
Student Number

Barker College

2024 YEAR 12 ASSESSMENT BLOCK
SEMESTER 1

Mathematics Advanced

Staff Involved: **DATE: Mon 8th April, 2024, 1:30pm**

- R Smith
- E Koorey
- A Davis
- D O'Byrne
- S Byun
- J Thomas *
- P Boyages
- J Iles
- E Barton
- G Fitzmaurice

140 copies

All Preliminary Topics	Graphs & Equations
Sequences & Series	Curve Sketching using Calculus
Series & Finance	Integration

General Instructions:

- Write your Student Number on every answer page
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A separate reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and / or calculations
- Marks may not be awarded for careless or poorly arranged working
- Diagrams are not to scale

Total marks: 80

Section I – 8 marks (pages 2 - 5)
- Attempt Questions 1 - 8
- Allow about 15 minutes for this section

Section II – 72 marks (pages 8 - 24)
- Attempt Questions 9 - 16
- Allow about 1 hours and 45 minutes for this section

Section I

8 marks
Attempt Questions 1 – 8
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 8.

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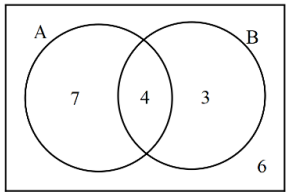
1 A survey of 20 students in a class asked whether students took Art (A) or Business Studies (B). The responses are represented in the Venn Diagram below. If a student is chosen randomly, find $P(A \cap B)$.



A. $\frac{1}{5}$

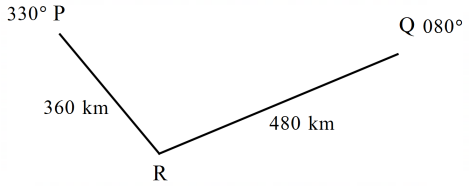
B. $\frac{7}{20}$

C. $\frac{7}{10}$

D. $\frac{13}{20}$

2 Peta and Quinn fly out of Rayburn at the same time. In the first two hours, Peta flies 360 km on a bearing of 330° and Quinn flies 480 km on a bearing of 080°.

How far apart are they after two hours (correct to the nearest kilometre)?



A. 548 km

B. 692 km

C. 360,000 km

D. 478,202 km

3 Which of the following is **NOT** an even function?

A. $f(x) = (x-2)^2$

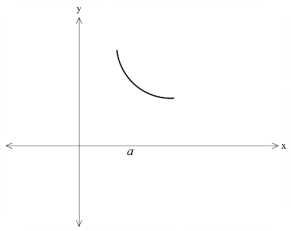
B. $f(x) = -|x|$

C. $f(x) = \cos 3x$

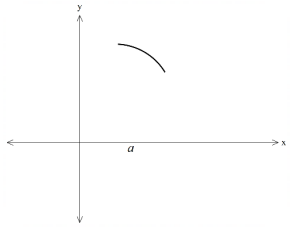
D. $f(x) = e^x + e^{-x}$

4 A function $f(x)$ at the point $x = a$ has the properties: $f(a) > 0, f'(a) < 0, f''(a) > 0$.
Which of the following represents the graph at a?

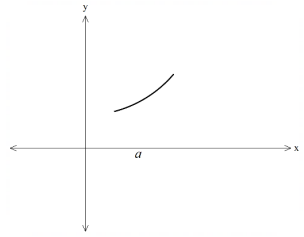
A.



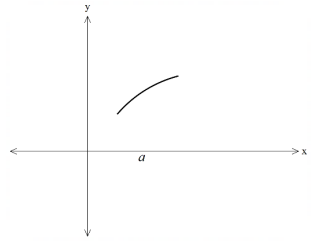
B.



C.



D.



5 Which is the simplified form of $\frac{2\sec^2 A - 2}{4\tan A}$?

A. $\frac{1}{4}\tan A$

B. $-\frac{1}{4}\tan A$

C. $\frac{1}{2}\tan A$

D. $-\frac{1}{2}\tan A$

6 Which of the following gives the domain and range of $y = \frac{2}{x^2+4}$?

A. Domain: $(-\infty, \infty)$ Range: $(0, \infty)$

B. Domain: $(-\infty, \infty)$ Range: $(0, \frac{1}{2}]$

C. Domain: $(-\infty, -2) \cup (-2,2) \cup (2, \infty)$ Range: $(0, \frac{1}{2}]$

D. Domain: $(-\infty, -2) \cup (-2,2) \cup (2, \infty)$ Range: $[\frac{1}{2}, \infty)$

7 Find the 10th term of the arithmetic series $\sqrt{3}+\sqrt{2},\ 2\sqrt{3},\ 3\sqrt{3}-\sqrt{2},\ldots$

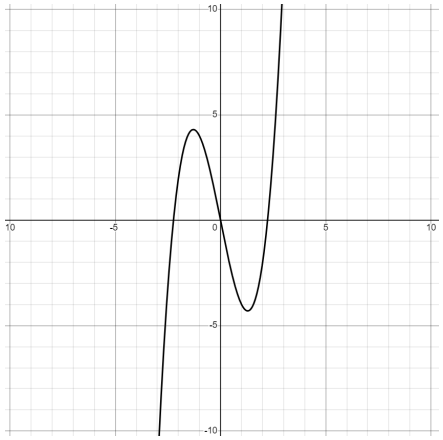
A. $10\sqrt{2} - 8\sqrt{3}$

B. $10\sqrt{3} + 10\sqrt{2}$

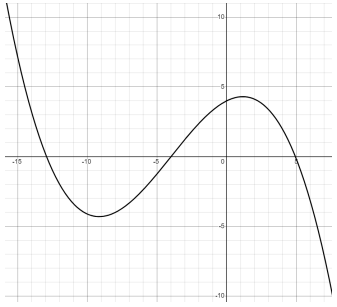
C. $10\sqrt{3} - 8\sqrt{2}$

D. $10\sqrt{2} + 8\sqrt{3}$

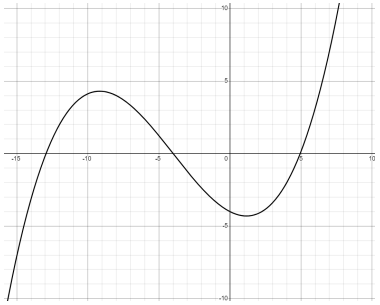
8 If the following is a graph of $y = f(x)$, which of the following represents $y = -f(\frac{x}{4} + 1)$?



A.



B.



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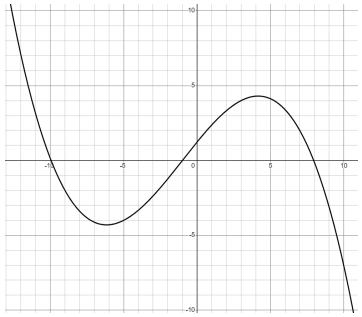
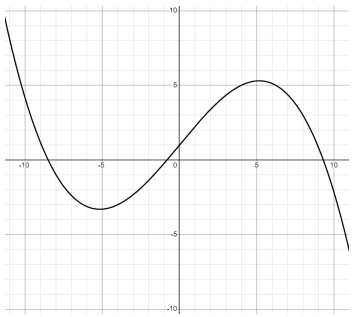


D.



End of Section I

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Student Number

Barker College

Mathematics Advanced

Section II Answer Booklet

72 marks
Attempt Questions 9 – 16
Allow about 1 hour and 45 minutes for this section

Instructions:

- Write your Student Number at the top of the page where indicated.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing paper is available.

Question 9 (9 marks)

a) Differentiate the following with respect to x:

(i) $f(x) = x^2 + \frac{1}{x}$ **1**

(ii) $y = e^{2x}(5x - 2)^3$ [You do not need to simplify your answer] **2**

b) Evaluate the following integral: $\int_{-1}^{2} x^2(3x^2 - 1)dx$ **3**

c) Give an expression for $\log_a\left(\frac{a^2}{54}\right)$ in terms of x and y, given that $x = \log_a 3$ and $y = \log_a 6$. **3**

Question 10 (9 marks)

a) Give the **gradient** of the normal to the curve $y = \frac{x}{1+x}$ at the point $\left(1, \frac{1}{2}\right)$ **2**

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b) Solve the equation $\tan^2(2x) - 1 = 0$ over the domain $0 \leq x \leq \pi$. **3**

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c) (i) Describe, in words, the graph of $x^2 - 6x + 8 + y^2 + 2y = 14$.
S3

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(ii) Give the domain of the graph in part (i) **1**

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Question 11 (10 marks)

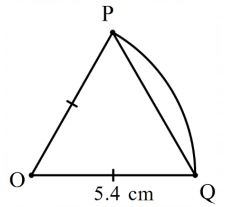
a) Solve the inequation $x^2 - 12x \geq -35$ **2**

b) A geometric series has $T_1 = 2$ and $T_6 = -\frac{64}{243}$. Calculate the limiting sum of the series. **3**

c) (i) Differentiate $f(x) = (2x^5 + 7)^6$ with respect to x. **1**

(ii) Hence, or otherwise, find an expression for $\int 3x^4(2x^5 + 7)^5 \, dx$ **1**

d) Below is a sector of a circle with centre O and radius 5.4 cm. The area of ΔOPQ is 7.29 cm^2. Calculate the exact area of the **sector** OPQ. **3**



Question 12 (8 marks)

For the following questions, the function $h(x) = f(g(x))$.

(i) Given that $f(x) = -x^2 - 8x$ and $g(x) = -x^2$, show that $h(x) = -x^4 + 8x^2$ **1**

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(ii) Find and determine the nature of any stationary points for $y = h(x)$. **3**

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(iii) Sketch $y = h(x)$ on the domain $[-2, 3]$, showing intercepts and stationary points. **3**

(iv) Give the range of $h(x)$ in the given domain. **1**

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Question 13 (9 marks)

a) Using your knowledge of graph transformations, sketch the curve $y = -\frac{1}{x+2} + 1$ in the box below. Show all key features. **3**

b) Use the trapezoidal rule with 2 trapeziums to approximate the area between the curve $y = (\sin x)^2$ and the x −axis between $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$. Give your answer in **exact** form. **3**

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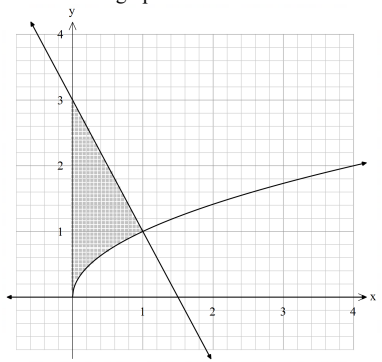
c) Calculate the shaded area bounded by the graphs $y = \sqrt{x}$, $y = -2x + 3$ and the y-axis. Note that the graphs intersect at $x = 1$. **3**



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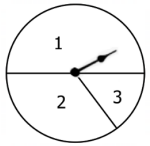
Question 14 (8 marks)

a) The spinner below is spun twice and the sum (x) of the two numbers spun is listed.



The probability distribution table for this event appears below.

x	2	3	4	5	6
$p(x)$	$\frac{1}{4}$	$3m$	$\frac{5}{18}$	m	$\frac{1}{36}$

(i) Show that $m = \frac{1}{9}$ **1**

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(ii) Calculate the standard deviation of the distribution, showing all working. **3**

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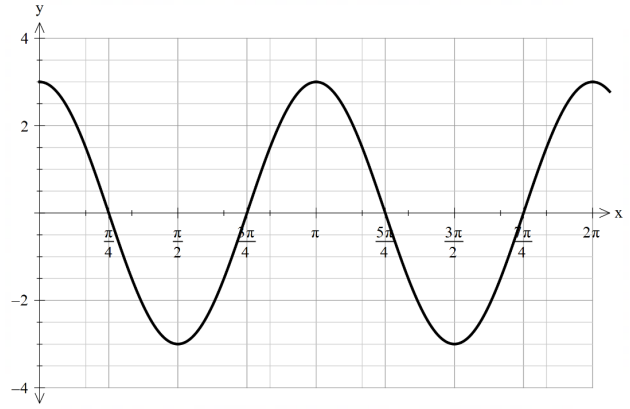
b) Below is the graph of a function in the form $f(x) = a\sin[b(x+\theta)]$ over the domain $[0, 2\pi]$.

(i) Use your knowledge of graph transformations to find the values of a, b and θ. **3**



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(ii) By sketching the line $g(x) = \frac{x}{2} - 2$ onto the graph, state the number of solutions for $f(x) = g(x)$ over the domain $[0, 2\pi]$. **1**

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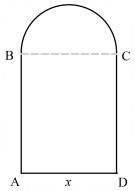
Question 15 (9 marks)

a) The diagram below shows the frame of a massive window, in the form of a semi-circle atop a rectangle. The perimeter of the window frame (which does not include BC) is 20 metres. Let the interval AD be x metres.



(i) Find an expression for CD in terms of x **2**

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(ii) Hence show that the total area of the frame is given by $A = 10x - \frac{x^2}{2}\left(1 + \frac{\pi}{4}\right)$ **1**

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(iii) Find the **exact** value of x which gives the maximum possible area of the window frame. **3**

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b) (i) An experiment involves events A and B. $P(A|B) = 0.18$, $P(A) = 0.4$ and $P(B) = 0.3$. Calculate $P(A \cap B)$. **2**

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(ii) Explain whether or not events A and B are independent. **1**

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Question 16 (10 marks)

a) Find the value of k for the following integral, given that $k > 0$. **3**

$$\int_1^k (k + 3x)\, dx = \frac{13}{2}$$

b) Taylor deposits \$8400 at the beginning of each quarter into an annuity, which earns 8% p.a. interest, compounding quarterly.

(i) The total value at the end of the investment is \$260 655.
By first building a series, calculate the number of deposits she has made. **4**

Question 16b continues on the next page.

(ii) Use the table below to calculate the single amount that Taylor could invest now to end up with the same amount at the end of the investment. **1**

The table gives the present value of $1 invested at given interest rates for given periods.

n / i	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264	0.8116	0.7972
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7312	0.7118
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6587	0.6355
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5935	0.5674
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645	0.5346	0.5066
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132	0.4817	0.4523
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665	0.4339	0.4039
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241	0.3909	0.3606
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855	0.3522	0.3220
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505	0.3173	0.2875
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186	0.2858	0.2567
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897	0.2575	0.2292
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633	0.2320	0.2046
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394	0.2090	0.1827
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176	0.1883	0.1631
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978	0.1696	0.1456
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799	0.1528	0.1300
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635	0.1377	0.1161
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486	0.1240	0.1037
21	0.8114	0.6598	0.5375	0.4388	0.3589	0.2942	0.2415	0.1987	0.1637	0.1351	0.1117	0.0926
22	0.8034	0.6468	0.5219	0.4220	0.3418	0.2775	0.2257	0.1839	0.1502	0.1228	0.1007	0.0826
23	0.7954	0.6342	0.5067	0.4057	0.3256	0.2618	0.2109	0.1703	0.1378	0.1117	0.0907	0.0738
24	0.7876	0.6217	0.4919	0.3901	0.3101	0.2470	0.1971	0.1577	0.1264	0.1015	0.0817	0.0659
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923	0.0736	0.0588
26	0.7720	0.5976	0.4637	0.3607	0.2812	0.2198	0.1722	0.1352	0.1064	0.0839	0.0663	0.0525
27	0.7644	0.5859	0.4502	0.3468	0.2678	0.2074	0.1609	0.1252	0.0976	0.0763	0.0597	0.0469
28	0.7568	0.5744	0.4371	0.3335	0.2551	0.1956	0.1504	0.1159	0.0895	0.0693	0.0538	0.0419
29	0.7493	0.5631	0.4243	0.3207	0.2429	0.1846	0.1406	0.1073	0.0822	0.0630	0.0485	0.0374
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573	0.0437	0.0334

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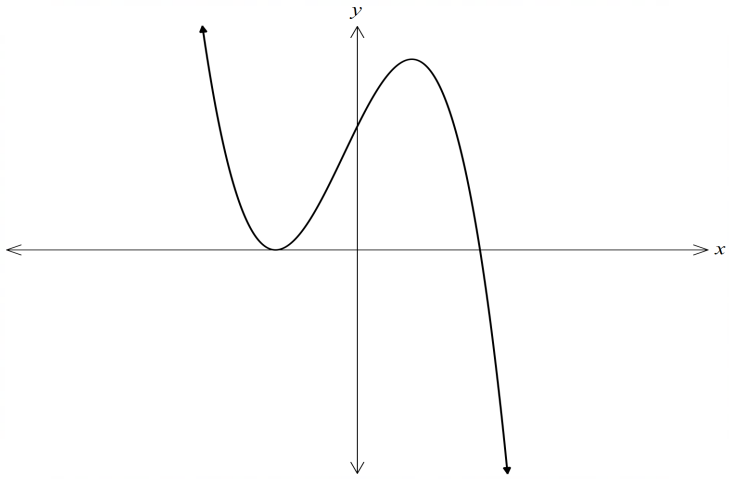
Question 16 continues on the next page

c) The diagram shows the graph of a function $y = f'(x)$. On the same diagram, sketch a possible graph of $y = f(x)$. **2**



End of Paper

Multiple Choice:
1. A 2. B 3. A 4. A 5. C 6. B 7. C 8. A

Q9 a) i) $f'(x) = 2x - \frac{1}{x^2}$ or $2x - x^{-2}$

ii) $u = e^{2x}$ $u' = 2e^{2x}$ $v = (5x-2)^3$ $v' = 15(5x-2)^2$
$y' = 15e^{2x}(5x-2)^2 + 2e^{2x}(5x-2)^3$

b) $\int_{-1}^{2}(3x^4 - x^2)dx = \left[\frac{3x^5}{5} - \frac{x^3}{3}\right]_{-1}^{2}$
$= \frac{3(2)^5}{5} - \frac{2^3}{3} - \left(\frac{3(-1)^5}{5} - \frac{(-1)^3}{3}\right)$
$= \frac{84}{5}$ or 16.8

c) $\log_a\left(\frac{a^2}{54}\right) = \log_a a^2 - \log_a 54$
$= 2 - (\log_a 9 + \log_a 6)$
$= 2 - (2x + y)$
$= 2 - 2x - y$

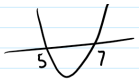
Q10 a) $u = x$ $u' = 1$ $v = 1+x$ $v' = 1$
$y' = \frac{1+x-x}{(1+x)^2} = \frac{1}{(1+x)^2}$
$m_{tan} = \frac{1}{(1+1)^2} = \frac{1}{4}$ $\therefore m_{normal} = -4$

b) $0 \le 2x \le 2\pi$ $\tan^2(2x) = 1$
$\tan(2x) = \pm 1$ $2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$

c) i) $(x-3)^2 - 1 + (y+1)^2 - 1 = 14$
$(x-3)^2 + (y+1)^2 = 16$
$\therefore$ Circle with radius 4, centre $(3,-1)$

ii) $[-1, 7]$

Q11 a) $x^2 - 12x + 35 \ge 0$
$(x-7)(x-5) \ge 0$
$\therefore x \le 5$ or $x \ge 7$
$(-\infty, 5] \cup [7, \infty)$



b) $a = 2$ $2r^5 = \frac{-64}{243}$
$r^5 = \frac{-32}{243}$
$r = -\frac{2}{3}$
$S_\infty = \frac{2}{1+\frac{2}{3}}$
$= \frac{6}{5}$

c) i) $f'(x) = 60x^4(2x^5+7)^5$

ii) $\frac{1}{20}\int 60x^4(2x^5+7)^5$
$= \frac{1}{20}(2x^5+7)^6 + C$

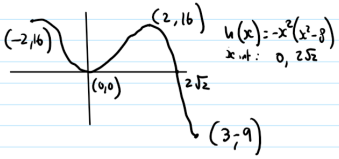
d) $7.29 = \frac{1}{2} \times 5.4^2 \times \sin C$
$C = \sin^{-1}\left(\frac{2 \times 7.29}{5.4^2}\right)$
$= 30°$ or $\frac{\pi}{6}$
$A_{sec} = \frac{1}{12} \times \pi \times 5.4^2$
$= \frac{243\pi}{100}$

Q12 i) $f(g(x)) = -(-x^2)^2 - 8(-x^2)$
$= -x^4 + 8x^2$

ii) $h'(x) = -4x^3 + 16x$
$0 = -4x(x^2 - 4)$
$= -4x(x+2)(x-2)$
$\therefore$ stat at $-2, 0, 2$

x	-3	-2	-1	0	1	2	3
$h'(x)$	60	0	-12	0	12	0	-60
		max		min		max	

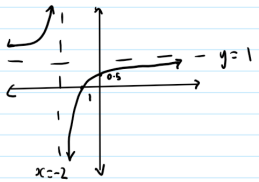
iii)



$h(x) = -x^2(x^2-8)$
x int: $0, 2\sqrt{2}$

iv) $-9 \le h(x) \le 16$ or $[-9, 16]$

Q13 a)



b) $A \approx \frac{\pi}{24}\left(\sin^2\left(\frac{\pi}{6}\right) + \sin^2\left(\frac{\pi}{3}\right) + 2\sin^2\left(\frac{\pi}{4}\right)\right)$
$= \frac{\pi}{24}\left(\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2\right)$
$= \frac{\pi}{12}$

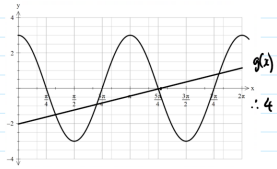
c) $\int_0^1 (-2x+3)dx - \int_0^1 \sqrt{x}\,dx$
area of trap.
$= \frac{1}{2}(3+1) - \left[\frac{2x^{\frac{3}{2}}}{3}\right]_0^1$
$= 2 - \frac{2}{3}$
$= \frac{4}{3}$

Q14 a) i) $4m + \frac{1}{4} + \frac{5}{18} + \frac{1}{36} = 1$
$4m = \frac{4}{9}$
$m = \frac{1}{9}$

ii) $\mu = \frac{10}{3}$, $\text{var}(X) = \frac{10}{9}$, $\sigma = \frac{\sqrt{10}}{3}$

b) i) $a = 3$ $b = 2$ $\theta = \frac{\pi}{4}$

ii)



$\therefore$ 4 solutions

Q15 a) i) $20 = \pi \times \frac{x}{2} + x + 2CD$
$2CD = 20 - x - \frac{\pi}{2}x$
$CD = 10 - \frac{x}{2} - \frac{\pi}{4}x$

ii) $A = \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 + x\left(10 - \frac{x}{2} - \frac{\pi}{4}x\right)$
$= \frac{\pi x^2}{8} + 10x - \frac{x^2}{2} - \frac{\pi}{4}x^2$
$= 10x - \frac{x^2}{2}\left(1 + \frac{\pi}{2} - \frac{\pi}{4}\right)$
$= 10x - \frac{x^2}{2}\left(1 + \frac{\pi}{4}\right)$

iii) $A' = 10 - x\left(1 + \frac{\pi}{4}\right)$
$0 = 10 - x\left(1 + \frac{\pi}{4}\right)$
$x = \frac{10}{1+\frac{\pi}{4}}$

x	$\frac{10}{1+\frac{\pi}{4}}$	5	6
A'	1.07	0	-0.71
	max		

$\therefore$ max at $x = \frac{10}{1+\frac{\pi}{4}}$

b) i) $P(A|B) = \frac{P(A \cap B)}{P(B)}$ $\therefore P(A \cap B) = 0.054$
$0.18 = \frac{P(A \cap B)}{0.3}$

ii) $P(A) \times P(B) = 0.12 \ne P(A \cap B)$
$\therefore$ not independent

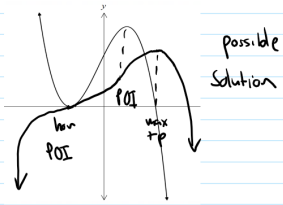
Q16 a) $\left[kx + \frac{3x^2}{2}\right]_1^k = \frac{13}{2}$
$k^2 + \frac{3k^2}{2} - \left(k + \frac{3}{2}\right) = \frac{13}{2}$
$5k^2 - 2k - 16 = 0$
$k = \frac{2 \pm \sqrt{2^2 - 4 \times 5 \times -16}}{10}$
$= \frac{2 \pm 18}{2}$ but $k > 0$
$\therefore k = 2$

Q16 a) $\left[kx + \frac{3x^2}{2}\right]_1^k = \frac{13}{2}$
$k^2 + \frac{3k^2}{2} - \left(k + \frac{3}{2}\right) = \frac{13}{2}$
$5k^2 - 2k - 16 = 0$
$k = \frac{2 \pm \sqrt{2^2 - 4 \times 5 \times -16}}{10}$
$= \frac{2 \pm 18}{2}$ but $k > 0$
$\therefore k = 2$

b) i) $A_1 = 8400 \times 1.02$ $A_2 = 8400 \times 1.02 + 8400 \times 1.02^2$
$A_n = 8400\left(1.02 + 1.02^2 + \ldots + 1.02^n\right)$
$260655 = 8400\left(\frac{1.02 \times (1.02^n - 1)}{0.02}\right)$
$260655 = 428400(1.02^n - 1)$
$\frac{260655}{428400} + 1 = 1.02^n$
$n = \log_{1.02} 1.608438$
$n = 24$ deposits

ii) $PV = 0.6217 \times 260655$
$= \$162049.21$

c)



possible Solution