



**BAULKHAM HILLS
HIGH SCHOOL**

HSC TASK 1 2020

Mathematics Advanced

**General
Instructions**

- Reading time – 5 minutes
- Working time – 1 hour
- Write using black pen
- NESA approved calculators may be used
- In Questions 6 – 14, show relevant mathematical reasoning and/or calculations

**Total marks:
37**

Section I – 5 marks (pages 2–3)

- Attempt Questions 1 – 5
- Allow about 8 minutes for this section

Section II – 32 marks (Answer Booklet pages 5–15)

- Attempt Questions 6 – 14
- Allow about 52 minutes for this section

Section I

5 marks

Attempt Questions 1 – 5

Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 5.

1 The limiting sum of an infinite geometric series exists when:

- A. $r > 1$
- B. $|r| > 1$
- C. $-1 < r < 1$
- D. $r < 1$

2 What is the domain of the function $f(x) = \sqrt{1 - x^2}$

- A. $(0, 1)$
- B. $[0, 1]$
- C. $(-1, 1)$
- D. $[-1, 1]$

- 3 Which of the following pairs of transformations commute?
- A. Vertical reflection and vertical translation.
 - B. Horizontal translation and horizontal dialation.
 - C. Reflection in the y –axis and reflection in the x –axis.
 - D. Vertical translation and vertical dialation.
- 4 The function $y = 3\cos 2x - 5$ has :
- A. Amplitude 3, period 2 and mean value -5
 - B. Amplitude 5, period $\frac{1}{2}$ and mean value 3
 - C. Amplitude 3, period π and mean value 5
 - D. Amplitude 3, period π and mean value -5
- 5 Consider the series $\sqrt{5} + \sqrt{45} + \sqrt{125} + \dots + z = 225\sqrt{5}$, the value of z is:
- A. $25\sqrt{5}$
 - B. $29\sqrt{5}$
 - C. $30\sqrt{5}$
 - D. $35\sqrt{5}$

End of Section I

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2020**HSC TASK 1**

Mathematics Advanced

Section II Answer Booklet

32 marks**Attempt Questions 6 – 14****Allow about 52 minutes for this section**

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your responses should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
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Question 6 (5 marks)

A gardener plants a bed of roses. The bed is planted so that the first row has 24 rose plants. The second row has 29 rose plants. Each succeeding row has 5 more rose plants than the previous row.

a) Calculate the number of rose plants in the 8th row.

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b) Which row would be the first to contain more than 150 rose plants?

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c) The gardener has planted 2895 rose plants altogether. Assuming that the above pattern has been continued, how many rows were planted?

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Question 7 (5 marks)

- (a) Solve the equation $x^2 - 6 = x$

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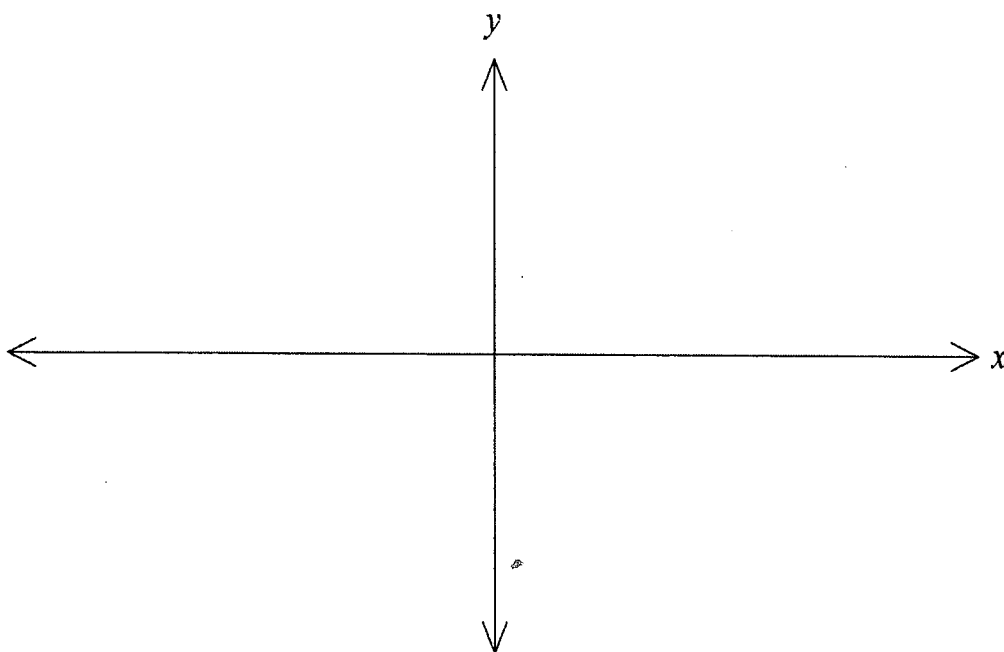
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- (b) In the space below, sketch the graph of $y = x^2 - 6$ and $y = x$. Label the coordinates of any point of intersection and any x or y intercepts of the graph.

2



- (c) Find the values of x , for which $x^2 - 6 > x$.

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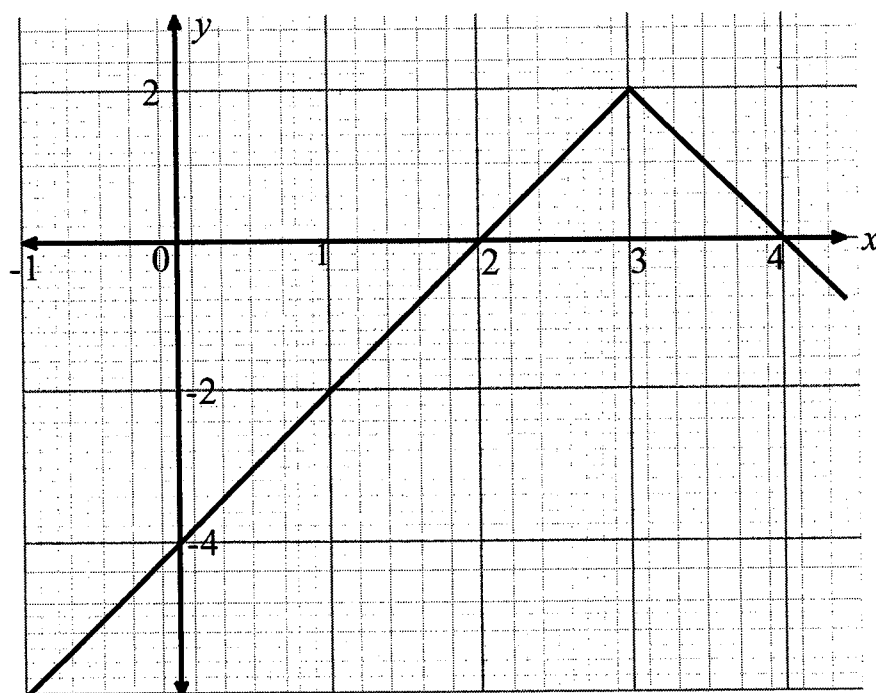
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Question 8 (3 marks)

The function $f(x) = |x|$ is transformed and the equation of the new function is of the form $y = kf(x + b) + c$, where k , b and c are constants. The graph of the new function is shown below. 3



What are the values of k , b and c ?

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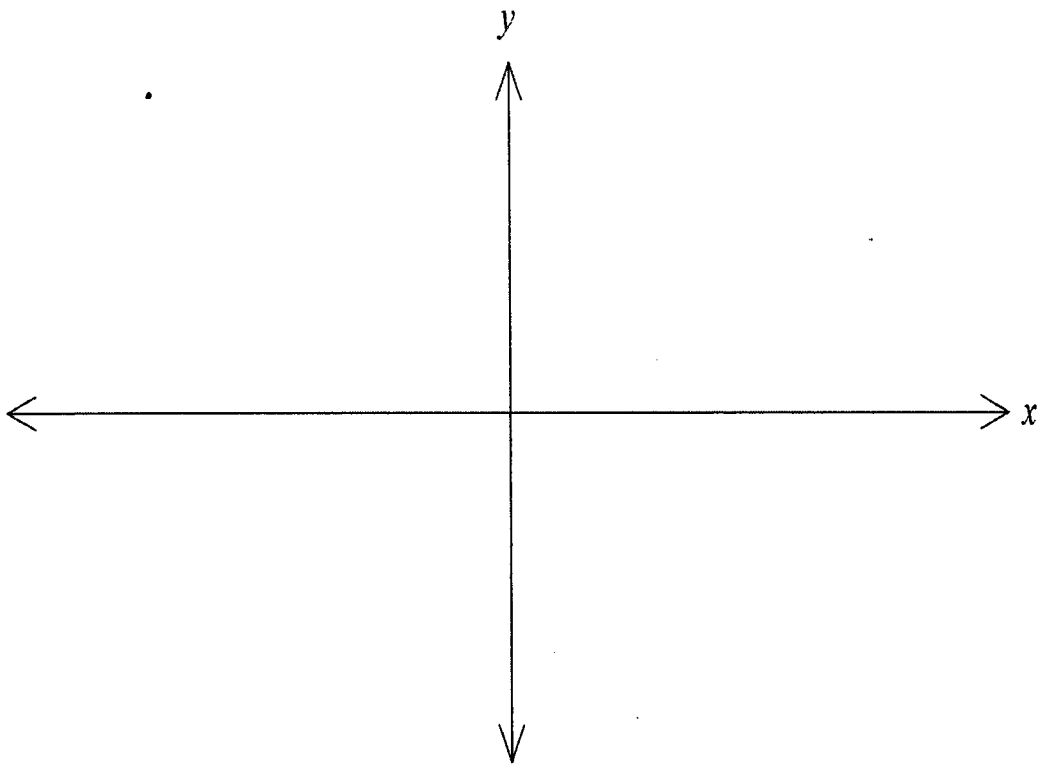
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Question 9 (3 marks)

- (a) Sketch $y = 3\cos x - 2$ and $y = \frac{x}{4} - 1$ in the domain $0 \leq x \leq 2\pi$. 2



- (b) Hence or otherwise, how many solutions are there to the equation 1

$$3\cos x - 2 = \frac{x}{4} - 1?$$

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Question 11 (5 marks)

Consider the function $y = \frac{8}{x^2-4}$

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- (a) Write down the domain.

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- (b) Find the vertical and horizontal asymptotes

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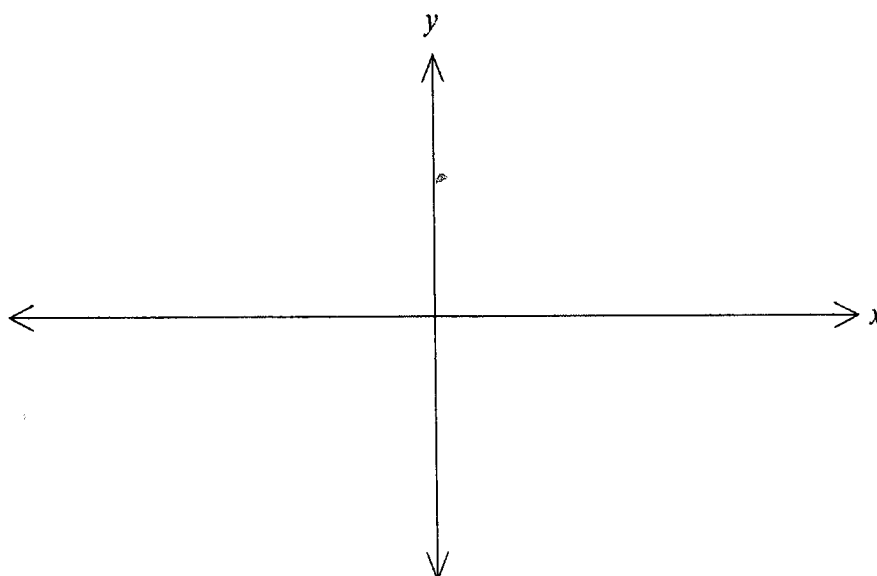
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- (c) Hence sketch the graph of $y = \frac{8}{x^2-4}$, showing all intercepts and asymptotes.

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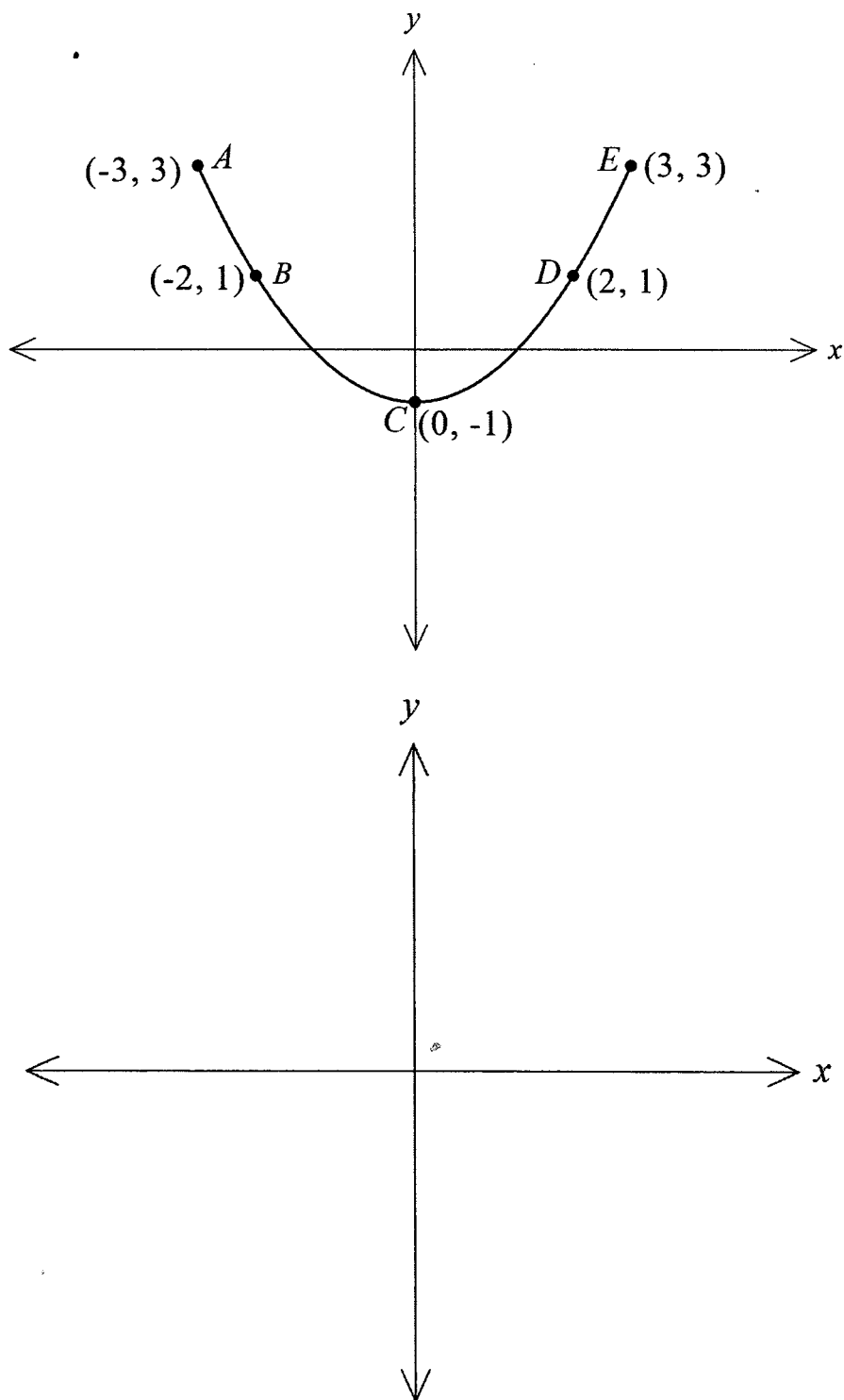
Question 12 (3 marks)

In a geometric progression $S_4 = 30$, while the sum to infinity is 32. Find the first three terms.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 13 (3 marks)

Given the sketch of the function $y = f(x)$, sketch the graph of $y = 3f(x + 3)$, clearly indicating the new coordinates for A, B, C, D and E on the graph. 3



Question 14 (3 marks)

For a given arithmetic sequence, the ratio of the sum of the first m terms to the sum of first n terms is $m^2 : n^2$, where $m \neq n$.

Prove that the ratio of the m^{th} term to the n^{th} term is $(2m - 1):(2n - 1)$.

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END OF EXAM



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- A. $(0, 1)$
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- A. $25\sqrt{5}$
- ☒ B. $29\sqrt{5}$
- C. $30\sqrt{5}$
- D. $35\sqrt{5}$

$$\begin{aligned} \sqrt{5} + 3\sqrt{5} + 5\sqrt{5} + \dots + z &= 225\sqrt{5} \\ a &= \sqrt{5}, d = 2 \\ 1 + 3 + 5 + \dots + n &= 225 \rightarrow a=1, d=2 \\ n &= a + (n-1)d \\ &= 1 + (n-1) \times 2 \\ &= 2n-1 \\ S_n &= 225 = \frac{n}{2} [a + l] \\ &= \frac{n}{2} [1 + 2n-1] \\ 225 &= n^2 \\ n &= \pm 15 \quad n > 0 \\ \therefore n &= 15 \\ 2\sqrt{5} (2n-1) \sqrt{5} &= (2 \times 15-1) \sqrt{5} \\ &= 29\sqrt{5} \end{aligned}$$

End of Section I

Question 6 (5 marks)

A gardener plants a bed of roses. The bed is planted so that the first row has 24 rose plants. The second row has 29 rose plants. Each succeeding row has 5 more rose plants than the previous row.

a) Calculate the number of rose plants in the 8th row.

$$24, 29, 34 \dots$$

1-correct answer

$$T_8 = a + (n-1)d$$

$$= 24 + (8-1) \times 5$$

$$= 59$$

b) Which row would be the first to contain more than 150 rose plants?

$$T_n > 150$$

$$24 + (n-1)d > 150$$

$$n-1 > \frac{126}{5}$$

$$n > 25.2 + 1 = 26.2$$

$$n = 27$$

2- correct solution

1- Establishing $T_n > 150$

c) The gardener has planted 2895 rose plants altogether. Assuming that the above pattern has been continued, how many rows were planted?

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$2895 = \frac{n}{2} (48 + 5n - 5)$$

$$5790 = \frac{n}{2} (43 + 5n)$$

$$5n^2 + 43n - 5790 = 0$$

$$n = \frac{-43 \pm \sqrt{43^2 - 4 \times 5 \times -5790}}{10}$$

$$= \frac{-43 \pm 343}{10}$$

$$n = 30 \text{ rows as } n > 0$$

2-correct solution

1-forming the quadratic equation or

Using the sum formula

Question 7 (5 marks)

- (a) Solve the equation $x^2 - 6 = x$

$$x^2 - x - 6 = 0$$

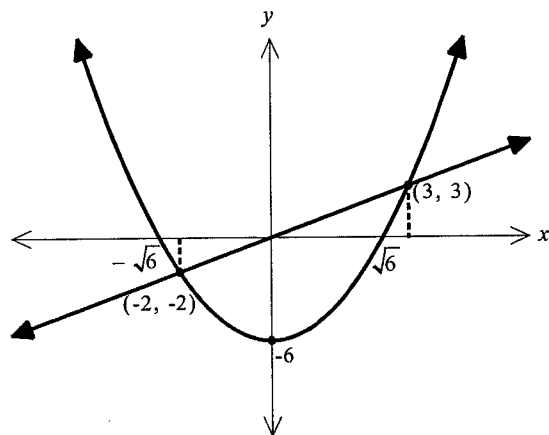
$$(x - 3)(x + 2) = 0$$

$$x = 3 \text{ or } x = -2$$

1- correct solution

1

- (b) In the space below, sketch the graph of $y = x^2 - 6$ and $y = x$. Label the coordinates of any point of intersection and any x or y intercepts of the graph.



2-Correct graphs with point of intersection

1- Correct graph without the label

2

- (c) Find the values for which $x^2 - 6 > x$.

$$x < -2 \text{ or } x > 3$$

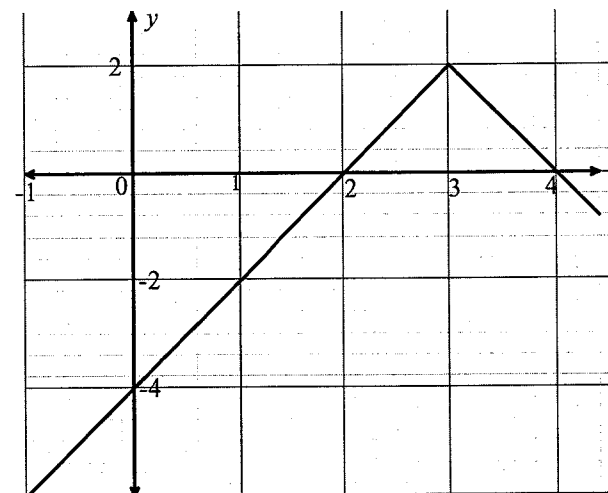
2-correct solutions

1-one correct inequality.

2

Question 8 (3 marks)

The function $f(x) = |x|$ is transformed and the equation of the new function is of the form $y = kf(x + b) + c$, where k, b and c are constants. The graph of the new function is shown below.



What are the values of k, b and c ?

$f(x) = |x|$ is translated to the right by 3 units. Reflected through the x -axis, multiplied by 2 and translated up by 2 units.

3- all correct answer

2- two correct answers

1- One correct answer

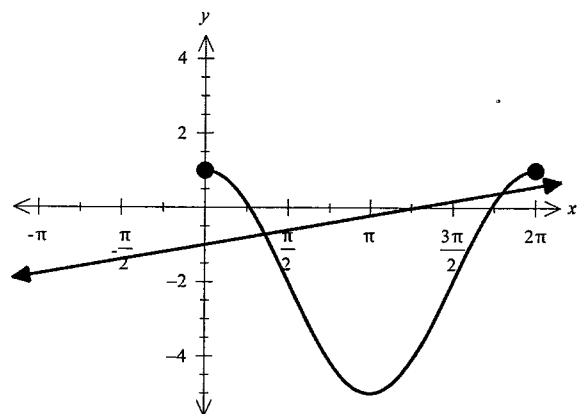
$$\therefore k = -2$$

$$b = -3$$

$$c = 2$$

Question 9 (3 marks)

- (a) Sketch $y = 3\cos x - 2$ and $y = \frac{x}{4} - 1$ in the domain $0 \leq x \leq 2\pi$.



2- Both the graphs are correct.
1- One of the them is correct

2

Question 10 (2 marks)

Express $0.4\dot{6}$ as infinite series and hence express as simplest fraction.

2

$$\begin{aligned} &0.4666 \dots \\ &= 0.4 + 0.06 + 0.006 + \dots \\ &= 0.4 + \frac{6}{100} + \frac{6}{1000} + \dots \\ &= 0.4 + \frac{\frac{6}{100}}{1 - \frac{1}{10}} \\ &= \frac{4}{10} + \frac{\frac{6}{100}}{\frac{9}{10}} \\ &= \frac{4}{10} + \frac{2}{30} \\ &= \frac{14}{30} \\ &= \frac{7}{15} \end{aligned}$$

2-correct solution

1- Forming the series correctly/
recognising a and r .

- (b) Hence or otherwise, how many solutions are there to the equation

1

$$3\cos x - 2 = \frac{x}{4} - 1?$$

2 solutions

1- Correct answer

Question 11 (5 marks)

Consider the function $y = \frac{8}{x^2-4}$.

(a) Write down the domain.

Domain : all real values of $x, x \neq \pm 2$

1- Correct answer

(b) Vertical asymptotes: $x = \pm 2$

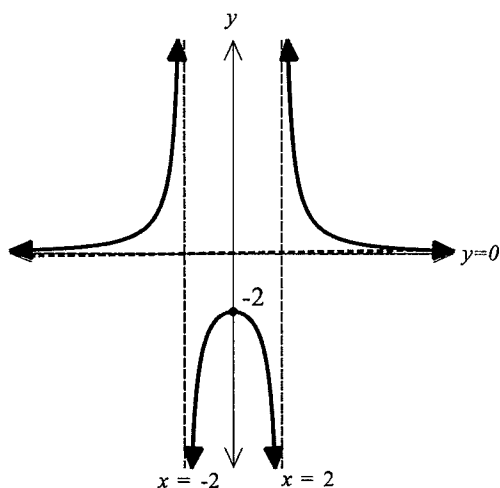
2- Both correct asymptotes

1- One correct asymptote

Horizontal asymptote: $\lim_{x \rightarrow \pm \infty} \frac{8}{(x^2-4)} = 0$

$\therefore y = 0$

(b) Hence sketch the graph of $y = \frac{8}{x^2-4}$, showing all intercepts and asymptotes.



2- Correct graph with labels

1- Correct asymptotes

Question 12 (3 marks)

$$30 = S_4 = \frac{a(1-r^4)}{1-r}$$

$$32 = S_{\infty} = \frac{a}{1-r}$$

$$a = 32(1-r)$$

$$30 = \frac{32(1-r)(1-r^4)}{1-r}$$

$$1-r^4 = \frac{30}{32}$$

$$r^4 = 1 - \frac{30}{32}$$

$$r^4 = \frac{1}{16}$$

$$r = \pm \frac{1}{2}$$

3- correct solution

2- finding the two values of r

1- finding a in terms of r .

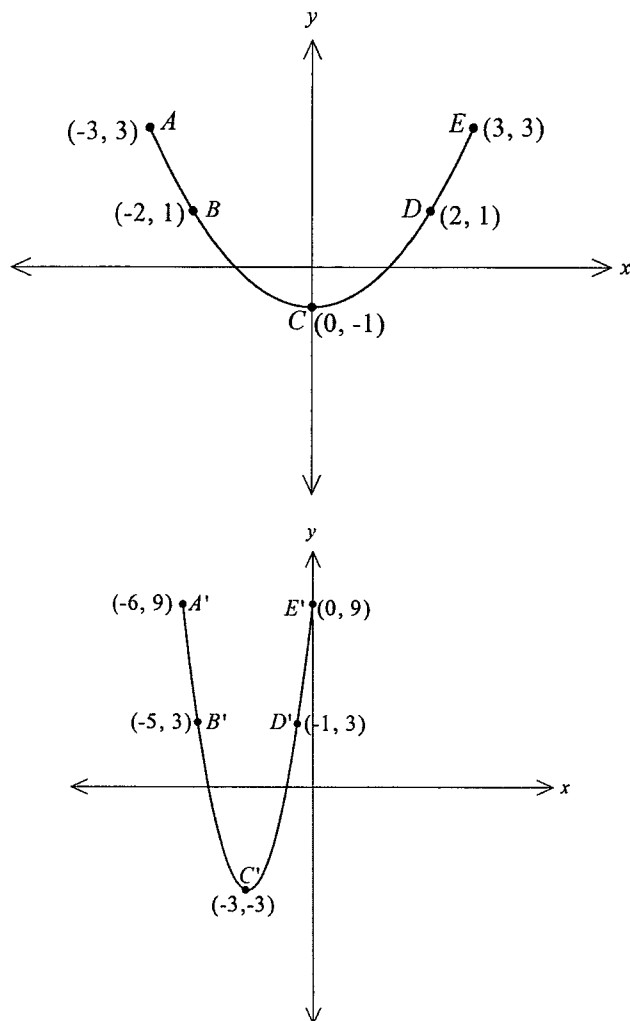
3

$r = \frac{1}{2}$	$r = -\frac{1}{2}$
$32 = \frac{a}{1-\frac{1}{2}}$	$32 = \frac{a}{1+\frac{1}{2}}$
$= \frac{a}{\frac{1}{2}}$	$32 = \frac{2a}{3}$
$a = 16$	$a = 48$
16, 8, 4...	48, -24, 12, ...

Question 13 (3 marks)

Given the sketch of the function $y = f(x)$, sketch the graph of $y = 3f(x + 3)$.

3



- 3- correct graph with label
 2- Correct horizontal transformation
 with correct vertex and y-intercept
 1- Correct horizontal transformation

Question 14 (3 marks)

For a given arithmetic sequence, the ratio of the sum of the first m terms to the sum of first n terms is $m^2 : n^2$, where $m \neq n$.

Prove that the ratio of the m^{th} term to the n^{th} term is $(2m - 1) : (2n - 1)$

$$\frac{\frac{m}{2}(2a + (m-1)d)}{\frac{n}{2}(2a + (n-1)d)} = \frac{m^2}{n^2}$$

$$\frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m}{n}$$

- 3- correct solution
 2-showing $a = 2d$
 1- sig. progress towards the proof

$$2an + mnd - nd = 2am + mnd - md$$

$$2an - 2am = nd - md$$

$$2a(n - m) = d(n - m)$$

$$d = 2a \text{ ----- (1)}$$

From (1) $\frac{a + (m-1)d}{a + (n-1)d} = \frac{a + (m-1)2a}{a + (n-1)2a}$

$$= \frac{1 + (m-1) \times 2}{1 + (n-1) \times 2}$$

$$= \frac{1 + 2m - 2}{1 + 2n - 2}$$

$$= \frac{2m - 1}{2n - 1}$$

$(2m - 1) : (2n - 1)$ Proved

END OF EXAM