
Student Number

Barker College

2022 YEAR 12 ASSESSMENT BLOCK

Mathematics Advanced

Staff Involved: **FRIDAY 01 APRIL 2022**

- ALY • PCB • RJW • RAS • LAK*
- ESP • AXD • GPF • JZT

160 copies

TOPICS COVERED:	
Year 11 Content	Sequences & Series
Series & Finance	Graphs & Equations
Curve Sketching using the Derivative	Integration
Differentiation of Trigonometric functions	

General Instructions:

- Working time - 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A separate reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and / or calculations
- Marks may not be awarded for careless or poorly arranged working
- Write your student number at the top of every page
- Diagrams not to scale

Total marks:
80

Section I - 8 marks (pages 2 - 4)
- Attempt Multiple Choice Questions 1 - 8
- Allow about 15 minutes for this section

Section II - 72 marks (pages 5 - 21)
- Attempt Questions 9 - 16
- Show all necessary working
- Allow about 1 hours and 45 minutes for this section

Section I

8 marks
Attempt Questions 1 – 8
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 8.

1 Given that $\sin\theta = \frac{5}{13}$ and $\tan\theta < 0$, what is the value of $\sec\theta$?

A. $\frac{13}{5}$

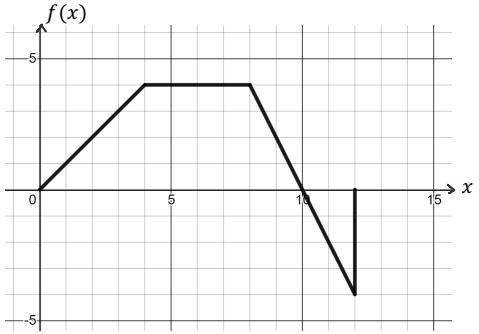
B. $-\frac{13}{12}$

C. $-\frac{13}{5}$

D. $\frac{13}{12}$

2 Consider the function, $f(x)$, depicted in the graph below.
Evaluate

$$\int_0^{12} f(x)\,dx$$



A. 24

B. 28

C. 32

D. 48

Student Number

3 Which of the following represents the range for the function $f(x) = e^{2x} - 1$

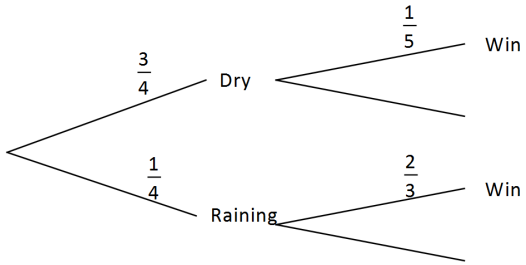
A. $[-1, \infty)$

B. $(-1, \infty)$

C. $(-\infty, -1]$

D. $(-\infty, -1)$

4 The manager of a team notices that the team has a probability of $\frac{2}{3}$ of winning the game if it is raining but, if it is dry, the probability of the team winning is $\frac{1}{5}$. The probability that it will rain on a day when they play is $\frac{1}{4}$. This information is partially summarised in the tree diagram below.



What is the probability that the team will not win their next game?

A. $\frac{13}{15}$

B. $\frac{17}{15}$

C. $\frac{19}{60}$

D. $\frac{41}{60}$

5 What is the solution to the following inequation?

$$x^2 \geq 16$$

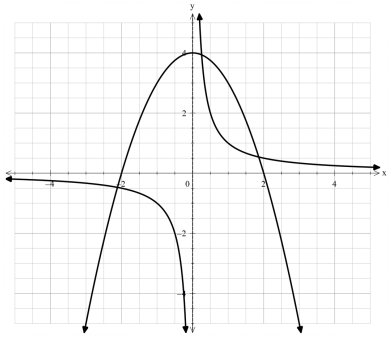
A. $x \geq 4$

B. $-4 \leq x \leq 4$

C. $x \leq -4 \; or \; x \geq 4$

D. $-4 \geq x \geq 4$

6 Consider the graphs of $y = \frac{1}{x}$ and $y = 4 - x^2$ shown below. How many solutions are there to the equation $\frac{1}{x} = 4 - x^2$?



A. 1 solution

B. 2 solutions

C. 3 solutions

D. 4 solutions

7 Given that the limiting sum of the series $5 - 5x + 5x^2 - \cdots$ is 15, what is x?

A. $x = \frac{2}{3}$

B. $x = -\frac{2}{3}$

C. $x = \frac{2}{15}$

D. $x = -\frac{2}{15}$

8 Solve $2\sin^2 x - 1 = \sin x$ for $0 \leq x \leq 2\pi$

A. $x = \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

B. $x = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

C. $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

D. $x = \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

End of Section I

Student Number

Section II

72 marks
Attempt Questions 9 – 16
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate space provided.

In Questions 9 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 9 (9 marks**)**

(a) Solve **2**

$$\frac{x+1}{5} - \frac{x-1}{3} = 1$$

(b) Differentiate

α. $f(x) = (2x^3 - 4)^5$ **1**

β. $y = 3e^{2x+5}$ **1**

Question 9 (cont.)

(c) Find $\int 9x^2 - 4 + \frac{3}{x^3}\,dx$, simplifying your answer using positive indices. **2**

(d) Given that $\log_a 3 = 0.477$ and $\log_a 5 = 0.699$ evaluate $\log_a 45$ **1**

(e) A circle is described by the expression $y^2 - 6y + x^2 + 2x + 2 = 0$.
What is the centre and radius of this circle? **2**

Student Number

Question 10 (9 marks)

(a) By considering the translations applied, and your knowledge of $y = \frac{1}{x}$, sketch $y = 3 - \frac{1}{x-2}$, showing important features and the x, y intercepts. **3**

Question 10 (cont.)

(b) Consider a function $f(x)$.

(i) Explain the sequence of translations and/or dilations that have been applied to $f(x)$ to produce $y = -2f(3x - 12)$. **3**

(ii) The point, $(-6, 8)$ lies on $f(x)$. What are the new co-ordinates of this point after the changes above have been applied? **2**

(c) In the game of Yahtzee 5 six-sided dice are rolled once.
What is the probability that at least one of the dice is a 6? **1**

Student Number

Question 11 (9 marks)

(a) The discrete probability distribution for an experiment is shown in the table below.

x	0	1	2	3	4
$P(X = x)$	$p =$	0.15	0.3	0.25	$2p =$

(i) Show that $p = 0.1$ and hence complete the discrete probability distribution table, assuming no other values of x are possible. **1**

(ii) Find $E(X)$. **2**

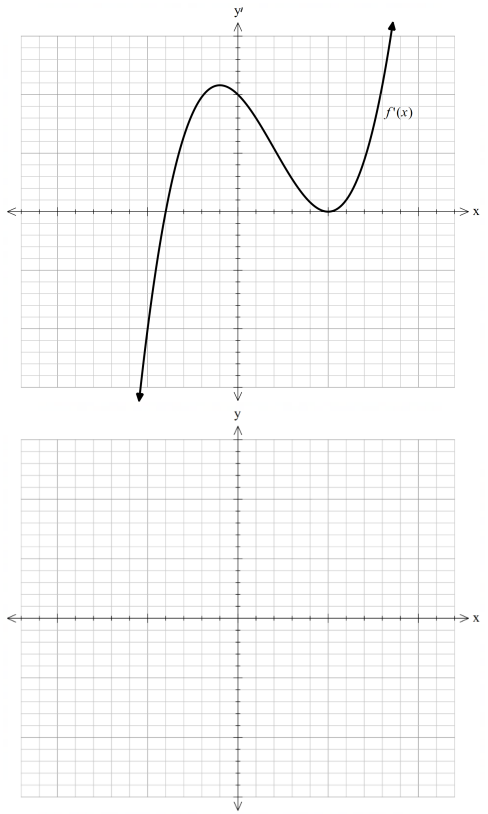
(iii) Calculate the standard deviation, to 2 d.p. **2**

(iv) What is the probability that an outcome for this experiment is odd? **1**

Question 11 (cont.)

(v) Given that the outcome for an experiment was odd, what is the probability that the outcome was a 3? **1**

(b) The graph below shows the gradient function, $f'(x)$. On the grid below sketch a possible curve for $f(x)$. **2**



Student Number

Question 12 (9 marks)

(a) Consider the function

$$f(x) = -\frac{x^3}{3} + x^2 + 3x - 9$$

(i) Find the coordinates of the turning points and determine their nature **3**

(ii) Factorise this function and identify the x −intercepts. **2**

Question 12 (cont.)

(iii) Hence, sketch the curve, showing all this information, and the y −intercept. **2**

(b) The sum of the first 8 terms of an arithmetic series is equal to 18. If the first term is -4, what is the value of the eighth term? **2**

Student Number

Question 13 (9 marks)

(a) The zoom function in a software package multiplies the dimensions of an image by 1.2.

In an image, the height of a building is 50 mm.

After the zoom function is applied once, the height of the building in the image is 60 mm.

After a second application, its height is 72 mm.

The height of the building in the image is required to be more than 400 mm.

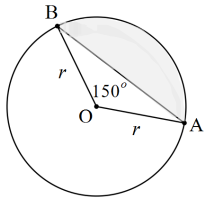
Starting from the original image, what is the least number of times the zoom function must be applied? **3**

(b) Evaluate **3**

$$\int_{-3}^{0} \sqrt{3x+9}\, dx$$

Question 13 (cont.)

(c) Consider the diagram below, where a triangle, ΔAOB, is inscribed in a circle of radius r.
The angle between the radii $0A$ and OB is $\angle AOB = 150°$.
Given that the area of the shaded minor segment is $16\ m^2$,
what is the length of the radius (2 d.p.)? **3**



Student Number

Question 14 (9 marks)

(a) Consider the curve

$$y = \frac{x^2}{4} + 2x - 4$$

(i) Find the equation of the normal to the curve at the point on the curve where $x = -6$ **3**

(ii) What are the coordinates of the point where the normal crosses the curve a second time? **2**

Question 14 (cont.)

(b) Find a and b if the curve $y = x^4 + ax^3 + bx^2$ has an inflection point at $(2, 0)$. **4**

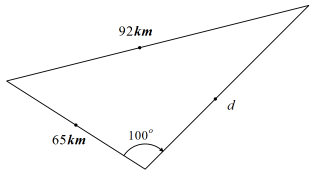
Student Number

Question 15 (9 marks)

(a) Differentiate $f(x)$, writing your answer in simplest form. **2**

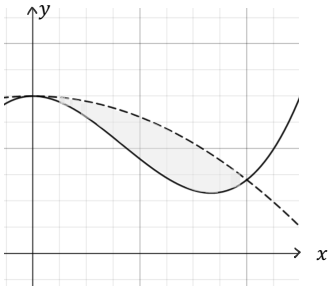
$$f(x) = \frac{x}{\sqrt{x^2+1}}$$

(b) Consider the triangle below. Evaluate the distance, d, to the nearest km. **3**



Question 15 (cont.)

(c) The functions, $f(x) = x^3 - 5x^2 + 30$ and $g(x) = 30 - x^2$ intersect as shown in the diagram below. Find the area of the shaded region. **4**



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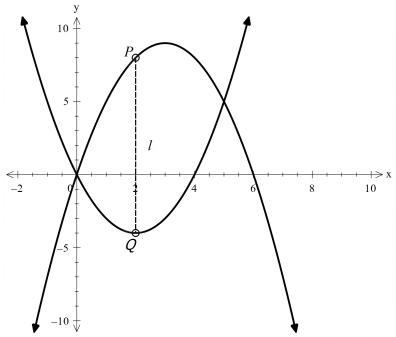
Question 16 (9 marks)

(a) In the diagram below, the points P and Q are on the curves, $y = x(6 - x)$ and $y = x(x - 4)$ respectively.

By showing that the length, $l\ units$, of PQ is given by $l = 10x - 2x^2$, find the maximum length of PQ, for $0 \leq x \leq 5$. **3**



Question 16 (cont.)

(b) As part of a retirement plan, Jo was putting aside $\$5000$ at the beginning of each year into an annuity, where the interest rate was $6\%\ p.a.$

Future value interest factors for an annuity of $1 for various interest rates (r) and numbers of periods (n) are given in the table.

Table of future value interest factors

number of periods (n)	Interest Rate per period (r) 0.5%	1%	2%	3%	4%	5%	6%	7%	8%
6	6.1059	6.2135	6.4343	6.6625	6.8983	7.1420	7.3938	7.6540	7.9228
7	7.1414	7.2857	7.5830	7.8923	8.2142	8.5491	8.8975	9.2598	9.6366
8	8.1821	8.3685	8.7546	9.1591	9.5828	10.0266	10.4913	10.9780	11.4876
9	9.2280	9.4622	9.9497	10.4639	11.0061	11.5779	12.1808	12.8164	13.4866
10	10.2792	10.5668	11.1687	11.8078	12.4864	13.2068	13.9716	14.7836	15.6455
11	11.3356	11.6825	12.4121	13.1920	14.0258	14.9171	15.8699	16.8885	17.9771
12	12.3972	12.8093	13.6803	14.6178	15.6268	16.7130	17.8821	19.1406	20.4953

(i) Using the table, calculate the value of Jo's annuity after 10 years. **1**

(ii) After 30 years the annuity has now grown to $\$419\ 008.50$ and Jo stops depositing money into it.

She then retires and begins to withdraw $\$2000$ at the end of each month.
The interest rate on the balance is $3\%\ p.a.$

Let A_n be the amount in the annuity at the end of n months after the n^{th} withdrawal.

By developing the first three terms, show that,

$A_n = 419\ 008.5 \times 1.0025^n - 800\ 000(1.0025^n - 1)$ **3**

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Question 16 (cont.)

(iii) How long will the annuity last (to the nearest month)? **2**

End of Paper

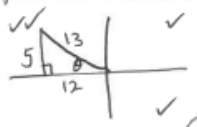
EXTRA WORKING

Student Number

EXTRA WORKING

2022 Year 12 Maths Advanced Assessment 2 solutions (1 April)

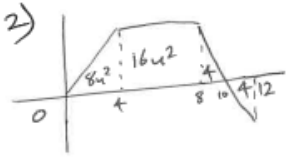
Multiple Choice - Section I

1) 

$\sec\theta = \frac{1}{\cos\theta}$

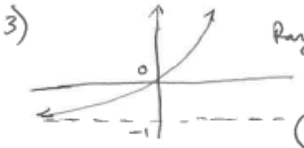
$= -\frac{13}{12}$

(B)

2) 

$\int_0^{12} f(x)\,dx = 8+16+4-4$

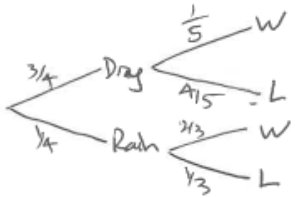
$= 24$ (A)

3) 

Range: $y > -1$

$(-1, \infty)$

(B)

4) 

$P(\text{lose}) = \frac{3}{4}\times\frac{4}{5} + \frac{1}{4}\times\frac{1}{3} = \frac{41}{60}$ (D)

5) $x^2 - 16 \geq 0$

$(x-4)(x+4) \geq 0$

$x \leq -4$ or $x \geq 4$ (C)

6) 3 points of intersection $\Rightarrow$ 3 solutions (C)

7) $a = 5$ $r = -x$

$15 = \frac{5}{1+x}$

$15 + 15x = 5$

$15x = -10$

$\therefore x = \frac{-10}{15} = -\frac{2}{3}$

(B)

8) $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$ (C)

Section II

Q9) (a) $3(x+1) - 5(x-1) = 15$

$3x + 3 - 5x + 5 = 15$

$-2x + 8 = 15$

$-2x = 7$ $\quad \therefore x = -\frac{7}{2}$

(b) (i) $f'(x) = 5\times(2x^3-4)^4 \times 6x^2$

$= 30x^2(2x^3-4)^4$

(ii) $\frac{dy}{dx} = 6e^{2x+5}$

(c) $\int 9x^2 - 4 + 3x^{-3}\,dx$

$= \frac{9x^3}{3} - 4x + \frac{3x^{-2}}{-2} + C$

$= 3x^3 - 4x - \frac{3}{2x^2} + C$

(d) $\log_a 45 = \log_a(9\times 5)$

$= \log_a 9 + \log_a 5$

$= 2\log_a 3 + \log_a 5$

$= 2\times 0.477 + 0.699$

$= 1.653$

(e) $x^2 + 2x + 1 + y^2 - 6y + 9 = -2 + 9 + 1$

$(x+1)^2 + (y-3)^2 = 8$

Centre $(-1, 3)$

Radius $= \sqrt{8}$ units

$= 2\sqrt{2}$ units

Q10) (a) $f(0) = 3 - \frac{1}{-2}$

$= 3\frac{1}{2}$

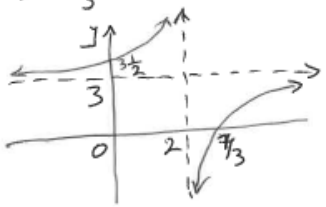
$0 = 3 - \frac{1}{x-2}$

$\therefore \frac{1}{x-2} = 3 \quad x \neq 2$

$\therefore 1 = 3x - 6 \quad y \neq 3$

$3x = 7$

$x = \frac{7}{3}$



(b)(i) $y = -2f(3(x-4))$

1. Horizontal stretch by a factor of $\frac{1}{3}$, ie. $f(3x)$
2. Horizontal shift right by 4 units, ie. $f(3(x-4))$
3. Vertical stretch by a factor of 2, ie. $2f(3(x-4))$
4. Reflection in x-axis ie. $-2f(3(x-4))$

(ii) 1. $(-6, 8) \rightarrow (-2, 8)$
2. $(-2, 8) \rightarrow (2, 8)$
3. $(2, 8) \rightarrow (2, 16)$
4. $(2, 16) \rightarrow (2, -16)$

$\therefore (2, -16)$

(c) P(at least one 6) $= 1 - P(\text{no 6s})$

$= 1 - \left(\frac{5}{6}\right)^5$

$= \frac{4651}{7776}$

Q11) (a)(i) $p + 2p + 0.15 + 0.3 + 0.25 = 1$

$3p + 0.7 = 1$

$3p = 0.3 \quad \therefore p = 0.1$

x	0	1	2	3	4
$P(X=x)$	0.1	0.15	0.3	0.25	0.2
$xp(x)$	0	0.15	0.6	0.75	0.8
$x^2p(x)$	0	0.15	1.2	2.25	3.2

(ii) $E(X) = 0 + 0.15 + 0.6 + 0.75 + 0.8$

$= 2.3$

(iii) $\text{Var}(X) = (0 + 0.15 + 1.2 + 2.25 + 3.2) - 2.3^2$

$= 1.51$

$\sigma = \sqrt{1.51}$

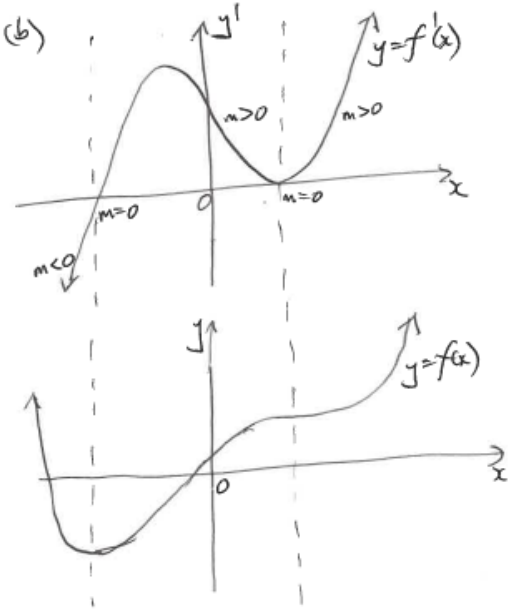
$\doteq 1.23$

(iv) $P(X=1 \text{ or } X=3) = 0.15 + 0.25$

$= 0.4$

(v) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.25}{0.4} = \frac{5}{8}$

(b)



Q12) (a) $f(x) = -\frac{x^3}{3} + x^2 + 3x - 9$

(i) $f'(x) = -x^2 + 2x + 3$

$f'(x) = 0$ for stat pts

$\therefore 0 = -x^2 + 2x + 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3 \text{ or } -1$

$y = 0 \text{ or } -\frac{32}{3}$

$f''(x) = -2x + 2$

$f''(3) = -6 + 2 < 0 \ \cap$

$\therefore$ Max turning pt @ $(3, 0)$

$f''(-1) = 2 + 2 > 0 \ \cup$

$\therefore$ Min turning pt @ $(-1, -\frac{32}{3})$

(ii) $-\frac{1}{3}(x^3 - 3x^2 - 9x + 27) = 0$

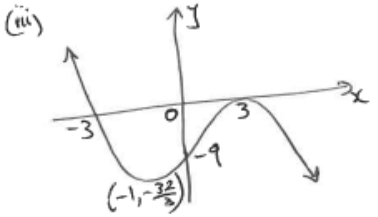
$x^2(x-3) - 9(x-3) = 0$

$(x-3)(x^2-9) = 0$

$(x-3)(x-3)(x+3) = 0$

$\therefore x = 3, 3 \text{ or } -3$

(iii)



(b) $S_8 = 18 \quad a = -4$

Using $S_n = \frac{n}{2}(a+l)$ gives

$18 = \frac{8}{2}(-4 + l)$

$18 = 4(l - 4)$

$18 = 4l - 16$

$4l = 34$

$l = T_8 = \frac{34}{4} = \frac{17}{2}$

Q13) (a) 50, 60, 72, ... G.P. with $a = 50, r = 1.2$

$400 = 50 \times 1.2^{n-1}$

$8 = 1.2^{n-1}$

$\log 8 = (n-1)\log 1.2$

$n - 1 = \frac{\log 8}{\log 1.2}$

$n = 1 + \frac{\log 8}{\log 1.2} \doteq 12.4$ ie. $T_{13} > 400$mm

$\therefore$ Need 12 zoom applications

(b) $\int_{-3}^{0} (3x+9)^{\frac{1}{2}} dx$

$= \left[\frac{(3x+9)^{3/2}}{3 \times 3/2}\right]_{-3}^{0}$

$= \frac{2}{9}\left[9^{3/2} - 0^{3/2}\right]$

$= \frac{2}{9} \times 27 = 6$

(c) $150° = 150 \times \frac{\pi}{180} = \frac{150\pi}{180} = \frac{5\pi}{6}$

$16 = \frac{1}{2} \times r^2 \times \frac{5\pi}{6} - \frac{1}{2} \times r^2 \times \sin\frac{5\pi}{6}$

$32 = \frac{5\pi}{6} r^2 - \frac{r^2}{2}$

$32 \times 6 = 5\pi r^2 - 3r^2$

$r^2(5\pi - 3) = 192$

$r^2 = \frac{192}{5\pi - 3}$

$r = \sqrt{\frac{192}{5\pi - 3}}$

$\doteq 3.89$ m

Q14) (a) $y = \frac{x^2}{4} + 2x - 4$

(i) $\frac{dy}{dx} = \frac{x}{2} + 2$

$f'(-6) = \frac{-6}{2} + 2$

$= -3 + 2$

$= -1$

$\therefore$ Grad of normal $= 1$

$f(-6) = \frac{36}{4} - 12 - 4$

$= 9 - 12 - 4$

$= -7 \quad \therefore (-6, -7)$

Eqn of normal is

$y + 7 = 1(x + 6)$

$\therefore y = x - 1$

(ii) $\left.\begin{matrix} y = x - 1 \\ y = \frac{x^2}{4} + 2x - 4 \end{matrix}\right\} x - 1 = \frac{x^2}{4} + 2x - 4$

$\therefore 4x - 4 = x^2 + 8x - 16$

$\therefore x^2 + 4x - 12 = 0$

$\therefore (x + 6)(x - 2) = 0$

$x = -6$ or 2

$y = -7$ or $1 \quad \therefore (2, 1)$

(b) $f(2) = 0 \quad \therefore 0 = 16 + 8a + 4b$

$\therefore 8a + 4b = -16$

$2a + b = -4 \quad (1)$

$\frac{dy}{dx} = 4x^3 + 3ax^2 + 2bx$

$\frac{d^2y}{dx^2} = 12x^2 + 6ax + 2b$

$f''(2) = 0 \quad \therefore 0 = 48 + 12a + 2b$

$12a + 2b = -48$

$6a + b = -24 \quad (2)$

$\left.\begin{matrix} 2a + b = -4 \\ 6a + b = -24 \end{matrix}\right\} \therefore 4a = -20$

$a = -5$

$\therefore -10 + b = -4$

$b = 6$

$\therefore a = -5, \; b = 6$

Q15) $f(x) = \frac{x}{\sqrt{x^2+1}} = \frac{u}{v}$

(a) $f'(x) = \frac{\sqrt{x^2+1} \times 1 - x \times \frac{1}{2} \times (x^2+1)^{-1/2} \times 2x}{\left(\sqrt{x^2+1}\right)^2}$

$= \frac{\left(\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}\right)}{x^2+1} \times \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$

$= \frac{x^2+1-x^2}{(x^2+1)\sqrt{x^2+1}} = \frac{1}{(x^2+1)\sqrt{x^2+1}}$

(b) $92^2 = 65^2 + d^2 - 2 \times 65d \cos 100^\circ$

$\therefore d^2 - 130 \cos 100^\circ d - 4239 = 0$

$d = \frac{+130\cos 100^\circ \pm \sqrt{(130\cos 100^\circ)^2 - 4 \times 1 \times -4239}}{2}$

$= \frac{130\cos 100^\circ \pm \sqrt{(130\cos 100^\circ)^2 + 16956}}{2}$

$\therefore d = 130\cos 100^\circ + \sqrt{(130\cos 100^\circ)^2 + 16956}$

$\doteq 55$ km

(c) $x^3 - 5x^2 + 30 = 30 - x^2$

$x^3 - 4x^2 = 0$

$x^2(x - 4) = 0$

$\therefore x = 0$ or 4

$A = \int_0^4 (30 - x^2) - (x^3 - 5x^2 + 30)\,dx$

$= \int_0^4 30 - x^2 - x^3 + 5x^2 - 30\,dx$

$= \int_0^4 4x^2 - x^3\,dx$

$= \left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^4$

$= \left[\frac{4 \times 4^3}{3} - \frac{4^4}{4} - (0 - 0)\right]$

$= \frac{64}{3}$ units^2

Q16) (a) $l = 6x - x^2 - (x^2 - 4x)$

$= 6x - x^2 - x^2 + 4x$

$\therefore l = 10x - 2x^2$

$\frac{dl}{dx} = 10 - 4x$

$10 - 4x = 0$

$4x = 10 \quad \therefore x = \frac{10}{4} = \frac{5}{2}$

$\frac{d^2l}{dx^2} = -4 < 0 \; \wedge \; \therefore$ Max when $x = \frac{5}{2}$

$\therefore$ Max length of $PQ = 10 \times \frac{5}{2} - 2 \times \left(\frac{5}{2}\right)^2$

$= 12.5$ units

(b)(i) 6% for 10 periods $= 13.9716$

$\therefore 13.9716 \times 5000 = \$69858$

(ii) $A_1 = 419008.50 \times 1.0025 - 2000$

$A_2 = A_1 \times 1.0025 - 2000$

$= 419008.50 \times 1.0025^2 - 2000 \times 1.0025 - 2000$

$A_3 = A_2 \times 1.0025 - 2000$

$= 419008.50 \times 1.0025^3 - 2000 \times 1.0025^2 - 2000 \times 1.0025 - 2000$

$\therefore A_n = 419008.50 \times 1.0025^n - 2000 \times 1.0025^{n-1} - \ldots - 2000 \times 1.0025 - 2000$

$= 419008.50 \times 1.0025^n - 2000(1 + 1.0025 + \ldots + 1.0025^{n-1})$

G.P. with $a = 1$, $r = 1.0025$, $n = n$

$= 419008.50 \times 1.0025^n - 2000 \times \frac{1 \times (1.0025^n - 1)}{1.0025 - 1}$

$= 419008.50 \times 1.0025^n - \frac{2000}{0.0025}(1.0025^n - 1)$

$A_n = 419008.50 \times 1.0025^n - 800000(1.0025^n - 1)$

(iii) $A_n = 0 \quad \therefore 0 = 419008.50 \times 1.0025^n - 800000 \times 1.0025^n + 800000$

$0 = -380991.50 \times 1.0025^n + 800000$

$380991.50 \times 1.0025^n = 800000$

$1.0025^n = \frac{800000}{380991.50}$

$1.0025^n = 2.099784\ldots$

$n \log 1.0025 = \log 2.099784\ldots$

$n = \frac{\log 2.099784\ldots}{\log 1.0025}$

$n \doteq 297.1$ months