
Student Number

Barker College

2022 YEAR 12

ASSESSMENT TASK 3

Mathematics Advanced

Staff Involved:

- PCB* • AXD • GPF • ESP
- RAS • JZT • RJW • ALY

Wednesday 1st June, 8:30am

160 copies

General Instructions

- Working time – 55 minutes
- Write using blue or black pen
- Write your Barker Student number in the spaces indicated
- Write your answers in the spaces provided
- Board approved calculators may be used
- The HSC Reference Sheet is provided.
- Attempt all questions
- ALL necessary working must be shown
- Marks may not be awarded for careless or badly arranged work
- Diagrams are NOT to scale

Topics Covered

- Exponential and Logarithmic Functions
- Trigonometric Functions

Section	Marks
Question 1	/ 10
Question 2	/ 10
Question 3	/ 10
Question 4	/ 10
Total Marks:	**/ 40**

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Question 1 (10 marks)

Student number

(a) Differentiate with respect to x:

(α) $y = \cos(4x)$ **1**

(β) $y = (2 + \ln x)^3$ **2**

(γ) $y = \frac{e^x}{x}$ **2**

(b) Find the following indefinite integrals:

(α) $\int \frac{1}{4x+1} dx$ **1**

(β) $\int x^3 e^{5x^4} dx$ **2**

(γ) $\int \frac{3x^2+x-2}{x} dx$ **2**

END OF QUESTION 1

Student number

Question 2 (10 marks)

(a) Evaluate $\int_{\frac{\pi}{4}}^{\pi} \cos x \, dx$ **2**

(b) Find the equation of the **normal** to the curve $y = \ln(3x + 4) + 2x$ **4**

at the point on the curve where $x = -1$.

(c) Find the area, in simplified exact form, between the curve $y = 2e^{x+1}$ and the x-axis bound by the lines $x = -1$ and $x = 1$. **2**

(d) It is given that $f'(x) = 2\sec^2(2x)$ and $f\left(\frac{\pi}{8}\right) = 5$. **2**

Find the equation of $y = f(x)$.

END OF QUESTION 2

Question 3 (10 marks)

Student number

(a) A curve has an equation $y = e^{3x}(x^2 - 5x + 3)$

(i) Show that $\frac{dy}{dx} = e^{3x}(3x^2 - 13x + 4)$ **2**

(ii) Hence, find the coordinates of the stationary points on the curve. **2**

(b) (i) Differentiate $(x + 1)^3 + 2$ **1**

(ii) Hence, find $\int \frac{(x+1)^2}{(x+1)^3+2} dx$ **2**

(iii) If $\int_0^{\sqrt[3]{22}-1} \frac{(x+1)^2}{(x+1)^3+2} dx = \ln b$, where b is an integer, find b. **3**

END OF QUESTION 3

Student number

Question 4 (10 marks)

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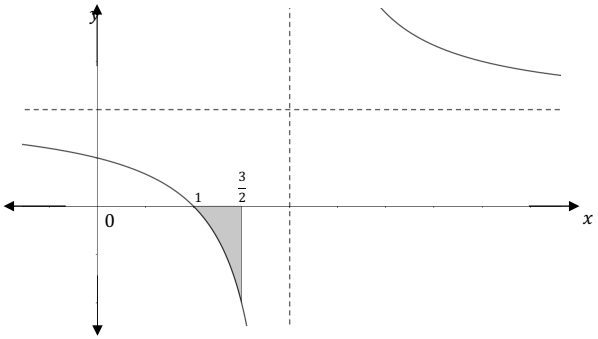
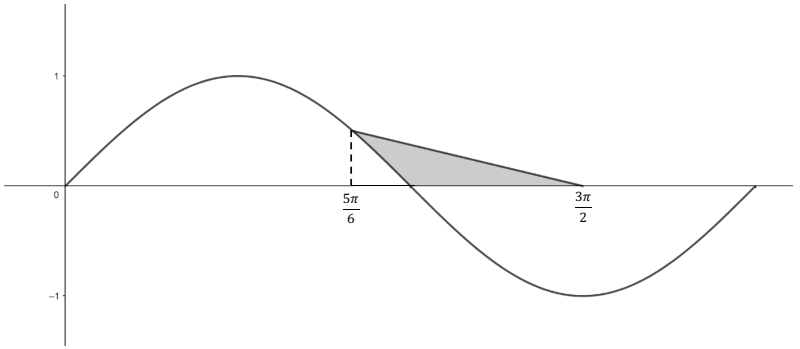
(a) The diagram shows the curve $y = \frac{1}{x-2} + 1$. **4**

Find the shaded area in simplest, exact form.

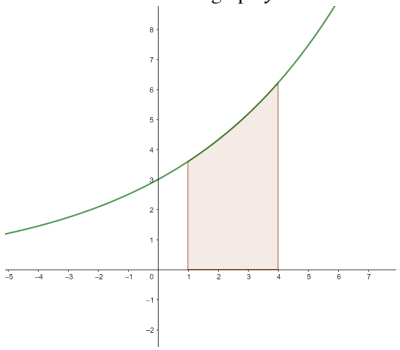


(b) Below is the graph $y = \sin x$. Find the shaded area, in simplest, exact form. **4**



Student number

(c) Below is a sketch of the graph $y = 3a^x$.



The value of $\int_1^4 3a^x dx = 14$.

Using graph transformations, find the value of $\int_2^5 3a^{x-1} - 3\, dx$ **2**

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END OF QUESTION 4

END OF ASSESSMENT

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Student number

Extra writing space

If you use this space, clearly indicate which question you are answering.

Yr 12 Advanced AT3 Solutions

1. a) α) $y' = -4\sin 4x$

β) $y' = 3(2+\ln x)^2 \times \frac{1}{x}$

$= \frac{3(2+\ln x)^2}{x}$

γ) $y' = \frac{x \cdot e^x - e^x \cdot 1}{x^2} = \frac{e^x(x-1)}{x^2}$

b) α) $\frac{1}{4}\int \frac{4}{4x+1}\,dx = \frac{1}{4}\ln|4x+1| + C$

β) $\frac{1}{20}\int 20x^3 e^{5x^4}\,dx = \frac{1}{20}e^{5x^4} + C$

γ) $\int 3x + 1 - \frac{2}{x}\,dx = \frac{3x^2}{2} + x - 2\ln|x| + C$

2. a) $\int_{\pi/4}^{\pi} \cos x\,dx = [\sin x]_{\pi/4}^{\pi}$

$= \sin\pi - \sin\frac{\pi}{4}$

$= -\frac{1}{\sqrt{2}}$

b) $y = \ln(3x+4) + 2x$ point $(-1, -2)$

$y' = \frac{3}{3x+4} + 2$

when $x = -1$ $y' = 5$ $\therefore m_{normal} = -\frac{1}{5}$

$y - -2 = -\frac{1}{5}(x - -1)$

$y = -\frac{1}{5}x - \frac{11}{5}$

c) Area $= \left|\int_{-1}^{1} 2e^{x+1}\,dx\right|$

$= \left|\left[2e^{x+1}\right]_{-1}^{1}\right|$

$= 2e^2 - 2e^0$

$= 2e^2 - 2$ or $2(e^2-1)$

d) $f'(x) = 2\sec^2(2x)$

$f(x) = \tan(2x) + c$ into $f\left(\frac{\pi}{8}\right) = 5$

$\therefore 5 = \tan\left(2\times\frac{\pi}{8}\right) + c$

$5 = 1 + c$

$c = 4$

$\therefore f(x) = \tan 2x + 4$

3. a) i) $y = e^{3x}(x^2 - 5x + 3)$ product rule

$y' = e^{3x}\cdot(2x-5) + 3e^{3x}\cdot(x^2-5x+3)$

$= e^{3x}(2x - 5 + 3x^2 - 15x + 9)$

$= e^{3x}(3x^2 - 13x + 4)$

ii) stat pts set $y' = 0$

$0 = e^{3x}(3x^2 - 13x + 4)$

$0 = e^{3x}(3x-1)(x-4)$

$x = \frac{1}{3}$ or $x = 4$

$y = \frac{13e}{9}$ $y = -e^{12}$

b) i) $\frac{d}{dx}(x+1)^3 + 2 = 3(x+1)^2$

ii) $\frac{1}{3}\int \frac{3(x+1)^2}{(x+1)^3+2}\,dx = \frac{1}{3}\ln\left|(x+1)^3+2\right| + c$

iii) LHS $= \int_0^{\sqrt[3]{11}-1} \frac{(x+1)^2}{(x+1)^3+2}\,dx$

$= \frac{1}{3}\left[\ln\left|(x+1)^3+2\right|\right]_0^{\sqrt[3]{11}-1}$

$= \frac{1}{3}(\ln 24 - \ln 3)$

$= \frac{1}{3}\ln 8$

$= \frac{1}{3}\ln 2^3$

$= \ln 2 \quad \therefore b = 2$

4. a) Area $= \left|\int_1^{3/2} \frac{1}{x-2} + 1\,dx\right|$

$= \left|\left[\ln|x-2| + x\right]_1^{3/2}\right|$

$= \left|\left(\ln\left|-\frac{1}{2}\right| + \frac{3}{2}\right) - (\ln|-1| + 1)\right|$

$= \left|\ln\frac{1}{2} + \frac{1}{2}\right|$

$= \left|-\ln 2 + \frac{1}{2}\right|$

$= \ln 2 - \frac{1}{2}$

b) Area $=$ triangle $- \int_{5\pi/6}^{\pi} \sin x\,dx$

triangle $= \frac{1}{2} \times \left(\frac{3\pi}{2} - \frac{5\pi}{6}\right) \times \frac{1}{2} = \frac{\pi}{6}$ u²

$\int_{\pi/6}^{\pi} \sin x\,dx = -\left[\cos x\right]_{\pi/6}^{\pi}$

$= -\left(-1 - -\frac{\sqrt{3}}{2}\right)$

$= 1 - \frac{\sqrt{3}}{2}$ u²

$\therefore$ Area $= \frac{\pi}{6} - 1 + \frac{\sqrt{3}}{2}$ u²

c) $\int_1^4 3a^x\,dx = 14$

$\int_1^4 3a^x\,dx = \int_2^5 3a^{x-1}\,dx$

$\int_2^5 3a^{x-1} - 3\,dx = 14 - 9$

$= 5$ u²