
Student Number

Barker College

2023 YEAR 12

ASSESSMENT TASK 3

Mathematics Advanced

Staff Involved:

- DZB*
- ARM
- DXC
- JAI
- LAK
- LMD
- ALY
- ARP
- PDJ
- RJW

Wednesday 31st May, 8:30am

160 copies

General Instructions

- Working time – 55 minutes
- Write using blue or black pen
- Write your Barker Student number in the spaces indicated
- Write your answers in the spaces provided
- Board approved calculators may be used
- The HSC Reference Sheet is provided.
- Attempt all questions
- ALL necessary working must be shown
- Marks may not be awarded for careless or badly arranged work
- Diagrams are NOT to scale

Topics Covered

- Exponential and Logarithmic Functions
- Trigonometric Functions
- Motion and Rates

Section	Marks
Question 1	/ 12
Question 2	/ 10
Question 3	/ 11
Question 4	/ 13
Total Marks:	**/ 46**

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Question 1 (12 marks)

Student number

(a) Differentiate with respect to x, leaving your answer in simplest form:

(α) $y = \ln 3x$ **1**

(β) $y = e^{(x^2)}$ **1**

(γ) $y = \frac{1}{2}\cos^2 x$ **2**

(δ) $y = \frac{\sin x}{e^{3x}}$ **3**

(b) Find the following indefinite integrals:

(α) $\int 2e^{2x+1}dx$ **1**

(β) $\int \frac{x^2}{x^3+1}dx$ **2**

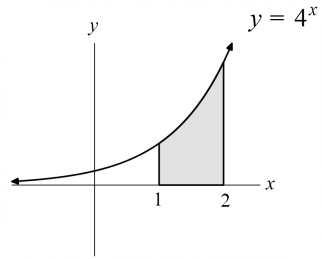
(γ) $\int 4x\sec^2(x^2)\,dx$ **2**

END OF QUESTION 1

Question 2 (10 marks)

Student number

(a) Evaluate the area under the curve $y = 4^x$, bounded by the x axis, $x = 1$ and $x = 2$. Round your answer to 2 decimal places. **2**



(b) Find the equation of the **normal** to the curve $y = \sin 2x$ at the point on the curve where $x = \frac{\pi}{2}$. **4**

(c) The height (h) of a cannonball, in metres, versus time (t), in seconds, is given by the equation $h = 4(t - 3)(10 - t)$.

i) Calculate the time(s) when the cannonball is at a height of 24 metres. **2**

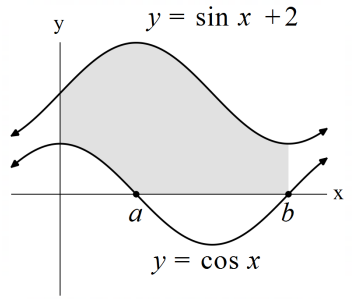
ii) Calculate the velocity the cannonball is travelling at 6 seconds. **2**

END OF QUESTION 2

Question 3 (11 marks)

Student number

(a) Consider the graph below.



i) Identify the exact values of a and b in radians. **1**

ii) Hence, calculate the exact shaded area. **3**

(b) Consider the function $f(x) = x - x \ln x$.

i) Find $f'(x)$. **2**

ii) Hence, find $\int -\ln(x^2)\, dx$. **2**

iii) Using your result from Part i), find the maximum value of $f(x)$. **3**

END OF QUESTION 3

Question 4 (13 marks)

Student number

(a) An ocean buoy's displacement (x), in metres above sea level, is given by the equation $x = 0.8\cos(\frac{\pi}{2}t)$, where t is measured in seconds.

i) Find the initial height of the buoy above sea level. **1**

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ii) Calculate the first time when the buoy's initial height is halved. **2**

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iii) Sketch the graph of the buoy's acceleration versus time for the first four seconds, labelling all key features. **4**

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(b) A scientist was observing the population of bacteria in a Petri dish modelled by the equation $P = 5te^{-0.01t} + 200e^{-0.01t}$,
where P is the population of bacteria, in millions, and t is the time in hours since the scientist started observing.

i) Find the population of bacteria when the scientist started observing. **1**

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ii) How long before the scientist started observing was the population zero? **1**

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iii) Calculate, to the nearest million, the maximum population that the bacteria reached. **4**

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END OF QUESTION 4

END OF ASSESSMENT

1a) α) $y = \ln 3x$

$\frac{dy}{dx} = \frac{1}{3x} \times 3 = \frac{1}{x}$

β) $y = e^{x^2}$

$y' = e^{x^2} \times 2x$

$y' = 2xe^{x^2}$

γ) $y = \frac{1}{2}\cos^2 x = \frac{1}{2}(\cos x)^2$

$y' = \frac{2}{2}(\cos x)\times -\sin x$

$= -\sin x \cos x$

δ) $y = \frac{\sin x}{e^{3x}}$ $u = \sin x$, $u' = \cos x$, $v = e^{3x}$, $v' = 3e^{3x}$

$y' = \frac{e^{3x}\cos x - 3e^{3x}\sin x}{(e^{3x})^2}$

$= \frac{\cos x - 3\sin x}{e^{3x}}$

b) α) $\int 2e^{2x+1}\,dx = \frac{2e^{2x+1}}{2} + c$

$= e^{2x+1} + c$

β) $\int \frac{x^2}{x^3+1}\,dx$ $y = x^3+1$, $y' = 2x^2$

$= \frac{1}{3}\int \frac{3x^2}{x^3+1}\,dx$

$= \frac{1}{3}\ln|x^3+1| + c$

γ) $\int 4x\sec^2(x^2)\,dx$

$= 2\int 2x\sec^2(x^2)\,dx$ $y = x^2$, $y' = 2x$

$= 2\tan x^2 + c$

2a) $\int_1^2 4^x\,dx = \left[\frac{4^x}{\ln 4}\right]_1^2$

$= \frac{16}{\ln 4} - \frac{4}{\ln 4} = \frac{12}{\ln 4}$

$= 8.65617\ldots$

$= 8.66$ (2 dec. places)

2b) $y = \sin 2x$

$y' = 2\cos 2x$

when $x = \frac{\pi}{2}$

$y' = 2\cos\pi = 2\times -1 = -2$

$\therefore \perp m = \frac{1}{2}$

$y = \sin 2\times\frac{\pi}{2} = \sin\pi = 0$

$y - 0 = \frac{1}{2}(x - \frac{\pi}{2})$

$y = \frac{1}{2}x - \frac{\pi}{4}$

$2x - 4y - \pi = 0$ (General form)

c) i) $24 = 4(t-3)(10-t)$

$6 = -t^2 + 13t - 30$

$t^2 - 13t + 36 = 0$

$(t-9)(t-4) = 0$

$\therefore t = 4$ or $t = 9$ secs

ii) $h = 4(-t^2 + 13t - 30)$

$= -4t^2 + 52t - 120$

$v = h' = -8t + 52$

When $t = 6$

$v = -8(6) + 52$

$= 4$ m/s.

3a) i) $a = \frac{\pi}{2}$, $b = \frac{3\pi}{2}$

ii) $A = \int_0^{\frac{3\pi}{2}} \sin x + 2\,dx - \int_0^{\frac{\pi}{2}} \cos x\,dx$

$= \left[-\cos x + 2x\right]_0^{\frac{3\pi}{2}} - \left[\sin x\right]_0^{\frac{\pi}{2}}$

$= \left((-\cos\frac{3\pi}{2} + 3\pi) - (-\cos 0 + 0)\right) - (\sin\frac{\pi}{2} - \sin 0)$

$= (0 + 3\pi + 1) - (1 - 0)$

$= 3\pi$ u²

b) $f(x) = x - x\ln x$

$f'(x) = 1 - (\ln x + x\times\frac{1}{x})$.

$= 1 - \ln x - 1$

i) $= -\ln x$

ii) $\int -\ln(x^2)\,dx$

$= 2\int -\ln x\,dx + c$

$= 2(x - x\ln x) + c$

3b(iii) $f'(x) = -\ln x$

possible max value when $-f'(x) = 0$

$0 = -\ln x$

$0 = \ln x$

$e^0 = x$ $\therefore x = 1$

x	$\frac{1}{2}$	1	2
$f'(x)$	0.69	0	−0.69

$\therefore$ max @ $x = 1$

When $x = 1$, $y = 1 - 1\times\ln 1$

$= 1$

$\therefore$ max value of $f(x) = 1$

4a) i) when $t = 0$ $x = 0.8\cos 0$

$= 0.8$ m

ii) $0.4 = 0.8\cos\frac{\pi}{2}t$

$\frac{1}{2} = \cos\frac{\pi}{2}t$

$\frac{\pi}{2}t = \frac{\pi}{3}$ $\therefore t = \frac{2}{3}$ secs

b) i) when $t = 0$ $P = 0 + 200$

$= 200$ million

ii) $0 = e^{-0.01t}(5t + 200)$

$5t + 200 = 0$

$5t = -200$

$t = -40$ hours

iii) $P' = 5te^{-0.01t}\times 0.01 + 5e^{-0.01t} - 2e^{-0.01t}$

$= -0.05te^{-0.01t} + 3e^{-0.01t}$

$= e^{-0.01t}(-0.05t + 3)$

T.P. when $P' = 0$

$0 = e^{-0.01t}(-0.05t + 3)$

$-0.05t + 3 = 0$

$-0.05t = -3$

$t = 60$ hours.

4b(ii) cont.

t	59	60	61
P'	0.028	0	−0.027

$\therefore$ max at $t = 60$

When $t = 60$

$P = 5\times 60e^{-0.01\times 60} + 200e^{-0.01\times 60}$

$= 274.405818$

$= 274$ million (nearest million)