series
Series & Finance	Graphs & Equations
Curve Sketching using the Derivative	Integration

General Instructions:

- Working time - 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A separate reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and / or calculations
- Marks may not be awarded for careless or poorly arranged working

Total marks: 80

Section I - 8 marks (pages 2 - 4)

- Attempt Multiple Choice Questions 1 - 8
- Allow about 15 minutes for this section

Section II - 72 marks (pages 5 - 22)

- Attempt Questions 9 - 17
- Show all necessary working
- Allow about 1 hours and 45 minutes for this section

Section I

8 marks
Attempt Questions 1 – 8
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 8.

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1 Using the log laws, fully simplify: $9\log_a(\sqrt[3]{a})$

A. 3

B. 27

C. $\sqrt[3]{9}$

D. $3\log_a(a)$

2 The sum of the first 10 terms of the series: $100 - 80 + 64 - \ldots$ is closest to:

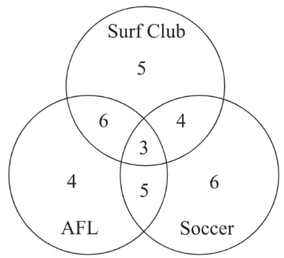
A. -55

B. 50

C. 55

D. 446

3 In a classroom, students are asked what sports club they are members of and the results are shown in the Venn diagram.



A student who is a member of a soccer club is chosen at random. What is the probability that he/she is also a member of a surf club?

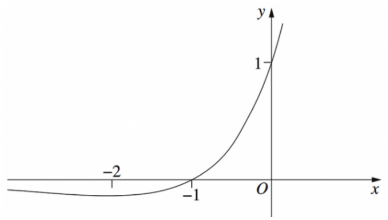
A. $\frac{2}{5}$

B. $\frac{4}{11}$

C. $\frac{2}{9}$

D. $\frac{7}{18}$

4 The diagram shows the graph of $y = e^x(1 + x)$



How many solutions are there to the equation $e^x(1 + x) = 1 - x^2$?

A. 0

B. 1

C. 2

D. 3

5 A circle with centre (*a*, -2) and radius 5 units has equation $x^2 - 6x + y^2 + 4y = b$ where *a* and *b* are real constants.

The values of *a* and *b* are respectively:

A. -3 and 38

B. 3 and 12

C. -3 and -8

D. 3 and 18

6 Let $f(x) = x^2$

Which one of the following is **not** true

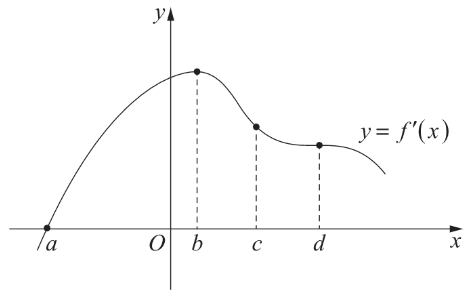
A. $f(ab) = f(a)f(b)$

B. $f(a) - f(-a) = 0$

C. $f(2a) = 4f(a)$

D. $f(a - b) = f(a) - f(b)$

7 The diagram shows the graph of $f'(x)$, the derivative of a function.



For what value of *x* does the graph of the function $f(x)$ have a point of inflexion?

A. $x = a$

B. $x = b$

C. $x = c$

D. $x = d$

8 The function $y = f(x)$ has a turning point at (4, 2). It is transformed to $y = -2f(2x + 6) + 3$. Where is the new turning point?

A. (-1, -1)

B. (-5, -2)

C. (-5, -1)

D. (1, -1)

End of Section I

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Section II

72 marks
Attempt Questions 9 – 17
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate space provided.

In Questions 9 – 17, your responses should include relevant mathematical reasoning and/or calculations.

Question 9 (8 marks)

(a) Find these derivatives

(α) $\frac{d}{dx}(3x^2-4)^6$ **1**

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(β) $\frac{d}{dx}xe^x$ **1**

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(γ) $\frac{d}{dx}\frac{(x-1)^4}{x^2}$, leaving your answer in simplest factorised form. **3**

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Question 9 (continued)

(b) Find the indefinite integrals

(α) $\int 6x-2x^3dx$ **1**

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(β) $\int (3x-2)^{-4}dx$ **2**

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Question 10 (8 marks)

On any given day, the number $\boldsymbol{X}$ of telephone calls that Luke receives is a random variable with probability distribution given in the table below. **2**

(i) Show that $p = 0.1$ and hence complete the discrete probability distribution table, assuming no other values of x are possible.

x	0	1	2	3	4	5
$P(X = x)$	$p =$	0.15	$2p =$	$3p =$	0.2	0.05

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(ii) Find $E(X)$. **2**

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(iii) What is the probability that Luke receives more than the mean number of phone calls on a given day? **1**

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(iv) Calculate the $Var(X)$. **2**

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(v) Calculate the standard deviation, to 2 decimal places. **1**

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Question 11 (8 marks)

(a)

(i) Sketch the curve $f(x) = (x-1)(x+1)(x+3)$, showing x and y intercepts, in the space below. No need to find turning points. **2**

(ii) Hence or otherwise, solve $f(x) \leq 0$, giving your answer in interval notation. **2**

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Question 11 (continued)

(b) Evaluate the following given that $f(x) = \sqrt{1-x}$ and $g(x) = \frac{1}{x}$

(i) $g(f(-3))$ **1**

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(ii) Determine the domain and range of $g(f(x))$ **3**

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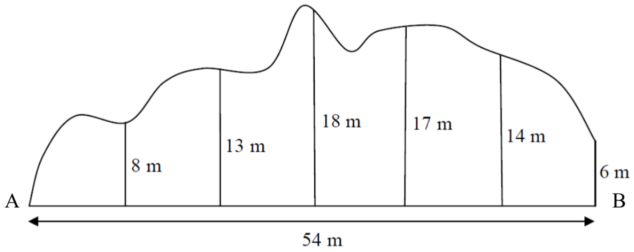
Question 12 (8 marks)

(a) The sketch shows a piece of land which borders the side of a straight road AB. **3**

The length of AB is 54m. At equal intervals along AB, perpendicular measurements are made to the boundary as shown in the sketch.



Use the trapezoidal rule with 7 function values to find the approximate area of the piece of land.

Question 12 (continued)

(b) (i) Prove the identity $(1-\sin\theta)(1+\sin\theta) = \cos^2\theta$ **2**

(ii) Hence solve the equation $(1-\sin\theta)(1+\sin\theta) = 0.5$ for $0° \le \theta \le 360°$ **3**

Student Number

Question 13 (8 marks)

Consider the curve $y = x^3 - \frac{3x^2}{2} - 6x + 5$

(i) Find the stationary points and determine their nature. **3**

(ii) Find the co-ordinates of the point of inflection and prove it is a point of inflection. **2**

Question 13 (continued)

(iii) Sketch the curve $y = x^3 - \frac{3x^2}{2} - 6x + 5$ in the interval $[-3, 4]$ in the space below, showing all important features. (No need to find the *x*-intercepts) **2**

(iv) Find the maximum value of $x^3 - \frac{3x^2}{2} - 6x + 5$ in the domain $[-3, 4]$. **1**

Student Number

Question 14 (8 marks)

(a) State the transformations that have occurred to the following graphs from the originals. The order may or may not be important.

(i) From $y = \ln x$ to $y = -\ln(x + 3)$ **1**

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(ii) From $y = e^x$ to $y = 2e^x - 5$ **1**

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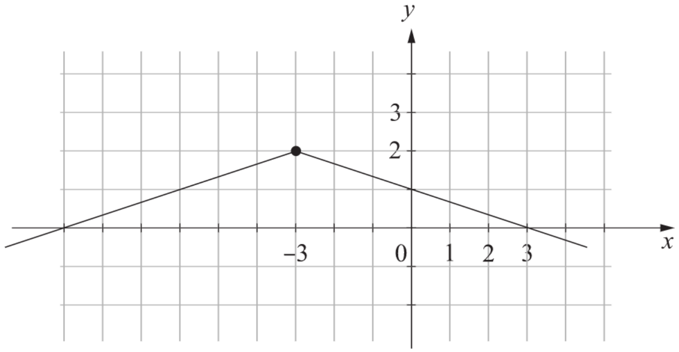
(b) The function $f(x) = |x|$ is transformed and the equation of the new function is $y = kf(x + b) + c$.

The graph of the new function is shown below.



Determine the values of k, b and c. **2**

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Question 14 (continued)

(c) (i) Use differentiation by first principles to find $f'(x)$ given $f(x) = 2x^2 + 5x$. **2**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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(ii) Hence, find the equation of the tangent to the curve $y = 2x^2 + 5x$ at the point on the curve where $x = 1$. **2**

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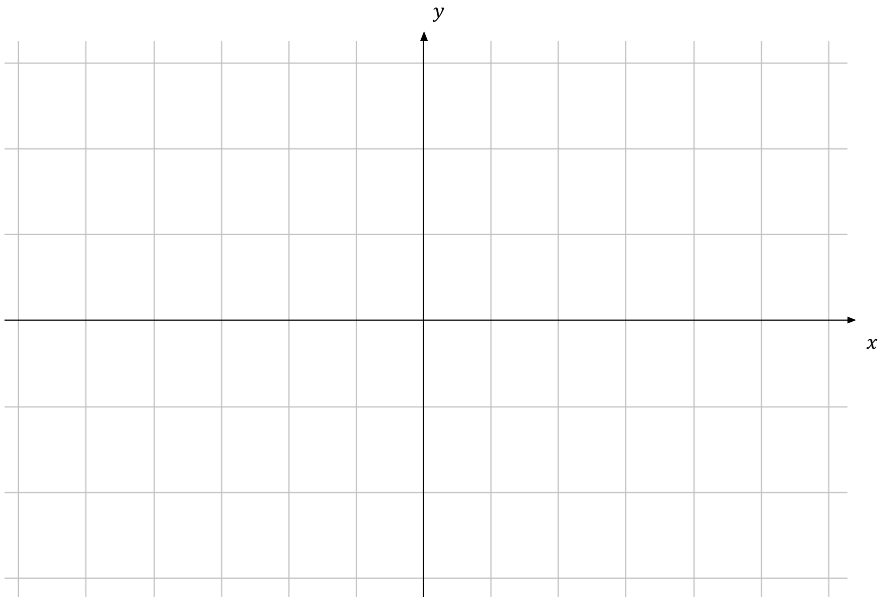
Question 15 (8 marks)

(a) Sketch the curve $y = 2\cos x$ in the interval [-2π, 2π]. Label x and y intercepts. **2**



Question 15 (continued)

(b) Lucy borrows \$500 000 to be repaid at a reducible interest rate of 0.6% per month. Let $\$A_n$ be the amount owing at the end of n months and $\$M$ be the monthly repayment.

(i) Show that $A_2 = 500000(1.006)^2 - M(1 + 1.006)$ **1**

(ii) Show that $A_n = 500000(1.006)^n - M(\frac{1.006^n - 1}{0.006})$ **2**

(iii) Lucy budgets a monthly repayment of \$3500. Determine how many months it would take her to pay off the loan. **3**

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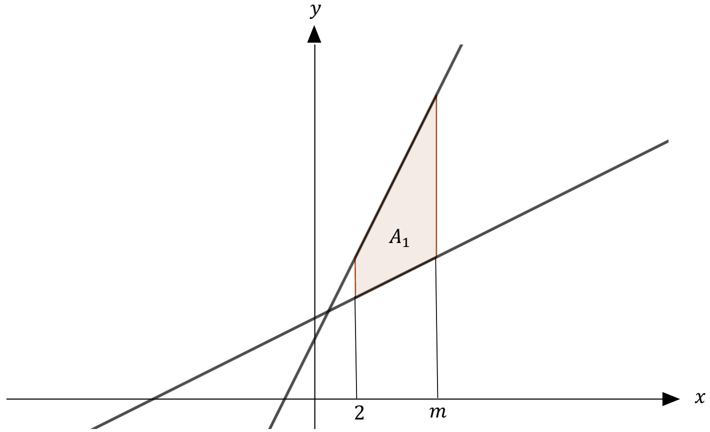
Question 16 (8 marks)

(a)

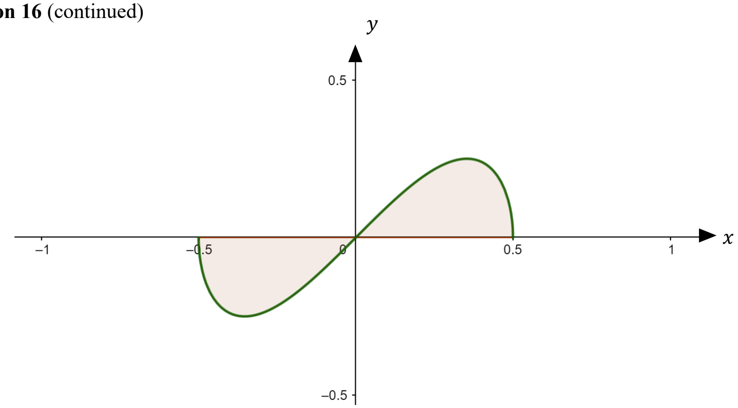


The diagram above shows part of the lines $y = 2x + 3$ and $y = \frac{x}{2} + 4$

Given that *area* $A_1 = 20\ units^2$, find the value of m marked on the diagram. **3**

Question 16 (continued)

(b)



The diagram above shows the curve $y = x\sqrt{1 - 4x^2}$.

(i) Find $\frac{d}{dx}(1 - 4x^2)^{\frac{3}{2}}$ and express your answer in surd form. **2**

(ii) Hence, find the shaded area shown on the diagram **3**

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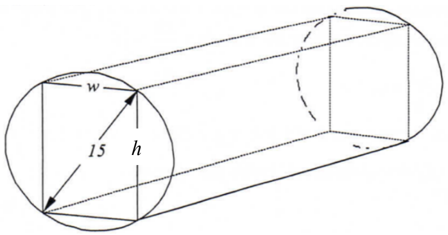
Question 17 (8 marks)



A rectangular beam of width w cm and height h cm is cut from a cylindrical pine log as shown. The diameter of the cross-section of the log (and hence the diagonal of the cross-section of the beam) is 15 cm. The strength S of the beam is proportional to the product of its width and the square of its height, so that $S = kh^2w$, where k is the constant of proportionality and $k > 0$.

(i) Show that $S = k(225w - w^3)$ **1**

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(ii) Hence, what dimensions will give a beam of maximum strength? Justify your answer. **4**

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Question 17 (continued)

(iii) A square beam with a diagonal of 15 cm could have been cut from the log. Show that the rectangular beam of maximum strength is more than 8% stronger than this square beam. **3**

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End of Paper

Year 12 Advanced Ass #2 Solutions 26/03/21

Section 1:

(1) $9\log_a(a^{1/3})$

$= \frac{1}{3}(9\log_a a)$

$= 3(1)$

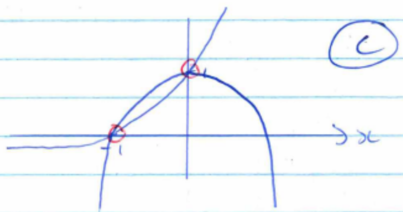
$= 3$ A

(2) $S_{10} = \dfrac{100(1-(-\frac{4}{5})^{10})}{1+4/5}$

$= 49.59\ldots$ B

(3) $7/18$ D

(4) C



(5) $x^2+6x+9+y^2+4y+4 = 13+b$

$(x-3)^2+(y+2)^2 = 13+b$

Centre $(3,-2)$ $R = 13+b$

$25-13 = b$

$12 = b$

B

(6) D

(7) B

(8) $y = -2f(2(x+3))+3$

$(4,2)$

$\times -2$: $(4,-4)$

$\times\frac{1}{2}$: $(2,-4)$

-3: $(-1,-4)$

$+3$: $(-1,-1)$ A

Section 2

(9)(α) (a) $\dfrac{dy}{dx} = 6(3x^2-4)^5 \times 6x$

$= 36x(3x^2-4)^5$

(β) $u = x$ $v = e^x$

$u' = 1$ $v' = e^x$

$\dfrac{dy}{dx} = xe^x + e^x$

$= e^x(x+1)$

(γ) $u = (x-1)^4$ $v = x^2$

$u' = 4(x-1)^3$ $v' = 2x$

$\dfrac{dy}{dx} = \dfrac{4x^2(x-1)^3 - 2x(x-1)^4}{x^4}$

$= \dfrac{2\cancel{x}(x-1)^3(2x-(x-1))}{\cancel{x^4}\ x^3}$

$= \dfrac{2(x-1)^3(x+1)}{x^3}$

9(b)(α) $3x^2 - \dfrac{x^4}{2} + C$

(β) $\dfrac{(3x-2)^{-3}}{-3\times 3} + C$

$= -\dfrac{(3x-2)^{-3}}{9} + C$

10(i) $p+0.15+2p+3p+0.2+0.05 = 1$

$6p = 0.6$

$p = 0.1$

$\therefore 2p = 0.2$ $3p = 0.3$

(ii) $E(X) = 2.5$

(iii) $P(X>2.5) = 0.3+0.2+0.05$

$= 0.55$

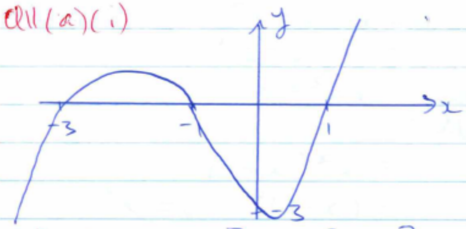
(iv) $Var(X) = 8.1-(2.5)^2$

$= 1.85$

(v) $\sigma = \sqrt{1.85}$

$= 1.36$

Q11(a)(i)



(ii) $(-\infty,-3] \cup [-1,1]$

(b)(i) $f(-3) = \sqrt{1+3} = 2$

$g(2) = 1/2$

(ii) $g(\sqrt{1-x}) = \dfrac{1}{\sqrt{1-x}}$

D: $x<1$

R: $y>0$

12(a) $\cancel{A} \approx \dfrac{b-a}{2n} = \dfrac{54-0}{2(6)} = 4.5$

$A \approx 4.5(0+2(8+13+18+17+14)+6)$

$\approx 4.5(146)$

$\approx 657\ m^2$

(b)(i) $1-\sin^2\theta = \cos^2\theta$

From Ref Sheet $\sin^2\theta+\cos^2\theta = 1$

$\cos^2\theta = 1-\sin^2\theta$

$\therefore \cos^2\theta = \cos^2\theta$

(ii) $\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm 1/\sqrt{2}$

$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

13(i) $y' = 3x^2-3x-6$

$x^2-x-2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

$x=2$ $y=-5$ $(2,-5)$ | $x=-1$ $y=8.5$ $(-1,8.5)$

$y'' = 6x-3$

$x = 2$

$y'' = 6(2)-3$

$= 9 > 0$ C.up

$(2,-5)$ L.Min

$x = -1$

$y'' = 6(-1)-3$

$= -9 < 0$ C.Down

$(-1,8.5)$ L.Max

(ii) $6x-3 = 0$

$x = 1/2$

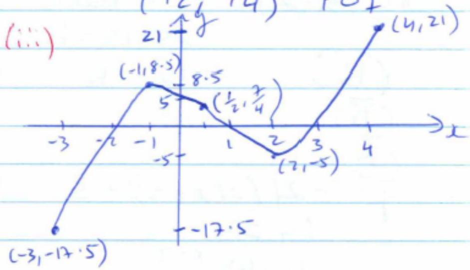
$y = 7/4$

x	0	1/2	1
y''	-3	0	3
	<0		>0

$\therefore$ Change in concavity

$(1/2, 7/4)$ POI

(iii)



(iv) $y = 21$

14(a)(i) Reflect x-axis, left 3

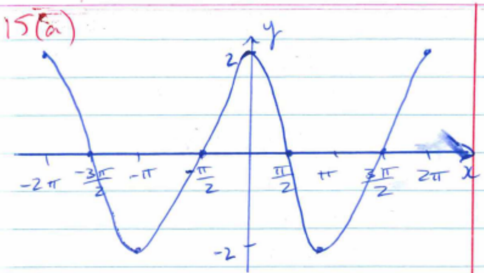
(ii) Vertical dilation by 2, down 5 spaces

(b) $k = -1/3$

$b = 3$

$c = 2$

15(a)



(b) (i) $A_1 = 500000(1.006) - M$

$A_2 = (500000(1.006) - M)(1.006) - M$

$= 500000(1.006)^2 - M(1.006) - M$

$= 500000(1.006)^2 - M(1 + 1.006)$

(ii) $A_N = 500000(1.006)^N - M(1 + 1.006 + \ldots 1.006^{N-1})$

$= 500000(1.006)^N - M\left(\frac{1(1.006^N - 1)}{1.006 - 1}\right)$

$A_N = 500000(1.006)^N - M\left(\frac{1.006^N - 1}{0.006}\right)$

(iii) $0 = 500000(1.006)^N - 3500\left(\frac{1.006^N - 1}{0.006}\right)$

$3500\left(\frac{1.006^N - 1}{0.006}\right) = 500000(1.006)^N$

$3500(1.006^N) - 3500 = 3000(1.006)^N$

$3500(1.006)^N - 3000(1.006)^N = 3500$

$500(1.006)^N = 3500$

$(1.006)^N = 7$

$N = \frac{\ln(7)}{\ln(1.006)}$

$N = 325.29\ldots$

$N = 326$

16(a) $\int_2^m \left(2x + 3 - \frac{1}{2}x - 4\right)dx = 20$

$= \int_2^m \left(\frac{3}{2}x - 1\right)dx = 20$

$= \left[\frac{3x^2}{4} - x\right]_2^m = 20$

$\frac{3m^2}{4} - m - \left(\frac{3(2)^2}{4} - 2\right) = 20$

$\frac{3m^2}{4} - m - 1 = 20$

$3m^2 - 4m - 84 = 0$

$(3m + 14)(m - 6) = 0$

$m = \frac{-14}{3}$ $\boxed{m = 6}$

(b)(i) $\frac{dy}{dx} = \frac{3}{\cancel{2}}(1 - 4x^2)^{1/2}(-\cancel{8}^{4}x)$

$= -12x\sqrt{1 - 4x^2}$

(ii) $2\left(\frac{-1}{12}\right)\int_0^{0.5} -12x\sqrt{1-4x^2}\,dx$

$= \frac{-1}{6}\left[(1 - 4x^2)^{3/2}\right]_0^{0.5}$

$= -1/6\left((1 - 4(0.5)^2)^{3/2} - (1 - 4(0)^2)^{3/2}\right)$

$= -1/6\,(0 - 1)$

$= 1/6$

OR

$2\int_0^{0.5} x\sqrt{1 - 4x^2}\,dx$

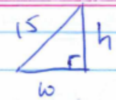
$= 2\left[\frac{\cancel{x}(1 - 4x^2)^{3/2}}{(-8\cancel{x})(3/2)}\right]_0^{0.5}$

$= \frac{2\left((1 - 4(0.5)^2)^{3/2} - (1 - 4(0))^{3/2}\right)}{-12}$

$= 2\left(\frac{0 - 1}{12}\right)$

$= 1/6$

Q17 (i)



$225 = w^2 + h^2$

$225 - w^2 = h^2$

$S = k(225 - w^2)(w)$

$= k(225w - w^3)$

(ii) $S = 225kw - kw^3$

$S' = 225k - 3kw^2$

$225k - 3kw^2 = 0$

$3kw^2 = 225k$

$w = \pm 5\sqrt{3}$

$S'' = -6kw$

$w = 5\sqrt{3}$

$= -6k(5\sqrt{3})$

$= -30k\sqrt{3}$

< 0 C. Down

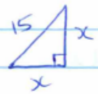
$w = 5\sqrt{3}$ is L. Max

$225 - (5\sqrt{3})^2 = d^2$

$150 = d^2$

$d = 5\sqrt{6}$

(iii)



$2x^2 = 225$

$x^2 = \frac{225}{2}$

$x = \frac{15\sqrt{2}}{2}$

Rectangle: $S = k(5\sqrt{6})^2(5\sqrt{3})$

$= 750\sqrt{3}k$

Square:

$S = k\left(\frac{15\sqrt{2}}{2}\right)^2\left(\frac{15\sqrt{2}}{2}\right)$

$= \frac{3375\sqrt{2}}{4}k$

$= \dfrac{750\sqrt{3}k}{\frac{3375\sqrt{2}k}{4}}$

$= 750\sqrt{3}\cancel{k} \times \frac{4}{3375\sqrt{2}\cancel{k}}$

$= 1.08866\ldots$

$\therefore$ 8.866% More