Barker College

Student Number

Year 11, 2024 Mathematics Advanced Assessment Task 1 (50 Marks)

In-class Component:

Algebra, Surds,

Functions and Transformations, Trigonometry

50 minutes

Period 4 or 5

THURSDAY 28th MARCH

235 copies

ALY	**A Young**
RJW	**R Williams**
DXC	**D Chua**
GDH	**G Hanlon**
BHC	**B Carruthers**
ARP	**A Perkins**
ECB	**E Barton**
DZB	**D Batchen**
SJB	**S Byun**
JZT	**J Thomas**
DZP	**D Peattie**
FHD	**F Deng***

General Instructions:

- Working time - 50 minutes
- Write using black pen
- NESA approved calculators may be used
- Diagrams are not to scale
- A copy of the diagram in Section II is on the last page and can be detached

Section I - 29 marks

- Attempt Questions 1 - 13

Section II - 21 marks

- Attempt Questions 14 - 17

Total 50 marks

Section I - 29 marks

Attempt Questions 1 – 13.

1. Which type of graph is classified as a **many to many**? 1 mark

(A) Straight line

(B) Quadratic

(C) Circle

(D) Hyperbola

2. Find the correct solutions of $|2x - 1| = 5$. 1 mark

(A) $x = \pm 3$

(B) $x = \pm 2$

(C) $x = -3, x = 2$

(D) $x = 3, x = -2$

3. Which of the following quadratic equation has only one real solution? 1 mark

(A) $x^2 + 1 = 0$

(B) $x^2 - 2x + 1 = 0$

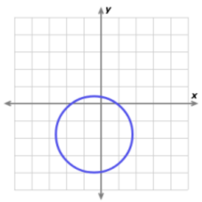
(C) $x^2 - 2x - 1 = 0$

(D) $x^2 + 2x - 1 = 0$

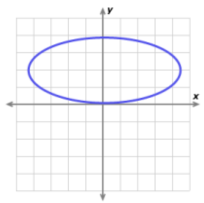
4. Using the vertical line test, select the graph that is a function.

1 mark

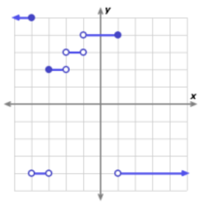
(A)



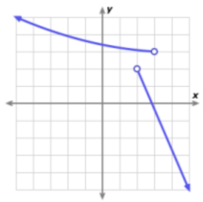
(B)



(C)



(D)



5. Choose the **INCORRECT** statement about the graph of the piecewise function below.

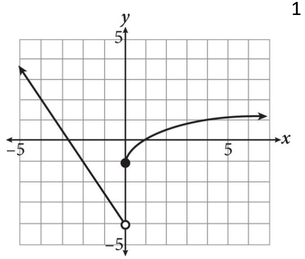
1 mark

(A) The domain is $x \neq 0$.

(B) The range is $y > -4$.

(C) $(-4, 2)$ is on the graph of this function.

(D) The function is neither odd nor even.



6. Explain how to translate the graph of $y = x^2$ to obtain the function $y = (x + 2)^2 - 5$.

2 marks

7. State the domain and range of

a. $x^2 + y^2 = 25$ 1 mark

Domain:

Range:

b. $y = \sqrt{9 - x^2}$ 2 marks

Domain:

Range:

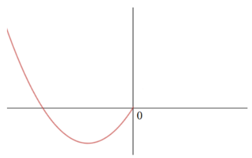
c. $y = 1 - 2x^2$, in interval form 2 marks

Domain:

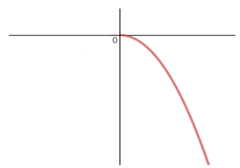
Range:

8. In each diagram below, complete the graph so that: 2 marks

a. f(x) is odd



b. f(x) is even



9. If $f(x) = x^2 + x + 10$ and $g(x) = -3x - 4$, find and simplify the following expressions.

i. $-f(x) + 5$ 1 mark

ii. $g(-2) + f(1)$ 1 mark

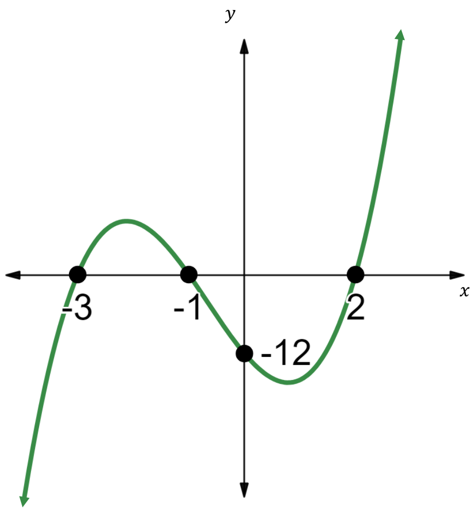
iii. $g(f(x^2))$ 2 marks

10. Show whether this function is odd, even or neither: $f(x) = \frac{2x}{x^2-1}$. 2 marks

11. Find the value of *a* and *b* given $\left(4 + \sqrt{2}\right)^2 = a + \sqrt{b}$. 2 marks

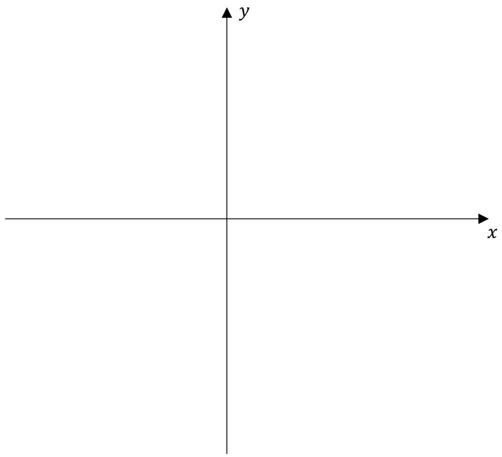
12. The graph of $y = g(x)$ is shown below.

Determine the equation of the function $y = g(x)$. 2 marks

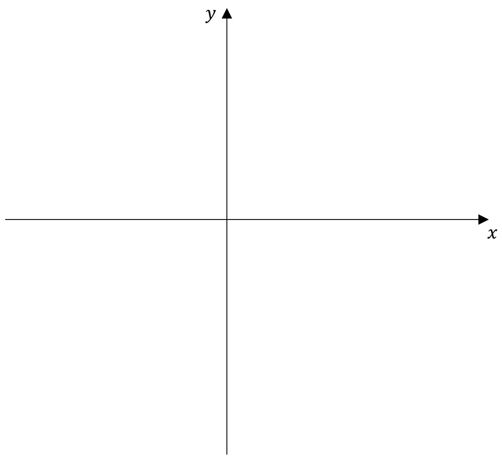


13. Sketch the following functions, showing all asymptotes and intercepts (if no intercepts, show the coordinates of a point on the curve).

i. $y = \frac{1}{x}$ 2 marks



ii. Hence $y = \frac{1}{x+2} + 1$ 3 marks



End of Section I

Section II - 21 marks

Attempt Questions 14 – 17

The Sydney Harbour Bridge is a well-known example of a *through arch bridge*, where the *base* of an *arch* structure is below the *deck* but the top rises above it.

The x-axis is the level of the water. The apex of the lower arch is shown with its coordinates below.

14. Answer the following questions for the *upper arch*.

i. The Aboriginal flag and the flag of Australia are flying above the bridge at its apex. Write down the coordinates of the apex using the equation of the *upper arch*:

$$y = -0.1(x-3)^2 + 1.6$$

1 mark

(,)

ii. Using algebra, find the x-intercepts of the *upper arch*. Show your working. 2 marks

iii. Hence, state the domain and range of the *upper arch*.

2 marks

iv. The Sydney Harbour Bridge is the world's tallest steel arch bridge, measuring 134 m from the apex to water level. Using the results from i and ii, find the *real-life horizontal distance* of the span of the *upper arch* at water level.

2 marks

15. Answer the following questions for the *lower arch.*

i. The *lower arch* starts at $(0, 0)$ and the coordinates of its apex are $(3, 1.35)$.

Using $y = a(x - h)^2 + k$, find the equation of the *lower arch*.

2 marks

ii. Hence, find the equation of the parabola representing the **reflection** of *the lower arch* in the water.

2 marks

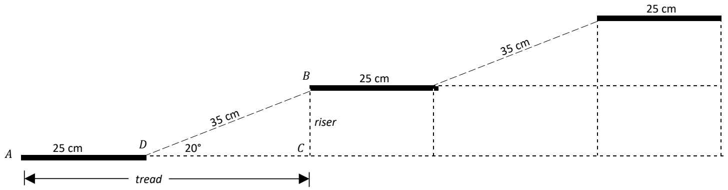
16. Answer the following questions in regard to some parts of the stairs.

Since 1998, Bridge Climb has made it possible for tourists to legally climb the bridge.

Before climbing, the guests are reminded that some parts of the stairs on the bridge have gaps between each stair, as shown in the photo on the right.

The straight-line gap between each stair is 35 cm and each step is 25 cm deep.

The angle of elevation between the closest edges of the stairs is 20°. AC is the tread and BC is the riser. See the diagram below.



i. In ΔBCD, find the length of BC (riser) and DC. Round to the nearest cm. 2 marks

ii. What is the gradient of one step? Use the values of riser and tread. 1 mark

iii. Draw a straight line connecting AB. 1 mark

Find the size of $\angle BAC$, rounded to the nearest minute.

17. Answer the following questions for the *deck* (on which the trains, cars, and the pedestrians travel).

i. A piecewise function is created to roughly describe the location of the *deck*.

$$f(x) = \begin{cases} 0.03x + b, & x < 3 \\ -0.03x + 0.78, & x \geq 3 \end{cases}$$

Find the value of b so that the piecewise function is continuous at $x = 3$ (meaning, the two parts of the piecewise function join at $x = 3$).

2 marks

ii. Groups of climbers start to climb the *upper arch* from the southern side of the *deck* (on the right side of the photo), close to the Sydney Opera House.

Using the function provided in (i) and the equation of the *upper arch* $y = -0.1(x-3)^2 + 1.6$, find the **coordinates** of the point where the bridge climbers enter the *upper arch*. Round the results to 2 decimal places.

4 marks

End of Assessment

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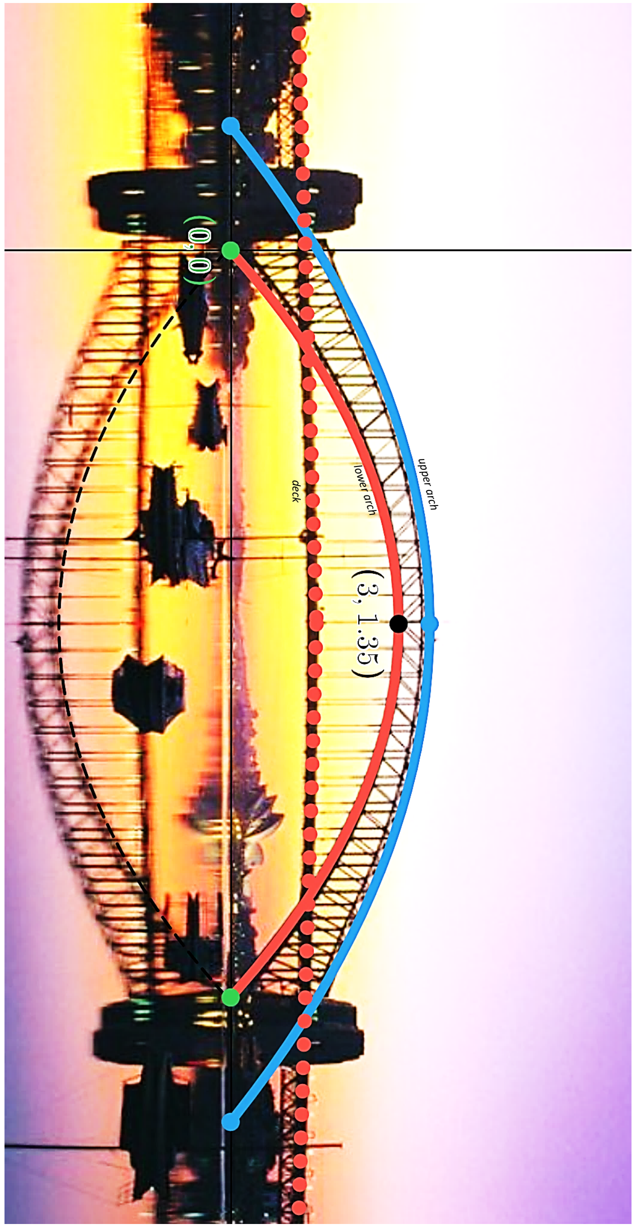


Use the following diagram to answer Question 14 to 17.

Y11 Advanced AT1 Student Solutions

1) C

2) D $2x-1=5$ $\quad$ $2x-1=-5$

$2x=6$ $\quad$ $2x=-4$

$x=3$ $\quad$ $x=-2$

3) B $b^2-4ac=0$

$(-2)^2-4(1)(1)=0$

$4-4=0$ ✓

4) C

5) Accepted A, B or C

6) Left 2 units, down 5 units

7a) D: $[-5,5]$

R: $[-5,5]$

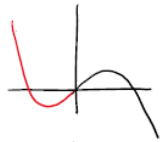
7b) D: $-3 \le x \le 3$

R: $0 \le y \le 3$

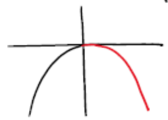
7c) D: $(-\infty, \infty)$

R: $(-\infty, 1]$

8a)



8b)



9i) $-(x^2+x+10)+5$

$= -x^2-x-5$

9ii) $g(-2)+f(1) = (-3(-2)-4)+(1^2+1+10)$

$= 14$

9iii) $f(x^2) = (x^2)^2+x^2+10$

$g(f(x^2)) = -3(x^4+x^2+10)-4$

$= -3x^4-3x^2-34$

10) $f(-x) = \dfrac{2(-x)}{(-x)^2-1}$

$= \dfrac{-2x}{x^2-1}$

$-f(x) = -\dfrac{2x}{x^2-1}$

$f(-x) = -f(x)$ $\therefore$ odd

11) $(4+\sqrt{2})(4+\sqrt{2})$

$18+8\sqrt{2}$

$18+\sqrt{128}$

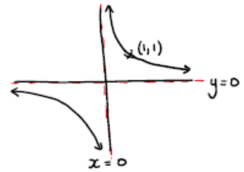
12) $y = a(x+3)(x+1)(x-2)$

$-12 = a(x+3)(x+1)(x-2)$

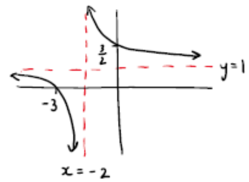
$\therefore a = 2$

$y = 2(x+3)(x+1)(x-2)$

13i)



13ii)



Section 2

14i) $(3,\ 1.6)$

14ii) $0 = -0.1(x-3)^2 + 1.6$

$-1.6 = -0.1(x-3)^2$

$16 = (x-3)^2$

$\pm 4 = x - 3$

$x = 3 \pm 4$

$x = 7 \quad x = -1$

14iii) $D: [-1, 7]$

$R: [0, 1.6]$

14iv) $\frac{134}{1.6} = \frac{x}{8}$

$x = 670\text{m}$

15i) $y = a(x-3)^2 + 1.35$

$0 = 9a + 1.35$

$a = -0.15$

$y = -0.15(x-3)^2 + 1.35$

15ii) $y = 0.15(x-3)^2 - 1.35$

16i) $\sin 20 = \frac{BC}{35}$

$BC = 11.97\text{cm}$

$\cos 20 = \frac{DC}{35}$

$DC = 32.89\text{ cm}$

16ii) $m = \frac{35\sin 20}{25 + 35\cos 20}$

$m = 0.21$

16iii) $\tan\theta = \frac{35\sin 20}{25 + 35\cos 20}$

$\theta = 11.683..$

$\theta = 11°41'$

17i) $y = -0.03(3) + 0.78$

$y = 0.69$

$0.69 = 0.03(3) + b$

$b = 0.6$

17ii) $-0.03x + 0.78 = -0.1(x-3)^2 + 1.6$

$0.1x^2 - 0.63x + 0.08 = 0$

$10x^2 - 63x + 8 = 0$

$x = \frac{-(-63) \pm \sqrt{(-63)^2 - 4(10)(8)}}{2(10)}$

$x = 6.17$

$\therefore\ y = -0.03(6.17...) + 0.78$

$y = 0.59$

$\therefore\ (6.17,\ 0.59)$