Barker College

Student Number

Year 12 HSC Mathematics Advanced
Assessment Task 1 2024
(35 Marks)

Sequences and Series
Series and Finance
Discrete Probability Distributions
Graphs and Equations

Staff Involved:
JAI GPF AXD
RAS* DOB PCB ECB EJK SJB

8:30am
Wednesday, 14th February
140 copies

General Instructions:

- Working time - 50 minutes
- Write using black pen
- Write your student number on all pages
- NESA approved calculators may be used
- A reference sheet is provided separately
- Show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for careless or poorly arranged working

Section I - 5 marks
- Attempt Questions 1 - 5
- Circle the most correct answer

Section II- 30 marks
- Attempt Questions 6 - 15
- Write your answers in the spaces provided

Section I
Questions 1 to 5 are multiple choice questions.
Circle the most correct answer.

1. The first three terms in a sequence are 20, 16 and 12.
What is the 11th term in this sequence?

A. 60

B. -20

C. -24

D. $-20\,971\,520$

2. The probability distribution of a random variable X is shown below:

x	0	1	2	3	4
$P(X = x)$	$\frac{3}{10}$	$\frac{1}{20}$	k	$\frac{1}{20}$	$2k$

What is the value of k?

A. $\frac{2}{15}$

B. $\frac{1}{5}$

C. $\frac{1}{10}$

D. $\frac{2}{5}$

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3. An infinite geometric series has a first term of 8 and a limiting sum of 24.
What is the common ratio?

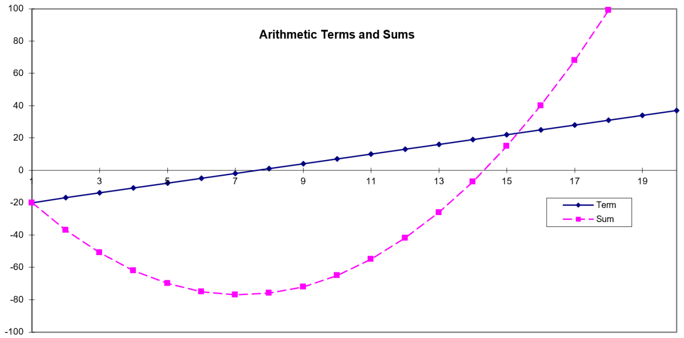
A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{24}{16}$

4. The following graph was produced from **ArithmeticSeries.xls** from the Investigation.



Which of the following statements is true about the graph?

A. $a < 0$ and $d > 0$

B. $a > 0$ and $d > 0$

C. $a > 0$ and $d < 0$

D. $a < 0$ and $d < 0$

5. A function $y = f(x)$ was vertically dilated by a factor of 2, translated 4 units to the right then horizontally dilated by a factor of 3.

Which of the following is the equation of the new function?

A. $y = 2f\left(\frac{x}{3} + 4\right)$

B. $y = 3f\left(\frac{x}{2} - 4\right)$

C. $y = 3f(2x + 4)$

D. $y = 2f\left(\frac{x}{3} - 4\right)$

End of Section I

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Section II

Questions 6 to 15 are extended response questions.
Answer in the spaces provided.

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6. Is 309 a term in the sequence 13, 19, 25, 31, … ?
Justify your answer with mathematical working. **2**

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7. 3, x, 108 are the first three terms of a geometric sequence. Find x. **2**

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8. By first rewriting as an expression including a series, convert $1.\dot{3}\dot{4}$ to an improper fraction. **2**

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9. Lionel and Sam are competing to fill the final place in a mixed soccer team.

Lionel created a discrete probability distribution where the random variable X is the number of goals he scores in a game.

x	0	1	2	3
$P(X = x)$	0.35	0.15	0.1	0.4

i. Calculate $E(X)$ of this distribution. **1**

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ii. Calculate $\mathrm{Var}(X)$, showing working. **2**

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Sam did similar calculations and found she had an expected value of 1.03, with a variance of 0.3.

iii. With reference to the above calculations, who should fill the position in the team? Justify your answer. **2**

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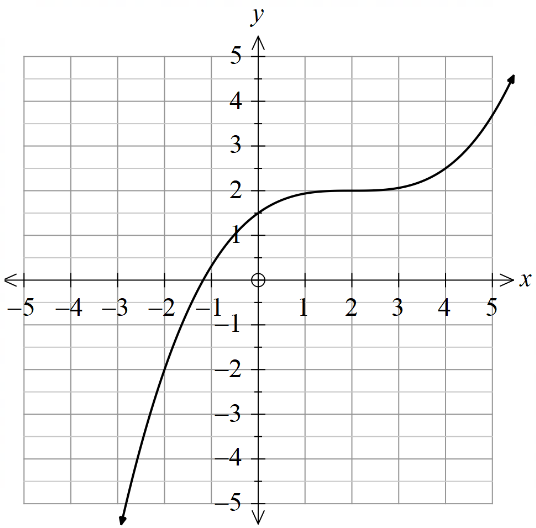
10. A function $y = f(x)$ was transformed to $y = 2f\left(\frac{x}{3}\right) - 4$. If the point $P(4, 5)$ was on the original function, what are the new coordinates of the image of P? **2**

11. Consider the graph of $y = f(x)$ as shown.



On the above number plane, draw a sketch of the function $y = 2f(x) - 4$.
Ensure that you show x and y intercepts. **3**

12. An underground dice game is played at school and has the following rules.

- An entry fee of $1 is paid and a dice is rolled.
- If you roll a 1, you receive nothing.
- If you roll a prime number, you are paid $2.
- If you roll a composite number, you pay an extra $1.

Let *x* be the net financial outcome from the game, including the entry fee.

Complete the following probability distribution table for this game.

x			
P(X = x)			

Find the expected value of the game and determine whether you would expect to win or lose money playing it. **3**

Student number

13. Given the condition for a limiting sum to occur is when $|r| < 1$, use the formula for the sum of a geometric series, $S_n = \frac{a(1-r^n)}{1-r}$, to derive the formula for a limiting sum, $S_\infty = \frac{a}{1-r}$

Ensure that you clearly explain your working. **2**

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14. The graph of the function $f(x) = x^2$ is translated p units to the left then dilated vertically by a factor of q.

The equation of the transformed function is now $g(x) = 3x^2 + 24x + 48$.

Find the values of p and q. **3**

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15. William is saving for a holiday. On the 1st January 2024, he deposited \$4000 into an account earning 2% per month. On the first of each month after this, he deposited \$2000 into the same account, making his final deposit on 1st January, 2025.

i. By first building a series, show that the amount in Willaim's account on 1st January, 2025 after the final deposit was made, is \$31 897.15. **3**

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Question 15 continues on the next page

Student number

ii. William decides to leave this \$31 897.15 in the same account, earning 2% interest per month. On 1st February, 2025 and on every first of the month afterwards, he withdraws \$3000 from the account.

By first building a series, determine how many withdrawals of \$3000 William will be able to make. **3**

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End of Assessment

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1
$a = 20,\ d = -4$
$T_n = a + (n-1)d$
$T_{11} = 20 + (11-1)(-4)$
$= -20$ $\boxed{B}$

2
$\frac{3}{10} + \frac{1}{20} + k + \frac{1}{20} + 2k = 1$
$3k + \frac{2}{5} = 1$
$3k = \frac{3}{5}$
$k = \frac{1}{5}$ $\boxed{B}$

3
$a = 8,\ S_\infty = 24$
$S_\infty = \frac{a}{1-r}$
$24 = \frac{8}{1-r}$
$24(1-r) = 8$
$1 - r = \frac{8}{24}$
$r = 1 - \frac{8}{24}$
$r = \frac{2}{3}$ $\boxed{C}$

4
The term line starts below the x-axis, $\therefore a < 0$
The sum of the terms begins to increase once the term becomes positive, $\therefore d > 0$. $\boxed{A}$

5
1 dilated vert. by a factor of 2 $\Rightarrow$ replace $y \to \frac{y}{2}$
2 translated 4 units right $\Rightarrow$ replace $x \to x-4$
3 dilated horiz. by a factor of 3 $\Rightarrow$ replace $x \to \frac{x}{3}$

1 $y = f(x) \Rightarrow \frac{y}{2} = f(x)$
$y = 2f(x)$
2 $y = 2f(x) \Rightarrow y = 2f(x-4)$
3 $y = 2f(x-4)$
$\Rightarrow y = 2f\left(\frac{x}{3} - 4\right)$
$\boxed{D}$

6
$a = 13,\ d = 6$
$T_n = 13 + (n-1)(6)$
$= 13 + 6n - 6$
$= 6n + 7$
Let $T_n = 309$
$309 = 6n + 7$
$302 = 6n$
$n = \frac{151}{3}$
$\therefore$ 309 is NOT a term in the sequence because n is not an integer.

7
$\frac{T_2}{T_1} = \frac{T_3}{T_2}$
$\frac{x}{3} = \frac{108}{x}$
$x^2 = 324$
$x = \pm 18$

8
$1.\dot{3}\dot{4} = 1 + \underbrace{0.34 + 0.0034 + \ldots}_{a = 0.34,\ r = 0.01}$
$= 1 + \frac{0.34}{1 - 0.01}$
$= 1 + \frac{34}{99}$
$= \frac{133}{99}$

9
(i) $E(X) = 0(0.35) + 1(0.15) + 2(0.1) + 3(0.4)$
$= 1.55$
(ii) $E(X^2) = 0^2(0.35) + 1^2(0.15) + 2^2(0.1) + 3^2(0.4)$
$= 4.15$
$Var(X) = E(X^2) - [E(X)]^2$
$= 4.15 - 1.55^2$
$= 1.75$
(iii)

	Lionel	Sam
$E(X)$	1.55	1.03
$Var(X)$	1.75	0.3

Answer could be either:
1 Lionel $\rightarrow$ Because even though his variance is larger, meaning he is less consistent than Sam, he is expected to score more goals than Sam each game.
2 Sam $\rightarrow$ Although Sam is expected to score fewer goals than Lionel, he is a more consistent player.

10
$y = 2f\left(\frac{x}{3}\right) - 4$
Horizontally:
* dilated by a factor of 3
* no translation

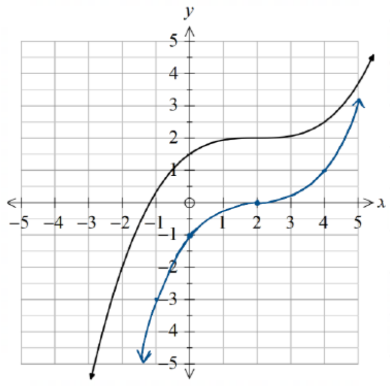
Vertically:
* dilated by a factor of 2
* translated down 4 units

$(4, 5) \longrightarrow (4 \times 3, 5 \times 2 - 4)$
$= (12, 6)$

11
$y = 2f(x) - 4$
Horizontally:
* no dilation or translation

Vertically:
* dilated by a factor of 2
* translated down 4 units

$(2, 2) \longrightarrow (2, 0) \Rightarrow$ new x-int.
$(0, 1.5) \longrightarrow (0, -1) \Rightarrow$ new y-int.
$(4, 2.5) \longrightarrow (4, 1)$



12

x	-2	-1	1
$P(X=x)$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{3}{6}$

$$E(X) = -2\left(\tfrac{2}{6}\right) - 1\left(\tfrac{1}{6}\right) + 1\left(\tfrac{3}{6}\right)$$
$$= -\tfrac{1}{3}$$

$\therefore$ It is expected that around 33 cents will be lost per game, taking into account the \$1 entry fee.

13

$$S_\infty = \frac{a(1-r^\infty)}{1-r}$$

Because $|r|<1$, $r^\infty \to 0$

$$S_\infty = \frac{a(1-0)}{1-r}$$
$$= \frac{a}{1-r}$$

14

Method 1

$$g(x) = 3x^2+24x+48$$
$$= 3(x^2+8x+16)$$
$$= 3(x+4)^2$$

In the form $y=kf(a(x+b))+c$

$\therefore$ $p=4$ and $q=3$

Method 2

$$g(x) = q(x+q)^2$$
$$= q(x^2+2px+p^2)$$
$$= qx^2+2pqx+p^2q$$

$$qx^2+2pqx+p^2q = 3x^2+24x+48$$

Equating coefficients:

$q=3$ and $2pq=24$

$$2p(3)=24$$
$$6p=24$$
$$p=4$$

15 (i) $A_1 = 4000(1.02)+2000$

$$A_2 = A_1\times 1.02+2000$$
$$= [4000(1.02)+2000](1.02)+2000$$
$$= 4000(1.02)^2+2000(1.02)+2000$$

$$A_3 = A_2\times 1.02+2000$$
$$= [4000(1.02)^2+2000(1.02) + 2000](1.02)+2000$$
$$= 4000(1.02)^3+2000(1.02)^2 + 2000(1.02)+2000$$

$\vdots$

$$A_{12} = 4000(1.02)^{12}+2000(1.02)^{11} + 2000(1.02)^{10}+\dots+2000$$
$$= 4000(1.02)^{12}+2000(1.02^{11} + 1.02^{10}+\dots+1)$$
$$= 4000(1.02)^{12}+2000\left(\frac{1.02^{12}-1}{0.02}\right)$$

$$= \$31\,897.15$$

(ii) A_1 now represents the 1st Feb 2025

$$A_1 = 31897.15(1.02)-3000$$
$$A_2 = A_1\times 1.02-3000$$
$$= [31897.15(1.02)-3000](1.02) - 3000$$
$$= 31897.15(1.02)^2-3000(1.02) - 3000$$
$$A_3 = A_2\times 1.02-3000$$
$$= [31897.15(1.02)^2-3000(1.02) - 3000](1.02)-3000$$
$$= 31897.15(1.02)^3-3000(1.02)^2 - 3000(1.02)-3000$$

$\vdots$

$\vdots$

$$A_n = 31897.15(1.02)^n - 3000(1.02)^{n-1} - 3000(1.02)^{n-2} - \dots - 3000$$
$$= 31897.15(1.02)^n - 3000(1.02^{n-1} + 1.02^{n-2}+\dots+1)$$
$$= 31897.15(1.02)^n - 3000\left(\frac{1.02^n-1}{0.02}\right)$$
$$= 31897.15(1.02)^n - \frac{3000}{0.02}(1.02^n-1)$$
$$= 31897.15(1.02)^n - 150000(1.02^n-1)$$
$$= 31897.15(1.02)^n - 150000(1.02)^n+150000$$
$$= -118102.85(1.02)^n+150000$$

When $A_n=0$

$$0 = -118102.85(1.02)^n+150000$$
$$118102.85(1.02)^n = 150000$$
$$(1.02)^n = \frac{150000}{118102.85}$$
$$n = \log_{1.02}\left(\frac{150000}{118102.85}\right)$$
$$= 12.07 \text{ (to 2dp)}$$

$\therefore$ William can make 12 withdrawals.