



BAULKHAM HILLS HIGH SCHOOL

Preliminary Task 1 2019

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 120 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11-14
- Marks may be deducted for careless or badly arranged work

Total marks – 70

Exam consists of 7 pages.

This paper consists of TWO sections.

Section 1 – Page 2-3 (10 marks)

Questions 1-10

- Attempt Questions 1-10
- Answer on answer sheet provided
- Allow about 15 minutes for this section

Section II – Pages 4-7 (60 marks)

- Attempt Questions 11-14
- Answer on answer sheet provided
- Allow about 1 hour and 45 minutes for this section

Section I - 10 marks**Use the multiple choice answer sheet for question 1-10**

1. $\sqrt{\frac{80}{27}} =$
- (A) $\frac{4\sqrt{5}}{9\sqrt{3}}$ (B) $\frac{4\sqrt{5}}{3\sqrt{3}}$
- (C) $\frac{8\sqrt{5}}{9\sqrt{3}}$ (D) $\frac{8\sqrt{5}}{3\sqrt{3}}$
2. $|-2|^2 - |-1| + |4| =$
- (A) 10 (B) 9
- (C) 7 (D) 6
3. Factorise fully $x^3 - 4x^2 - x + 4$
- (A) $(x^2 - 1)(x - 4)$ (B) $(x^2 + 1)(x - 4)$
- (C) $x^2(x - 4)$ (D) $(x - 4)(x - 1)(x + 1)$
4. For two events A and B , conditional probability $P(A|B)$ is given by:
- (A) $\frac{P(A \cup B)}{P(B)}$ (B) $\frac{P(A \cap B)}{P(A)}$
- (C) $\frac{P(A \cup B)}{P(A)}$ (D) $\frac{P(A \cap B)}{P(B)}$
5. A database is used to record the names of all the people in Australia and the doctors they have visited. This is an example of a relation which is
- (A) one-to-one (B) many-to-one
- (C) many-to-many (D) one-to-many
6. In a group of 30 students, 19 catch a train to school and 21 catch a bus and 5 catch neither. If one of these student is chosen at random, find the probability that the student catches a bus but not a train to school.
- (A) $\frac{6}{30}$ (B) $\frac{21}{30}$
- (C) $\frac{3}{5}$ (D) $\frac{3}{30}$
7. $P(X) = 0.2$ and $P(X \cap Y) = 0.06$. What is the value of $P(Y)$, given that X and Y are independent events.
- (A) 0.6 (B) 0.3
- (C) 0.02 (D) 0.04

8. Let $f(n) = 1 \times 2 \times 3 \times \dots \times n - 1 \times n$. Simplify $\frac{f(n)}{f(n-1)}$ (where n is a positive integer).
- (A) $n - 1$ (B) $n + 1$
(C) 1 (D) n
9. Given the functions $f(x) = 2x - 10$ and $g(x) = \sqrt{x + 2}$, what is the value of a for which $f(g(a)) = 0$?
- (A) 4 (B) 25
(C) 23 (D) -7
10. If x is a negative number, which of the following statements is **FALSE**?
- (A) $|x| = -x$ (B) $2x < x$
(C) $-x > x$ (D) $\sqrt{x^2} = x$

End of Section I

Section II – Extended Response(56 marks)

Attempt questions 11-14. All necessary working should be shown in every question. Answer each question on the appropriate page in the writing booklet.

Question 11 (15 marks) Use the Question 11 section of the writing booklet.

Marks

a) Simplify

2

$$18 - 5(3y - 2) + 4y$$

b) Solve simultaneously for x and y .

2

$$\begin{aligned} 3x + 2y &= 5 \\ 2x - y &= -6 \end{aligned}$$

c) Find the value(s) of n if

2

$$(x - 2n)(x + 2n) = x^2 - 16$$

d) By rationalising the denominator find the values of x and y , such that

3

$$\frac{\sqrt{3}}{3 + 2\sqrt{3}} = x - y\sqrt{3}$$

e) Factorise and simplify

3

$$\frac{2st - t^2}{2s^2 + st - t^2} \div \frac{s - t}{s^2 - t^2}$$

f) Simplify by expressing as one fraction

2

$$\frac{7x + 1}{3x - 6} - \frac{2x + 1}{x - 2}$$

g) If $f(x) = 3kx + 1$ and $f(-2) = 7$, find the value of k

1

End of Question 11

Question 12 (15 marks) Use the Question 12 section of the writing booklet.

- a) Find the domain and the range of the following function

$$y = \frac{2}{\sqrt{3x-2}} \quad 2$$

- b) State whether the following function is even, odd or neither. Justify your answer.

$$f(x) = x^3 + 3x \quad 2$$

- c) The frequency, f kilohertz, of a radio wave is inversely proportional to its wavelength, w metres. When $w = 200$ and $f = 1500$

- (i) Express f in terms of w . 1

- (ii) On the axes provided, sketch the graph of f against w . 1

- (iii) The wavelength of a radio wave is 1250 m, calculate the frequency. 1

- d) (i) Completely factorise the polynomial $P(x) = x^4 + 2x^3 - 3x^2$. 2

- (ii) Sketch the graph of $y = P(x)$. 2

- e) Make x the subject of the formula 2

$$y = \frac{x-a}{x+b}$$

- f) Find the centre and radius of the circle given by $x^2 + y^2 + 10x - 6y + 30 = 0$ 2

End of Question 12

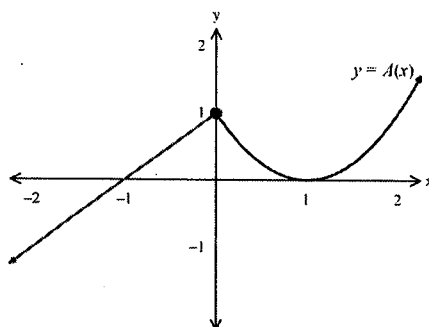
Question 13 (15 marks) Use the Question 13 section of the writing booklet.**Marks**

a) Solve

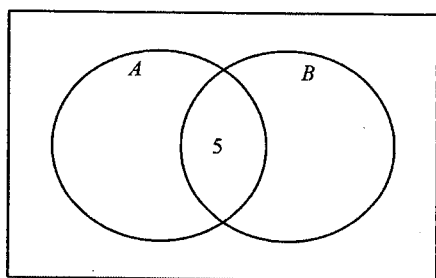
(i) $|1 - 2x| = 5$

2

(ii) $\frac{3x+2}{5} - \frac{x-3}{4} = 1$

2b) Draw $y = -f(x - 1)$ for the graph of $y = f(x)$ given below.**2**c) Sketch the parabola given by $y = -x^2 + 3x - 2$. Showing the x and y -intercepts and coordinates of vertex.**3**d) If $f(x) = \begin{cases} 2x + 4 & \text{for } x > 0 \\ -x - 4 & \text{for } x \leq 0 \end{cases}$ **2**evaluate $f(2) + f(0) + f(-2)$ e) For the two events A and B we are given that $n(A \cap B) = 5$, $n(A) = 11$,
 $n(A \cup B) = 12$ and $n(\bar{B}) = 8$.

(i) Complete the Venn diagram:

2(ii) Hence find $P(A \cup B)$ **1**f) The function $y = f(x)$ is an odd function. It is known that the point $(3, -5)$ lies on the graph of $y = f(x)$. State the coordinates of another point that you can be sure lies on the graph.**1****End of Question 13**

Question 14 (15 marks) Use the Question 14 section of the writing booklet.**Marks**

a) Factorise the following

2

(i) $x^2 + 2x + 1 - y^2$

(ii) $8x^2 - 16x - 42$

2

b) Ryan has a chance of 0.2 of winning a prize in Taekwondo competition. If he enters 3 competitions, find the probability of him winning a prize for

(i) exactly two competitions

1

(ii) at least one competition.

2

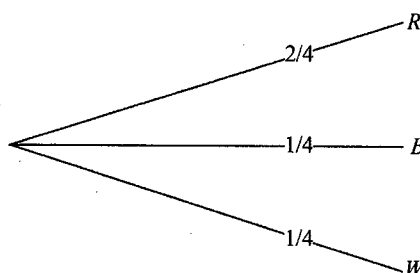
c) Simplify

2

$$\frac{3^{n+3} - 3^n}{3^n}$$

d) A bag contains two red balls, one black ball and one white ball. Varun selects one ball from the bag and keeps it hidden. He then selects a second ball, also keeping it hidden.

(i) Complete the probability tree to show all the possible outcomes.

1

(ii) Find the probability that both the selected balls are red.

1

(iii) Find the probability that at least one of the selected balls is red.

1

(iv) Varun drops one of the selected balls and we can see that it is red. What is the probability that the ball that is still hidden is also red?

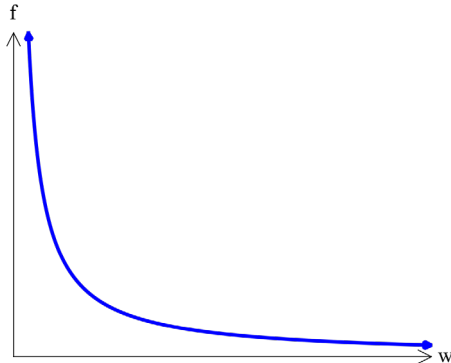
1e) For every pair of numbers a and b the function $f(x)$ satisfies**2**

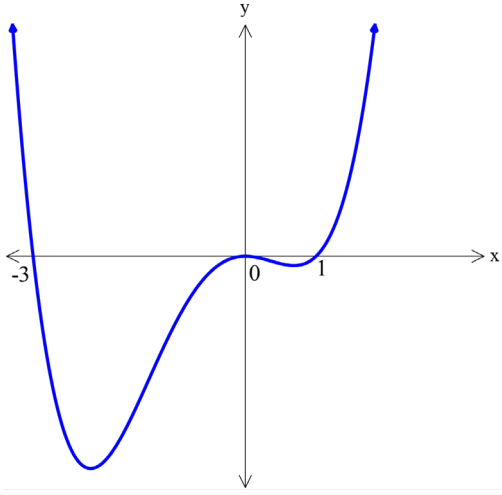
$$b^2 f(a) = a^2 f(b)$$

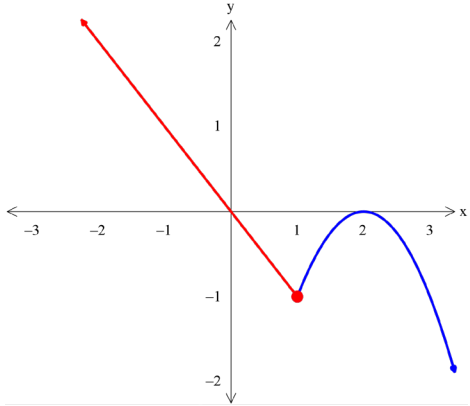
Find the value of $\frac{f(5)-f(1)}{f(2)}$ given that $f(2) \neq 0$.**End of Exam**

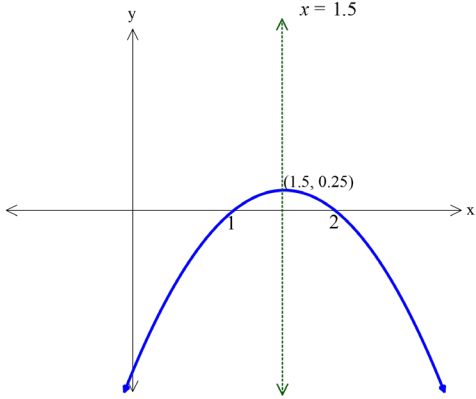
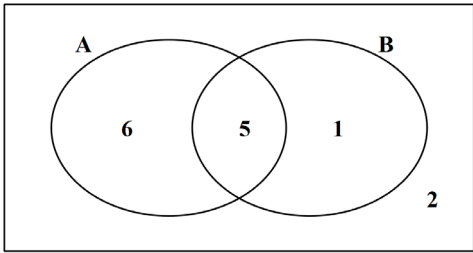
BAULKHAM HILLS HIGH SCHOOL - Mathematics
2019 Preliminary Task 1 SOLUTIONS

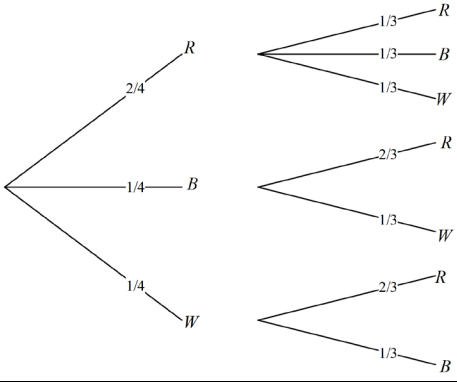
Solution		Mks	Comments
QUESTION 11			
11a.	$= 18 - 15y + 10 + 4$ $= 28 - 11y$	2	2 mark • Correct answer 1 mark • Expanding correctly. • Collects like terms correctly
11b.	$3x + 2y = 5$ $2x - y = -6 \times 2$ $3x + 2y = 5$ $4x - 2y = -12$ $7x = -7$ $x = -1$ $y = 4$	2	2 mark • Correct answer. 1 mark • Finding only x or y correctly
11c.	$x^2 - 4n^2 = x^2 - 16$ $n^2 = 4$ $n = \pm 2$	2	2 mark • Correct answer. 1 mark • If $n^2 = 4$.
11d.	$\frac{\sqrt{3}}{3 + 2\sqrt{3}} \times \frac{3 - 2\sqrt{3}}{3 - 2\sqrt{3}}$ $= \frac{3\sqrt{3} - 6}{3^2 - (2\sqrt{3})^2}$ $= \frac{3\sqrt{3} - 6}{-3}$ $= -\sqrt{3} + 2 = x - y\sqrt{3}$ $x = 2, y = 1$	3	3 mark • Correct answer. 2 marks • Significant progress towards solution. 1 marks • Multiply numerator and denominator by its conjugate.
11e.	$\frac{t(2s - t)}{(s + t)(2s - t)} \times \frac{(s - t)(s + t)}{s - t}$ $= t$	3	3 marks • Correct solution 2 marks • Factorise all terms 1 mark Factorise two expressions correctly
11f.	$\frac{7x + 1}{3(x - 2)} - \frac{2x + 1}{x - 2}$ $= \frac{7x + 1 - 6x - 3}{3(x - 2)}$ $= \frac{x - 2}{3(x - 2)}$ $= \frac{1}{3}$	2	2 mark • Correct solution 1 mark • Finds a correct expression as a single fraction
11g.	$f(-2) = 3k \times -2 + 1$ $7 = -6k + 1$ $k = -1$	1	1 mark • Correct solution

	Solution	Mks	Comments
QUESTION 12			
12a.	D: all real values of $x > \frac{2}{3}$ or $(\frac{2}{3}, \infty)$ R: all real values of $y > 0$ or $(0, \infty)$	2	2 marks • Correct solution 1 mark • Finds domain or range correctly.
12b	$f(x) = x^3 + 3x$ $f(-x) = (-x)^3 + 3 \times (-x)$ $= -x^3 - 3x$ $= -f(x) \text{ Hence it is an odd function.}$	2	2 marks • Correct solution 1 mark • Substitute $-x$ into $f(x)$ Or equivalent
12c(i)	$f \propto \frac{1}{w}$ $f = \frac{k}{w}$ $1500 = \frac{k}{200}$ $k = 300\,000$ $f = \frac{300\,000}{w}$	1	1 mark • Correct solution
12c(ii)		1	1 marks • Correct solution
12c(iii)	$f = \frac{300\,000}{1250}$ $= 240$	1	1 marks • Correct solution
12d(i)	$y = x^2(x + 3)(x - 1)$	2	2 marks • Correct solution 1 mark • If two factors are correct.

12d(ii)		2	2 marks • Correct solution 1 mark • Roots are correct but shape is not correct.
12e	$yx + yb = x - a$ $yx - x = -(yb + a)$ $x(y - 1) = -(yb + a)$ $x = -\frac{yb+a}{y-1} \text{ or } x = \frac{yb+a}{1-y}$	2	2 mark • Correct solution 1 mark • If collected all x terms on one side.
12g	$x^2 + 10x + 5^2 + y^2 - 6y + 3^2 = -30 + 25 + 9$ $(x + 5)^2 + (y - 3)^2 = 4$ $\therefore c(-5,3), r = 2$	2	2 marks • Correct solution. 1 mark • Finds the perfect square for x or y. • Writes the coordinates of centre or writes the radius

QUESTION 13			
	Solution	Mks	Comments
13a(i).	$1 - 2x = 5$ $-2x = 4$ $x = -2$	2	2 marks • Correct Solution. 1 mark • Successfully finding one x value.
13a(ii)	$4(3x + 2) - 5(x - 3) = 20$ $12x + 8 - 5x + 15 = 20$ $7x = -3$ $x = -\frac{3}{7}$	2	2 marks • Correct answer. 1 mark • Significant progress towards solution
13b.		2	2 marks • Correct solution. 1 marks • If one of the transformation is correct.

13c.	y –intercept is -2 x –intercept $-x^2 + 3x - 2 = 0$ $x = 1, x = 2$ Axis of symmetry $x = \frac{3}{2}$ Coordinates of vertex $(\frac{3}{2}, \frac{1}{4})$ 	3	3 marks • Correct solution with correct scale. 2 marks Correct intercepts. 1 marks • Shape.
13d.	$f(2) = 2 \times 2 + 4$ $= 8$ $f(0) = -4$ $f(-2) = -1 \times -2 - 4$ $= -2$ $f(2) + f(0) + f(-2) = 2$	2	2 marks • Correct answer. 1 mark • If two of the values are correct
13e(i)		2	2 marks • Correct solution. 1 mark • If two of the values are correct
13e(ii)	$P(A \cup B) = \frac{12}{14} = \frac{6}{7}$	1	1 marks • Correct solution.
13f	$(-3, 5)$	1	1 marks • Correct solution.
Solution Question 14		Mks	Comments
14a(i).	$(x + 1)^2 - y^2$ $(x + 1 - y)(x + 1 + y)$	2	2 marks • Correct solution. 1 marks • If finds the perfect square.
14a(ii)	$2(4x^2 - 8x - 21)$ $2(2x + 3)(2x - 7)$		2 marks • Correct solution. 1 marks • If finds one correct factors.
14b(i)	$P(w) = 0.2$ $P(l) = 0.8$ $P(\text{two win}) = P(wwl) + P(wlw) + P(lww)$ $= 3 \times 0.2 \times 0.2 \times 0.8 = 0.096$ or $\frac{12}{125}$	2	2 marks • Correct solution. 1 marks • If worked out $0.2 \times 0.2 \times 0.8$
14b(ii)	$P(\geq 1w) = 1 - P(lll)$ $= 0.488$ or $\frac{61}{125}$	1	1 marks • Correct solution.

14c.	$\frac{3^n \times 3^3 - 3^n}{3^n}$ $\frac{3^n(3^3 - 1)}{3^n} = 26$	2	2 marks • Correct solution. 1 marks • Using indices rule correctly.
14d(i)		1	1 marks • Correct answer.
14d(ii)	$P(RR) = \frac{2}{4} \times \frac{1}{3}$ $= \frac{1}{6}$	1	1 mark • Significant progress towards solution
14d(iii).	$P(\text{atleast one red}) = 1 - P(\text{no red})$ $= 1 - [P(BW) + P(WB)]$ $= 1 - \left(\frac{1}{4} \times \frac{1}{3} + \frac{1}{4} \times \frac{1}{3}\right)$ $= \frac{5}{6}$	1	1 marks • Correct solution.
14d(iv).	Our sample space was $RR, RB, RW, BR, BW, WR, WB$ Since we know one of the balls is Red, we now only have 5 possible outcomes. $P(RR) = \frac{1}{5}$	1	1 marks • Correct solution.
14e.	$\frac{f(a)}{f(b)} = \frac{a^2}{b^2}$ $\frac{f(5)}{f(2)} - \frac{f(1)}{f(2)} = \frac{25}{4} - \frac{1}{4}$ $= 6$	2	2 marks • Correct answer. 1 marks Recognise $\frac{f(a)}{f(b)} = \frac{a^2}{b^2}$
	Multiple Choice		
1	$\sqrt{\frac{16 \times 5}{9 \times 3}} = \frac{4\sqrt{5}}{3\sqrt{3}}$	B	
2	$2^2 - 1 + 4 = 7$	C	
3	$x^2(x - 4) - (x - 4)$ $(x^2 - 1)(x - 4)$ $(x - 1)(x + 1)(x - 4)$	D	
4	$P(A/B) = \frac{P(A \cap B)}{P(B)}$	D	
5		C	
6	$P(\text{catches bus}) = \frac{6}{30} = \frac{1}{5}$	A	
7	$P(X/Y) = \frac{P(X \cap Y)}{P(Y)}$ but X and Y are independent $\therefore P(X/Y) = P(X)$ $P(X \cap Y) = 0.2 \times P(Y)$ $P(Y) = \frac{0.06}{0.2} = 0.3$	B	

8	$\frac{f(n)}{f(n-1)} = \frac{1 \times 2 \times 3 \times \dots n}{1 \times 2 \times 3 \times \dots (n-1)} = n$	D	
9	$g(a) = \sqrt{a+2}$ $f(g(a)) = 0$ $2\sqrt{a+2} - 10 = 0$ $\sqrt{a+2} = 5$ $a = 23$	C	
10	(A) $ -x = -(-x)$ $x = x$ True (B) $2 \times -x < -x$ $-2x < -x$ True (C) $-(-x) > -x$ $x > -x$ True (D) $\sqrt{(-x)^2} = -x$ $\sqrt{x^2} = -x$ False	D	