Barker College

Student Number

Year 12, 2023 HSC Mathematics Advanced Assessment Task 1 (38 Marks)

Sequences and Series
Series and Finance
Discrete Probability Distributions
Graphs and Equations

Staff Involved: **8:30 AM**

DXC JAI LAK LMD ARM DZB*

PDJ ARP ALY RJW **WEDNESDAY 8th FEBRUARY**

155 copies

General Instructions:

- Working time - 55 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided separately
- Show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for careless or poorly arranged working

1. Circle all tables below that are **NOT** valid discrete probability distributions. **2 marks**

x	0.1	0.2	0.3	0.4
$P(X = x)$	0.2	0.3	0.5	0.1

x	1	2	3	4
$P(X = x)$	0.25	0.25	0.25	0.25

x	5	10	15	20
$P(X = x)$	-0.1	0.2	0.4	0.5

x	-9	-3	3	9
$P(X = x)$	0.5	0	0	0.5

2. The overall financial return in dollars on a game of chance is shown in the complete discrete probability table below. Positive x values indicate financial gain, and negative x values indicate financial loss.

x	-4	0	3	9
$P(X = x)$	$4a$	$3a$	$2a$	a

i) Find a as a decimal. **1 mark**

.....

.....

.....

ii) Calculate the expected value of the distribution.
Would you expect to make money playing this game? **2 marks**

.....

.....

.....

.....

Student number

iii) Calculate the standard deviation of the distribution, correct to 3 decimal places. **3 marks**

iv) One player wanted to raise the stakes and double the financial returns of each outcome. Will this version of the game lead to a better scenario for the player compared to the original version of the game? Justify with calculations. **2 marks**

3. The first 3 terms of an arithmetic sequence are 4, 9 and 14.
Calculate T_{10}. **1 mark**

4. In a particular geometric sequence, T_2 is 12 and T_5 is 1.5.
Calculate the common ratio in the sequence. **2 marks**

5. Given that $S_n = \frac{n}{2}(a + l)$, use the formula for the n^{th} term of a sequence to prove that **1 mark**

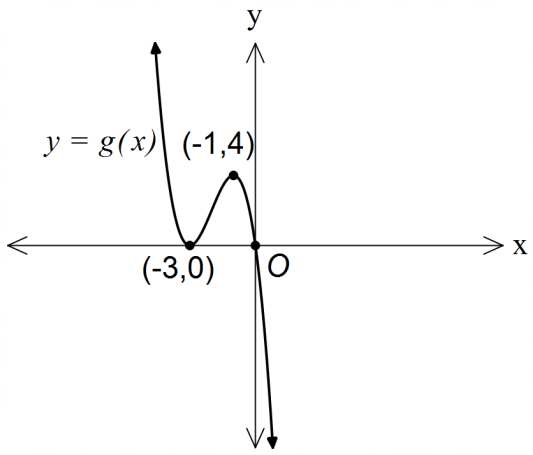
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

6. For what values of x does the geometric series $3 + 2x + \frac{4x^2}{3} + \frac{8x^3}{9} + \cdots$ have a limiting sum? **2 marks**

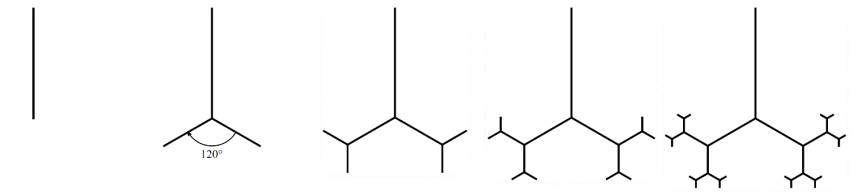
Student number

7. Consider the function $f(x) = (x+2)^2$
Sketch the function $y = f(x-3) - 1$, labelling the vertex and all intercepts. **2 marks**

8. The graph of $y = g(x)$ is shown below.
On the same axis, sketch the function $y = g(\frac{-x}{2})$, showing the coordinates of any x-intercepts and coordinates of any stationary points **2 marks**



9. Below is an example of a fractal, different to the one in your investigation.



Stage 0 Stage 1 Stage 2 Stage 3 Stage 4

The initial root (at Stage 0) is vertical and of length 1. The additional branches at each stage are half as long as the previous branch, and grow at 120° from each other, each a 60° rotation from the previous branch. The increase in vertical height of the total system in stage 1 is

$\frac{1}{2}\cos 60° = \frac{1}{4}$. (You do **NOT** need to prove this)

i) Calculate the total vertical height of the system at Stage 3. **2 marks**

ii) Find the limiting total vertical height of the fractal system. **3 marks**

Student number

10. Sketch the graph of $y = -\log_e(1 + 2x)$, labelling all key features. **3 marks**

11. The sum of T_2, T_5 and T_{10} in an arithmetic sequence is 16.
If the value of T_5 is 4, find the value of T_{10}. **3 marks**

12. Zoe took out a reducible interest loan of \$760 000 with an interest rate of 3% p.a, compounding monthly, that is paid off in monthly instalments of \$3604.01, paid at the end of the month.

i) Show that the amount owing after 2 months immediately after the instalment is paid is: **2 marks**

$$A_2 = 760\,000 \times 1.0025^2 - 3604.01(1 + 1.0025)$$

ii) Calculate how many years until the loan is paid off. **3 marks**

Student number

iii) The interest for each month, I, can be represented as

$$I = A_n \times r,$$

where A_n is the amount owing after the previous instalment and r is the monthly interest rate.

If Zoe made a monthly instalment where 50% of it erased the monthly interest and 50% of it went towards reducing the principal, calculate the amount still owing immediately after the instalment is paid.

2 marks

.....

.....

.....

.....

.....

.....

.....

End of Paper

2 Adv 2023 Ass 1 Solutions

① | x | 0·1 | 0·2 | 0·3 | 0·4 |
|---|---|---|---|---|
| P(x) | 0·2 | 0·3 | 0·5 | 0·1 |

has sum of P(x) = 1·1 ∴ not a D.P.D

x	5	10	15	20
P(x)	−0·1	0·2	0·4	0·5

is not ($P(x) \geq 0$).

② (i) $\Sigma P(x) = 1$

$4a + 3a + 2a + a = 10a = 1$

$\therefore a = 0{\cdot}1$

ii) $E(X) = \Sigma x P(x)$

$= (-4)^{a} \times 0{\cdot}4 + 3^{a} \times 0{\cdot}2 + 9^{a} \times 0{\cdot}1$

$= -0{\cdot}1$

∴ No you will not make money

iii) $Var(X) = \Sigma E(X^2) - \mu^2$

$= (-4)^2 \times 0{\cdot}4 + 3^2 \times 0{\cdot}2 + 9^2 \times 0{\cdot}1 - (-0{\cdot}1)^2$

$= 16{\cdot}29$

$\therefore \sigma = \sqrt{16{\cdot}29} = 4{\cdot}036$ (3dp)

(iv)

x	−8	0	6	18
P(X)	0·4	0·3	0·2	0·1

$xP(x) = -3{\cdot}2 + 0 + 1{\cdot}2 + 1{\cdot}8 = -0{\cdot}2$

∴ worse scenario

③ $a = 4, d = 5, T_{10} = a + 9d$

$= 4 + 45 = 49$

④ $ar = 12$ ①, $ar^4 = 15$ ②

② / ① gives: $\frac{1{\cdot}5}{12} = r^3$

$\therefore r^3 = \frac{1}{8}, r = \frac{1}{2}$

⑤ $L = T_n = a + (n-1)d$

$\therefore S_n = \frac{n}{2}(a + L)$

$= \frac{n}{2}(a + a + (n-1)d)$

$= \frac{n}{2}(2a + (n-1)d)$

⑥ $r = \frac{T_2}{T_1} = \frac{2x}{3}$ Limiting sum exists for $|r| < 1$ i.e. $\left|\frac{2x}{3}\right| < 1$

$\therefore -1 < \frac{2x}{3} < 1$

$-3 < 2x < 3$

$-\frac{3}{2} < x < \frac{3}{2}$

⑦ $f(x) = (x+2)^2$

$f(x-3) - 1 = ((x-3)+2)^2 - 1$

$= (x-1)^2 * - 1$

∴ vertex = $(1, -1)$

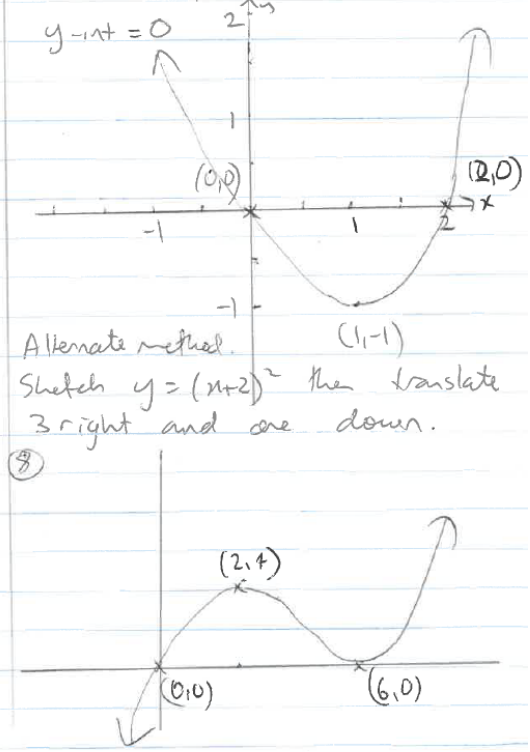
x-ints when $y = 0$

$\therefore (x-1)^2 - 1 = 0$

$(x-1)^2 = 1$

$x - 1 = \pm 1$

$x = 1 \pm 1$ i.e. $x = 0$ or 2.



Alternate method.

Sketch $y = (x+2)^2$ then translate 3 right and one down.

⑧

12 Adv AS 2023 Solutions p2.

(9)(c) Vertical height

S1 $1 + \frac{1}{4}$

S2 $1\frac{1}{4} + \frac{1}{4} = 1\frac{1}{2}$

S3 

$y = \frac{1}{8}\cos 60°$

$= \frac{1}{16}$

$\therefore$ vert height $= 1\frac{1}{2} + \frac{1}{16} = 1\frac{9}{16}$

(ii) Limiting height

$= 1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{16} + \frac{1}{16} + \frac{1}{32} + \frac{1}{32} + \ldots$

$= 1 + 2\left(\frac{1}{4} + \frac{1}{16} + \frac{1}{32} + \ldots\right)$

$S_\infty, a = \frac{1}{4}, r = \frac{1}{4}$

$\therefore S_\infty = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{3}$

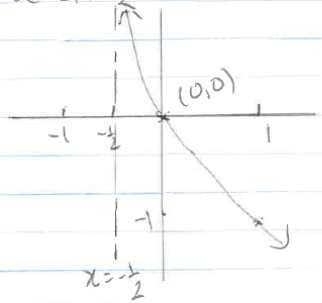
$\therefore$ limiting height $= 1 + 2\left(\frac{1}{3}\right)$

$= \frac{5}{3}$

(10) Reflect in x-axis

Scale horiz $\times \frac{1}{2}$

Move left $\frac{1}{2}$



(11) $T_5 = a + 4d = 4$ ①

$T_2 + T_5 + T_{10} = a + d + 4 + a + 9d = 16$

$2a + 10d = 12$

$a + 5d = 6$ ②

$a + 4d = 4$ ①

② − ① $d = 2$

(11) continued. Sub $d = 2$ into ①

$T_5 = a + 4d = 4$

$a + 8 = 4$

$a = -4$

$\therefore T_{10} = a + 9d$

$= -4 + 18 = \underline{14}$

(12) $A_1 = 760000(1.0025) - 3604.01$ $\left(r = \frac{0.03}{12} = 0.0025\right)$

$A_2 = A_1 \times 1.0025 - 3604.01$

$= (760000(1.0025) - 3604.01) \times 1.0025 - 3604.01$

$= 760000(1.0025)^2 - 3604.01(1.0025 - 3604.01$

$= 760\,000(1.0025)^2 - 3604.01(1 + 1.0025)$

(ii) $A_n = 760000(1.0025)^n - 3604.01(1 + 1.0025 + \ldots + 1.0025^{n-1})$

$n = n, r = 1.0025, a = 1$ in S_n

and $A_n = 0$ when loan is paid off.

$\therefore 0 = 760000(1.0025)^n - 3604.01\left(\frac{1.0025^n - 1}{0.0025}\right)$

$\therefore 1441604 \times 1.0025^n - 1441604$

$= 760000(1.0025)^n$

$\therefore 681604 \times 1.0025^n = 1441604$

$1.0025^n = \frac{1441604}{681604} = 2.1149\ldots$

$\therefore \frac{\ln 1.0025}{\ln\left(\frac{1441604}{681604}\right)} = n = 299.99\ldots$ $\therefore$ 300 months $= 25$ years.

(iii) $I = A_{n-1} \times r$, $r = 0.0025$, $I = \frac{1}{2}(3604.01)$

$\therefore 1802.005 = A_{n-1} \times 0.0025$

$\therefore A_{n-1}$ \$720802 is amount owing after previous instalment.

$\therefore$ Amount owing after this instalment is

$A + I - M = 720802 + 1802.005 - 3604.01$

$= 718999.99$