Student Number

Barker College

2024 YEAR 12

ASSESSMENT TASK 3

Mathematics Advanced

Staff Involved:

Wednesday 5th June, 8:30am

- **PCB*** • **AXD** • **GPF** • **ECB** • **EJK**
- **RAS** • **DOB** • **JAI** • **SJB**

145 copies

General Instructions

- Working time – 50 minutes
- Write using blue or black pen
- Write your Barker Student number in the spaces indicated
- Write your answers in the spaces provided
- Board approved calculators may be used
- The HSC Reference Sheet is provided.
- Attempt all questions
- ALL necessary working must be shown
- Marks may not be awarded for careless or badly arranged work
- Diagrams are NOT to scale

Topics Covered

- Exponential and Logarithmic Functions
- Trigonometric Functions

Section	Marks
Question 1	/ 11
Question 2	/ 12
Question 3	/ 8
Question 4	/ 9
Total Marks:	**/ 40**

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Question 1 (11 marks)

Student number

(a) Differentiate with respect to x:

(α) $y = 5\tan\left(\frac{x}{4}\right)$ **1**

(β) $y = e^{3x^3+5}$ **1**

(γ) $y = 3x^2\ln(2x)$, fully simplify your answer. **2**

(b) Find the following indefinite integrals:

(α) $\int \frac{7}{3x+1}\,dx$ **2**

(β) $\int 3\sin(\pi x)\,dx$ **2**

(c) Evaluate, fully simplifying your answer: **3**

$$\int_{\ln 2}^{\ln 3} e^{3x} + 2e^{x}\,dx$$

End of Question 1

Student number

Question 2 (12 marks)

(a) A curve $y = g(x)$ passes through $(3, 4)$ and has a gradient function: **3**

$$g'(x) = e^{2x-6} + x.$$

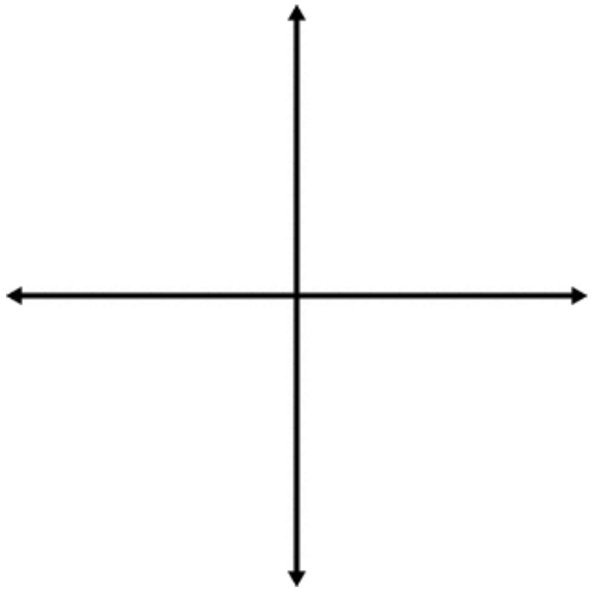
Find the equation of $y = g(x)$.

(b) Find the **gradient** of the **normal** to the curve $y = \sin^3 x$ at the point on the curve where $x = \frac{\pi}{3}$. **3**

(c) A curve has an equation of $y = \frac{6}{x} - 3$

(i) Show that the coordinates of the x-intercept is $(2, 0)$ **1**

(ii) Draw a sketch of $y = \frac{6}{x} - 3$, labelling all key features. **2**



Question 2(c) continues on page 7

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(iii) Find the area bound by the curve $y = \frac{6}{x} - 3$ and the x-axis between $x = 1$ and $x = 5$, giving your answer to **<u>2 decimal places</u>**. **3**

End of Question 2

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Student number

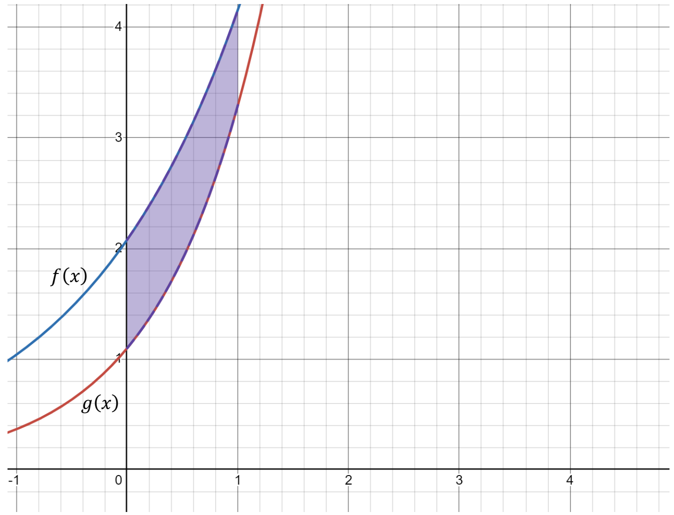
Question 3 (8 marks)

(a) A curve has an equation $y = \ln(x^8) - 6x + \frac{x^2}{2}$:

(i) Find $\frac{dy}{dx}$ in simplest form. **1**

(ii) Find the ***x*-coordinates** of the stationary points and determine their nature. **4**

(b) Below are the graphs of $f(x) = \ln 8 \times 2^x$ and $g(x) = \ln 3 \times 3^x$



Find the shaded area bound by $f(x)$ and $g(x)$ between $x = 0$ and $x = 1$. **3**

End of Question 3

Question 4 (9 marks)

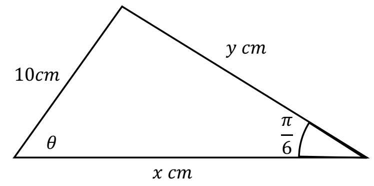
Student number

(a) Consider the function $y = \sec x$

(i) Show, with clear working, that $\frac{dy}{dx} = \sec x \tan x$ **2**

(ii) Hence, find $\int \tan x \, (1 + \sec x) \, dx$ **2**

(b) A triangle has a side length of $10\ cm$ and angle of $\frac{\pi}{6}$ radians as shown.



(i) Show that the length of y can be written as $y = 20 \sin \theta$ **1**

(ii) Hence, show that the perimeter, P, of the triangle can be written as **1**

$$P = 10 + 20 \sin \theta + 20 \sin(\frac{5\pi}{6} - \theta)$$

Question 4(b) continues on page 13

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(iii) Find the value of θ that maximises the perimeter. 3

End of Question 4

End of Assessment

Year 12 Mathematics Advanced - Assessment 3 - 2024 - Solutions

Q1 a)

$\alpha)$ $y' = \frac{5}{4}\sec^2\left(\frac{x}{4}\right)$

$\beta)$ $y' = (9x^2)e^{3x^3+5}$

$\gamma)$ $y' = (6x)(\ln 2x) + \left(\frac{2}{2x}\right)(3x^2)$

$= 6x\ln 2x + 3x$

b)

$\alpha)$ $\frac{7}{3}\int \frac{3}{3x+1}\,dx$

$= \frac{7}{3}\ln|3x+1| + C$

$\beta)$ $\frac{3}{\pi}\int \pi\sin(\pi x)\,dx$

$= \frac{-3}{\pi}\cos(\pi x) + C$

c) $\frac{1}{3}\int_{\ln 2}^{\ln 3} 3e^{3x}\,dx + 2\int_{\ln 2}^{\ln 3} e^{x}\,dx$

$= \frac{1}{3}\left[e^{3x}\right]_{\ln 2}^{\ln 3} + 2\left[e^{x}\right]_{\ln 2}^{\ln 3}$

$= \frac{1}{3}\left(e^{3\ln 3} - e^{3\ln 2}\right) + 2\left(e^{\ln 3} - e^{\ln 2}\right)$

$= \frac{25}{3}$

Q2 a)

$g(x) = \int e^{2x-6} + x\,dx$

$= \frac{1}{2}\int 2e^{2x-6}\,dx + \int x\,dx$

$= \frac{1}{2}e^{2x-6} + \frac{x^2}{2} + C$

$4 = \frac{1}{2}e^{0} + \frac{9}{2} + C$

$-1 = C$

$g(x) = \frac{1}{2}e^{2x-6} + \frac{x^2}{2} - 1$

b) $y' = 3\cos x\sin^2 x$

$m_T = 3\cos\left(\frac{\pi}{3}\right)\sin^2\left(\frac{\pi}{3}\right)$

$= \frac{9}{8}$

$m_N = -\frac{8}{9}$

c)

i) $0 = \frac{6}{x} - 3$

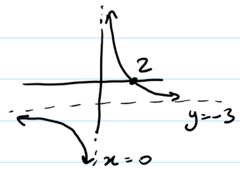
$3 = \frac{6}{x}$

$x = \frac{6}{3}$

$x = 2$

$\therefore (2,0)$

ii)



iii) $A = \int_1^2 \frac{6}{x} - 3\,dx + \left|\int_2^5 \frac{6}{x} - 3\,dx\right|$

$= \left[6\ln|x| - 3x\right]_1^2 + \left|\left[6\ln|x| - 3x\right]_2^5\right|$

$= (6\ln 2 - 6) - (6\ln 1 - 3) + \left|(6\ln 5 - 15) - (6\ln 2 - 6)\right|$

$= 4.66$ units^2

Q3 a)

i) $y = 8\ln x - 6x + \frac{x^2}{2}$

$y' = \frac{8}{x} - 6 + x$

ii) stat pts when $y' = 0$

$0 = \frac{8}{x} - 6 + x$

$0 = x^2 - 6x + 8$

$0 = (x-4)(x-2)$

$x = 2, 4$

x	1	2	3	4	5
y'	3	0	$-\frac{1}{3}$	0	$\frac{3}{5}$

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$x = 2$ max TP

$x = 4$ min TP

b) $A = \int_0^1 \ln 8(2^x)\,dx - \int_0^1 \ln 3(3^x)\,dx$

$= \left[\ln 8\left(\frac{2^x}{\ln 2}\right)\right]_0^1 - \left[\ln 3\left(\frac{3^x}{\ln 3}\right)\right]_0^1$

$= \frac{\ln 8}{\ln 2}(2-1) - (3-1)$

$= 1$

Q4 a)

i) $y = \frac{1}{\cos x} = (\cos x)^{-1}$

$y' = (-1)(-\sin x)(\cos x)^{-2}$

$= \frac{\sin x}{\cos^2 x}$

$= \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$

$= \sec x \tan x$

ii) $\int \tan x\,dx + \int \sec x \tan x\,dx$

$= -\int \frac{-\sin x}{\cos x}\,dx + \sec x + C$

$= -\ln|\cos x| + \sec x + C$

b) i) $\frac{y}{\sin\theta} = \frac{10}{\sin\frac{\pi}{6}}$

$y = \frac{10\sin\theta}{\frac{1}{2}}$

$y = 20\sin\theta$

ii) $\frac{x}{\sin(\pi - \frac{\pi}{6} - \theta)} = \frac{10}{\sin\frac{\pi}{6}}$

$x = 20\sin\left(\frac{5\pi}{6} - \theta\right)$

$P = 10 + y + x$

$= 10 + 20\sin\theta + 20\sin\left(\frac{5\pi}{6} - \theta\right)$

iii) $P' = 20\cos\theta - 20\cos\left(\frac{5\pi}{6} - \theta\right)$

possible max when $P' = 0$

$\cos\left(\frac{5\pi}{6} - \theta\right) = \cos\theta$

$\frac{5\pi}{6} - \theta = \theta$

$\frac{5\pi}{6} = 2\theta$

$\frac{5\pi}{12} = \theta$

θ	1.3	$\frac{5\pi}{12}$	1.4
P'	0.34	0	-3.51

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$\therefore$ max at $\theta = \frac{5\pi}{12}$