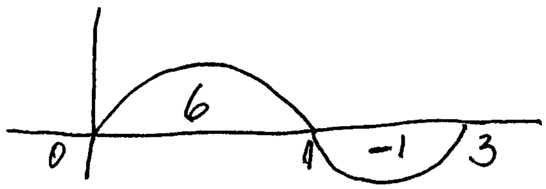
Multiple choice 2020 - HSC - 2unit Task 2 (Answers)

Q.1. $y = \sin\left(x \times \frac{\pi}{180}\right)$ (C)

$\frac{dy}{dx} = \frac{\pi}{180} \times \cos\frac{\pi}{180}x$

Q.2. $\lim_{x \to 0} \frac{\sin 2x}{3x} = \frac{2}{3}$ (A)

Q.3




$\int_0^1 f(x)dx = 6$ (A)

$\int_0^3 f(x)dx = 5 \therefore \int_1^3 f(x)dx = -1$

Q.4 $y = 4\sin\left(\frac{x}{3}\right)$ $T = \frac{2\pi}{\frac{1}{3}} = 6\pi$ (A)

Q.5 $y = \log_e(2x-1) \therefore 2x - 1 > 0$ (C)

$x > \frac{1}{2}$

Q.6 $\int_0^1 (x-1)^2 - (2x-2)\,dx + \int_1^3 (2x-2) - (x-1)^2\,dx$ (D)

Q.7 $f(x) = \ln(\tan x)$ (C)

$f'(x) = \frac{\sec^2 x \to f'(x)}{\tan x \to f(x)} = \frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \frac{1}{\cos x} \cdot \frac{1}{\sin x} = \sec x . \operatorname{cosec} x$

(8) $\int_1^d \frac{2}{x}dx = 2 \therefore 2\left[\ln|x|\right]_1^d = 2$ (A)

$d > 0$ $\ln d - \ln 1 = 1 \therefore \ln d = 1 \therefore e^1 = d$

(9) $f(x) < 0 \therefore A, C, D$ & $f''(x) < 0 \therefore$ concave down

$f'(x) > 0 \therefore$ increasing $\therefore A, D$ $\therefore$ (A)

(10) $y = x^2 + bx + c$, $x = \frac{-b}{2a}$ axis passes through vertex which is min. at $x = -2$ since $f'(-2) = 0$

$\therefore -2 = \frac{-b}{2 \times 1} \therefore b = 4$

now: $y = x^2 + 4x + c$ & min. value $(-2, 6)$

$6 = (-2)^2 + 4(-2) + c \therefore c = 10$ (D)

**Section II- Extended Response**

Attempt Questions 11-16.

Allow about 75 minutes for this section.

**Question 11(12 Marks)**

a) Differentiate the following

Marking scale

(i) $y = \log_e(2x-1)^4$ **2**

$y = 4\log_e(2x-1)$

$\frac{dy}{dx} = 4 \times \frac{2}{2x-1}$

$\frac{dy}{dx} = \frac{8}{2x-1}$

(2 marks)
- correct solns. - (2)
- applies correctly $\frac{d}{dx}\ln f(x)$ - (1)

(ii) $y = \frac{x}{\sin^2 x}$ **2**

$u = x \quad u' = 1$

$v = \sin^2 x \quad v' = 2\sin x\cos x$

$\therefore \frac{dy}{dx} = \frac{1 \times \sin^2 x - 2x\sin x\cos x}{\sin^4 x}$

or simplifies further $\frac{dy}{dx} = \frac{\sin x - 2x\cos x}{\sin^3 x}$

(2 marks)
- correct solns - (2)
- applies quotient rule correctly - (1)

(iii) $y = 3^x$ **1**

$\frac{dy}{dx} = 3^x \times \ln 3$

(1 mark)
- correct solns.

**Question 11 continued on the next page**

b) Integrate the following

$$\int 3e^{5x+1} - \frac{1}{x^3}\,dx$$

2

$= \frac{3}{5}e^{5x+1} - \frac{x^{-2}}{-2} + c$

$= \frac{3}{5}e^{5x+1} + \frac{1}{2x^2} + C$

2 marks
- correct solns.

1 mark
- Finds the correct primitive of $3e^{5x+1}$

1 mark
- Finds the correct primitive of $\frac{1}{x^2}$

c) Evaluate $\int_0^{\frac{\pi}{8}} \sec^2 2x dx$

2

$= \frac{1}{2}\left[\tan 2x\right]_0^{\pi/8}$

$= \frac{1}{2}\left[\tan\frac{\pi}{4} - \tan 0\right]$

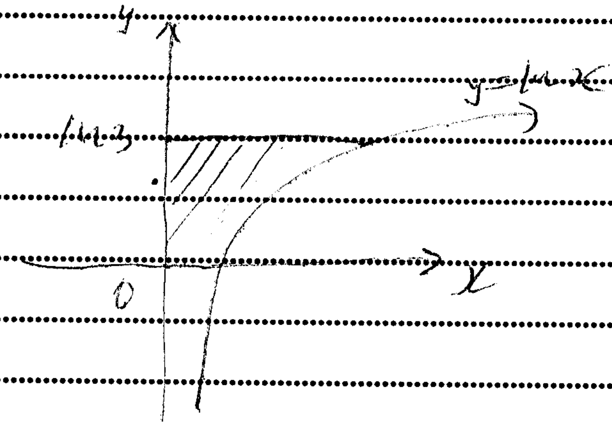
$= \frac{1}{2}\left[1-0\right] = \frac{1}{2}$

2 marks
- correct solns.

1 mark
- Find the correct primitive
- correctly evaluates definite integral

Question 11 continued on the next page

**d)** Find the area bounded by $y = \ln x$, y-axis between $y = 0$ and $y = \ln 3$. **3**




$$\text{Area} = \int_0^{\ln 3} x \, dy$$

$y = \ln x$

where $x = e^y$

$$\therefore \text{Area} = \int_0^{\ln 3} e^y \, dy$$

$$= \left[ e^y \right]_0^{\ln 3}$$

$$= e^{\ln 3} - e^0$$

$$= 3 - 1 = 2 \text{ (units}^2\text{)}$$

(3 marks)
- correct solns.

(2 marks)
- correctly makes $x$ the subject and establishes the area towards y-axis

(1 mark)
- correctly makes $x$ the subject
- finds the correct primitive

**End of Question 11**

**Question 12 (7 Marks)**

(a)

(i) Differentiate $\cos 3x^3$ $\rightarrow$ 2

$\frac{d}{dx}\cos 3x^3 = 9x^2 \times (-\sin 3x^3)$

$\therefore \frac{d}{dx}\cos 3x^3 = -9x^2 \sin(3x^3)$

(2 marks)
- correct solns

(1 mark)
- applies chain rule correctly
- correctly differentiates cos-function

(ii) Hence $\int x^2 \sin 3x^3 dx$ **2**

from (i) $\frac{d}{dx}\cos 3x^3 = -9x^2 \sin 3x^3$

$\therefore \cos 3x^3 = \int -9x^2 \sin 3x^3 dx$

$\cos 3x^3 = -9\int x^2 \sin 3x^3 dx$

$\therefore -\frac{1}{9}\cos 3x^3 + c = \int x^2 \sin 3x^3 dx$

(2 marks)
- correct solns

(1 mark)
- correctly applies reverse chain rule
- ignore +c

**Question 12 continued on the next page**

**(b)** Solve the equation given by $2\ln x = \ln(3 - 2x)$ 3

$2\ln x = \ln(3-2x)$

$\ln x^2 = \ln(3-2x)$

$x^2 = 3-2x$

$x^2+2x-3=0$

$(x+3)(x-1) = 0$

$x=-3$ or $x=1$

but $x=-3$ is not a soln. since $2\ln(-3)$ is not defined

$\therefore$ soln: $x=1$

(3 marks) correct solns.

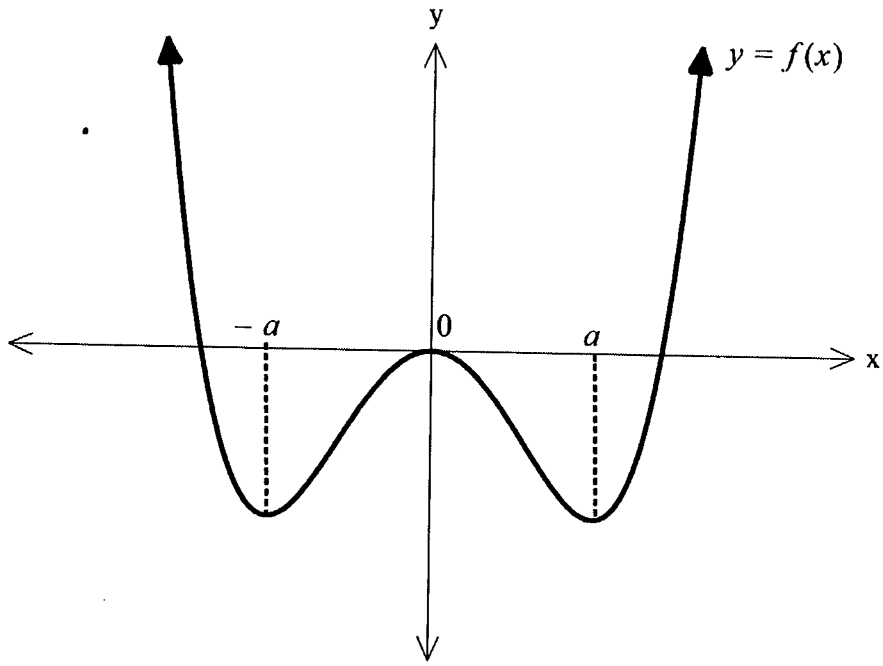
(2 marks)
- gets both solns correctly but doesn't exclude $x=-3$

(1 mark)
- applies log. rules correctly
- solves quadratic equation correctly

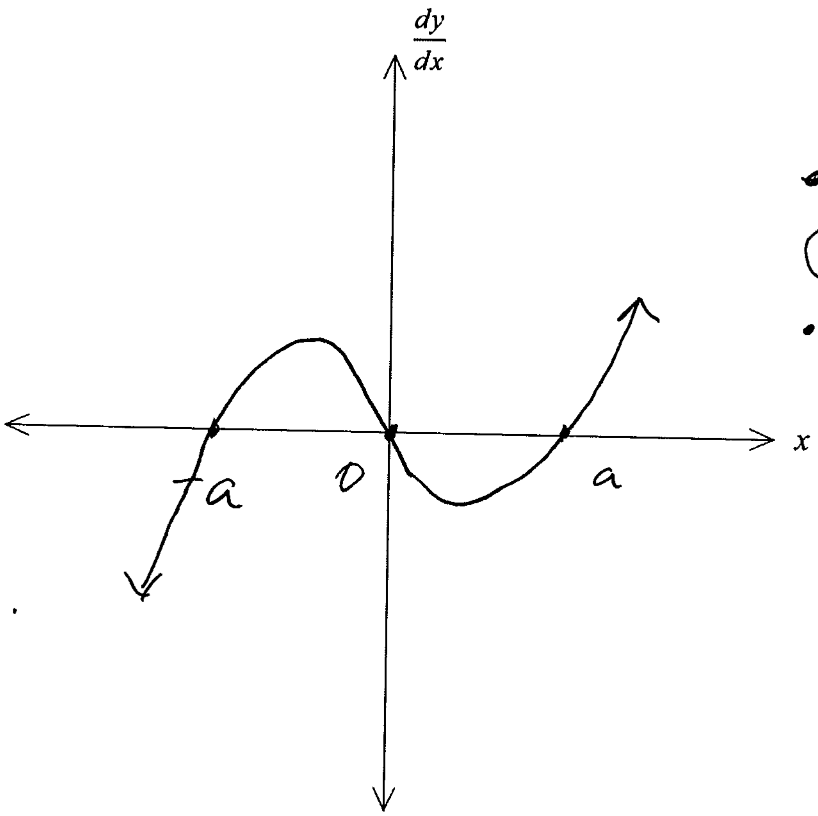
End of Question 12

**Question 13 (5 Marks)**

(a) Given the graph of $y = f(x)$ below **2**




Sketch a possible graph of $\frac{dy}{dx}$ on the axes provided.




2 marks

- correct graph

1 mark

- All 3 x-inter. correct
- correct shape

**Question 13 continued on the next page**

(b) Water is flowing out of a pool at a rate given by $R = 4t - t^2$ cubic metres/minut **3**

If the volume of water in the pool is initially 10 cubic metres, find the amount of the water in the pool after 4 minutes.

$R = -\frac{dV}{dt}$

$\therefore V = -\int 4t - t^2 \, dt$

$V = -2t^2 + \frac{t^3}{3} + C$

given $t = 0$ $V = 10\ m^3$

$\therefore 10 = 2(0) - \frac{0}{3} + C$

$\therefore C = 10$

$\therefore V = -2t^2 + \frac{t^3}{3} + 10$

Now after $t = 4$ minutes

sub. $t = 4$

$V = -2(4)^2 + \frac{4^3}{3} + 10$

$V = \frac{62}{3}\ m^3$ or $= 0.6\ m^3$

$\therefore$ pool is empty

OR

(3 marks)

• correct solns.

(2 marks)

• finds the correct primitive and evaluates the constant using the initial values

(1 mark)

• finds the correct primitive

• finds Volumes correctly

**End of Question 13**

**Question 14 ( 8 marks)**

For the curve given by $f(x) = 2xe^{-x}$

(i) Show that $f'(x) = 2e^{-x}(1-x)$. **1**

$f(x) = 2xe^{-x}$

$u = 2x \qquad v = e^{-x}$

$u' = 2 \qquad v' = -e^{-x}$

$\therefore f'(x) = 2e^{-x} + 2x(-e^{-x})$

$= 2e^{-x} - 2xe^{-x}$

$= 2e^{-x}(1-x)$

1 mark

• correct solns.

(ii) Hence or otherwise find any stationary point(s) and determine their nature. **2**

$f'(x) = 0$

$2e^{-x}(1-x) = 0$

$2e^{-x} \neq 0 \qquad 1 - x = 0$

$x = 1$

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f'(x)$ | 2 | 0 | $-0.27$ |

$+ \quad x=1 \quad -$ $\therefore (1, 2e^{-1})$ Max. turning pt.

2 marks

• correct solns.

1 mark

• solves correctly $f'(x) = 0$

• justifies the nature of stat. pt. correctly

(iii) Determine $\lim_{x \to \infty} f(x)$. **1**

$\lim_{x \to \infty} 2xe^{-x} = 0$

1 mark

• correct solns.

**Question 14 continued on the next page**

(iv) Find any possible point of inflection. **2**

$f'(x) = 2e^{-x} - 2xe^{-x}$

$f''(x) = -2e^{-x} - (\underbrace{2e^{-x} - 2xe^{-x}}_{f'(x)})$

$\therefore f''(x) = -2e^{-x} - 2e^{-x} + 2xe^{-x}$

$= -4e^{-x} + 2xe^{-x}$

$= -2e^{-x}(2 - x)$

$0 = -2e^{-x}(2-x)$ at $x=+2$

$-2e^{-x} \neq 0$ $\quad x = +2$ possible pt. of infl.

| $x$ | 1 | +2 | 3 |
|---|---|---|---|
| $f''$ | -0.7 | 0 | 0.099 |

change in concavity $\quad \therefore (2, 4e^{-2})$ pt. of inflexion

2 marks
- correct solns.

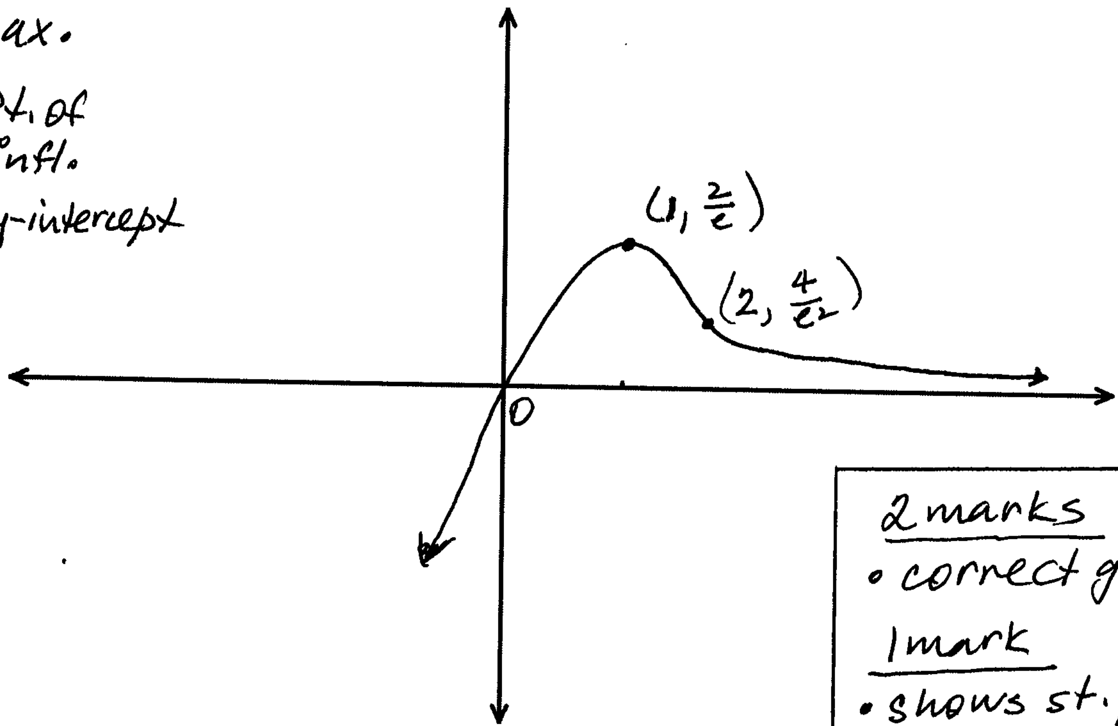
1 mark
- solves correctly $f''(x) = 0$
- shows change in concavity

(v) Hence sketch the curve. **2**

$\left(1, \frac{2}{e}\right)$ Max.

$\left(2, \frac{4}{e^2}\right)$ Pt. of infl.

$(0, 0)$ - $x, y$-intercept




2 marks
- correct graph

1 mark
- shows st. point & x-intercept
- correct shape showing asymptote

**End of Question 14**

**Question 15 (10 marks)**

The velocity of the particle is given by $v = 2\sin\left(t - \frac{\pi}{3}\right)$ metres per second.

a) Find:

(i) the equation of the displacement x of the particle as a function of time if the particle is initially at the origin. **2**

$x = \int v\,dt$

$x = \int 2\sin\left(t - \frac{\pi}{3}\right)dt$

$x = -2\cos\left(t - \frac{\pi}{3}\right) + c$

at $t=0$ $x=0$

$\therefore 0 = -2\cos\left(0 - \frac{\pi}{3}\right) + c$

$c = 1$

$\therefore x = -2\cos\left(t - \frac{\pi}{3}\right) + 1$

2 marks
- correct solns.

1 mark
- correctly finds primitive of $2\sin\left(t - \frac{\pi}{3}\right)$
- substitutes initial conditions & finds c

(ii) When does the particle come to the rest first time? **1**

rest $\therefore V = 0$

$0 = 2\sin\left(t - \frac{\pi}{3}\right)$

$t = \frac{\pi}{3}$ (seconds)

1 mark
- correct solns.

Question 15 continued on the next page

(iii) the exact initial velocity.

$t=0 \therefore V = 2\sin\left(0-\frac{\pi}{3}\right)$

$V = 2\times \sin\left(-\frac{\pi}{3}\right)$

$V = 2\times -\frac{\sqrt{3}}{2}$

$\therefore V = -\sqrt{3}$ m/s

1 mark

• correct solns.

(iv) the maximum speed of the particle.

speed = |velocity|

velocity graph is a function of $2\sin\left(t-\frac{\pi}{3}\right)$

$\therefore$ speed$_{MAX}$ = amplitude = 2 m/s

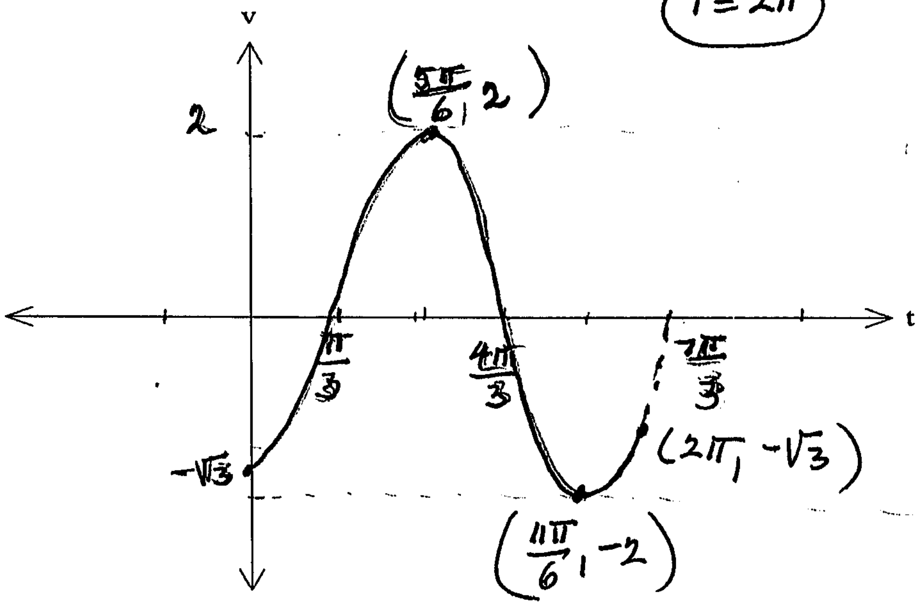
1 mark

• correct solns.

(v) Draw the graph of the velocity $v = 2\sin\left(t-\frac{\pi}{3}\right)$ for $0 \leq t \leq 2\pi$.

Show $x, y$ intercepts and stationary points. **3**

$T = 2\pi$




3 marks

• correct graph

2 marks

• correct shape showing $T = 2\pi$, a= and initial V=[illegible] and zero velocity at $t = \frac{\pi}{3}$

1 mark $T = 2\pi, a =$

• correct shape

• initial velocity ~~x~~ V=0 correctly

**Question 15 continued on the next page**

(vi) Using your graph above or otherwise, find the total distance travelled by the particle in the first $\pi$ seconds. **2**

$$d = \left| \int_0^{\pi/3} v \, dt \right| + \int_{\pi/3}^{\pi} v \, dt$$

$$d = \left| \left[ -2\cos\left(t - \frac{\pi}{3}\right) \right]_0^{\pi/3} \right|$$

$$+ \left[ -2\cos\left(t - \frac{\pi}{3}\right) \right]_{\pi/3}^{\pi}$$

$$= \left| -2\left(\cos 0 - \cos\left(\frac{-\pi}{3}\right)\right) \right|$$

$$+ -2\left[\cos\frac{2\pi}{3} - \cos 0\right]$$

$$= 1 + 3 = 4 \text{ metres}$$

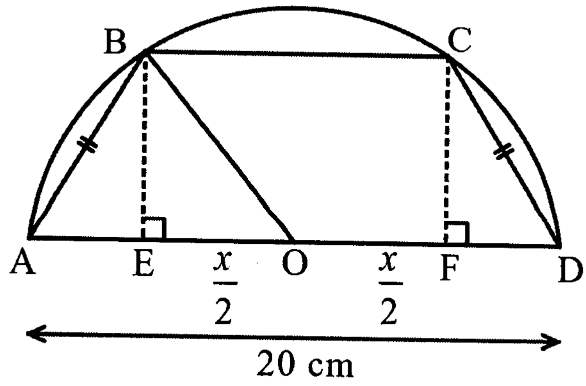
2 marks
- correct solns

1 mark
- correctly puts the first section $0 \le t \le \frac{\pi}{3}$ into absolute value
- Finds $\int_0^{\pi} v \, dt = 2$

End of Question 15

## Question 16 (7 Marks)

An isosceles trapezium $ABCD$ is drawn with its vertices on a semicircle centre $O$ and diameter 20 cm. Perpendiculars $BE$ and $CF$ are drawn to meet the diameter $AD$ as shown in the diagram below.




(i) If $EO = \frac{x}{2}$, show that $BE = \frac{1}{2}\sqrt{400 - x^2}$. **2**

$BO = \text{radius} = 10\text{cm}$

in $\Delta BEO$: $BE^2 = BO^2 - EO^2$

$BE^2 = 10^2 - \left(\frac{x}{2}\right)^2$

$BE^2 = 100 - \frac{x^2}{4} = \frac{400-x^2}{4}$

$\therefore BE = (+)\sqrt{\frac{400-x^2}{4}} = \frac{1}{2}\sqrt{400-x^2}$ only

$BE > 0$

(2 marks
• correct solns.
(1 mark
• applies Pythagoras' theorem correctly

(ii) Show that the area of the trapezium $ABCD$ is given by

$$A = \frac{1}{4}(x + 20)\sqrt{400 - x^2}.$$

**2**

$BC = x$

$A = \frac{1}{2}\text{height}(BC + AD)$

height $= BE$ (part (i))

$\therefore A = \frac{1}{2} \times \frac{1}{2}\sqrt{400-x^2}\,(x + 20)$

$\therefore A = \frac{1}{4}(x+20)\sqrt{400-x^2}$

(2 marks
• correct solns.
(1 mark
• applies correctly the area formula of trapezium
• recognises $BC = x$ and height $= BE$

**Question 16 continued on the next page**

(iii) Hence find the length of $BC$ so that the area of the trapezium $ABCD$ is maximum. **3**

find $BC = x$

$$A = \frac{1}{4}(x+20)\sqrt{400-x^2}$$

$$u = \frac{1}{4}(x+20) \qquad v = (400-x^2)^{\frac{1}{2}}$$

$$u' = \frac{1}{4} \qquad v' = \frac{1}{2} \times -2x(400-x^2)^{-\frac{1}{2}}$$

$$v' = -x(400-x^2)^{-\frac{1}{2}}$$

$$\frac{dA}{dx} = \frac{1}{4}(400-x^2)^{\frac{1}{2}} + \frac{1}{4}(x+20) \cdot -x(400-x^2)^{-\frac{1}{2}}$$

$$0 = \frac{1}{4}(400-x^2)^{\frac{1}{2}} - \frac{x(x+20)}{4(400-x^2)^{\frac{1}{2}}}$$

$$0 = \frac{(400-x^2) - x(x+20)}{4(400-x^2)^{\frac{1}{2}}}$$

$$0 = 400 - x^2 - x^2 - 20x$$

$$0 = 2x^2 + 20x - 400$$

$$0 = x^2 + 10x - 200$$

$$0 = (x+20)(x-10)$$

$x = -20$ $\quad$ $x = 10$

but $x > 0$

$\therefore$ impossible

justifies Maximum

| $x$ | 9 | 10 | 11 |
|---|---|---|---|
| $\frac{dA}{dx}$ | 0.81 (+) | 0 | −0.927 |

$x = 10$

(+) $\quad$ (−)

$\therefore$ at $x = BC = 10$ cm

Area is Maximum

(3 marks)

• correct solns.

(2 marks)

• applies product rule correctly and solves for $\frac{dA}{dx} = 0$

(1 mark)

• finds $\frac{dA}{dx}$ correctly

~~& attempts~~ to solve $\frac{dA}{dx} = 0$

• ~~find a solution to $\frac{dA}{dx} = 0$ &~~

justifies the maximum

**End of Exam**

**BAULKHAM HILLS HIGH SCHOOL**

**HSC Assessment Task 2 2020**

# Mathematics Advanced

**General Instructions**

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black pen only
- Board-approved calculators may be used
- Show all necessary working in Questions 11-16
- Marks may be deducted for careless or badly arranged work
- Reference sheet is provided at the end.

**Total marks – 59**

This paper consists of TWO sections.

**Section 1 – Page 2-5 (10 marks)**
**Questions 1-10**

- Attempt questions 1-10
- Allow about 15 minutes for this section

**Section II – Pages 6-19 (49 marks)**

- Attempt questions 11-19
- Allow about 75 minutes for this section

## Section I - 10 marks  Answer on the multiple choice page

**1.** The derivative of $y = \sin x°$ is

(A) $\dfrac{dy}{dx} = \cos x°$

(B) $\dfrac{dy}{dx} = -\sin x°$

(C) $\dfrac{dy}{dx} = \dfrac{\pi}{180}\cos\dfrac{\pi}{180}x$

(D) $\dfrac{dy}{dx} = -\dfrac{\pi}{180}\cos\dfrac{\pi}{180}x$

**2.** $\lim\limits_{x \to 0} \dfrac{\sin 2x}{3x} =$

(A) $\dfrac{2}{3}$

(B) 0

(C) $\dfrac{3}{2}$

(D) 1

**3.** The function $y = f(x)$ is continuous for all $x$.

Given that $\int_0^1 f(x)dx = 6$ and $\int_0^3 f(x)\,dx = 5$, what is the value of $\int_1^3 f(x)dx$ ?

(A) -1

(B) 0

(C) 1

(D) 2

**4.** What is the period of the function $y = 4\sin\left(\dfrac{x}{3}\right)$?

(A) $6\pi$

(B) 4

(C) $\dfrac{2\pi}{3}$

(D) $\dfrac{1}{4}$

**4.** The domain for the function $y = \log_e(2x - 1)$ is:

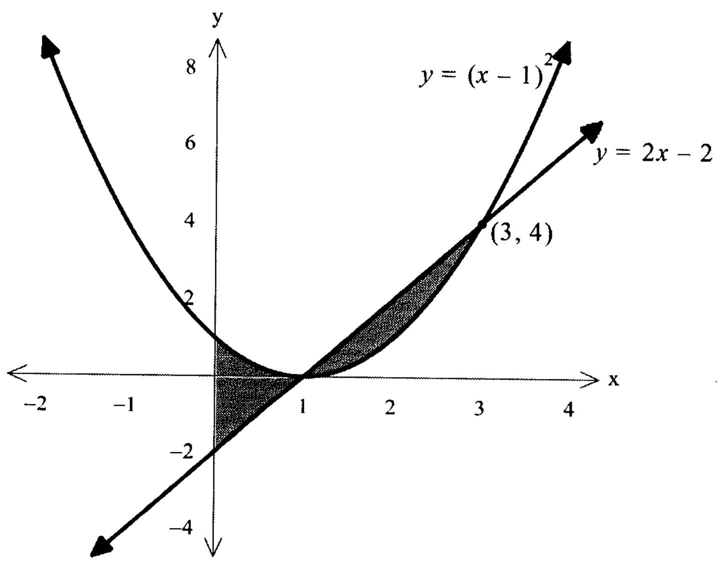
(A) All real $x, x \neq \dfrac{1}{2}$

(B) $x \geq \dfrac{1}{2}$

(C) $x > \dfrac{1}{2}$

(D) $x \leq \dfrac{1}{2}$

**5.** The area of the shaded region in the diagram below is given by:




(A) $\displaystyle\int_0^3 2x - 2 - (x-1)^2 dx$

(B) $\displaystyle\int_1^3 (x-1)^2 - 2x - 2dx + \int_0^1 2x - 2 - (x-1)^2 dx$

(C) $\displaystyle\int_0^3 2x - 2 - (x-1)^2 \, dx$

(D) $\displaystyle\int_0^1 ((x-1)^2 - 2x + 2)dx + \int_1^3 (2x - 2) - (x-1)^2 \, dx$

**7.** What is $f'(x)$ if $f(x) = \ln(\tan x)$?

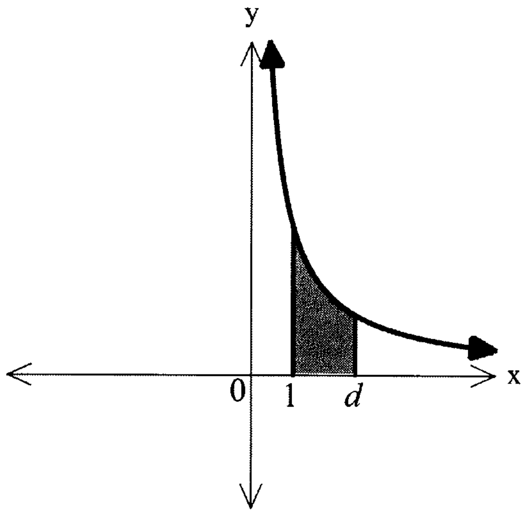
(A) $\dfrac{1}{\sec^2 x}$

(B) $\dfrac{\sec x}{\tan x}$

(C) secx.cosecx

(D) $\cot x$

**8.** The diagram shows the area under the curve $y = \dfrac{2}{x}$ from $x = 1$ to $x = d$.




What value of $d$ makes the shaded area equal to 2 square units?

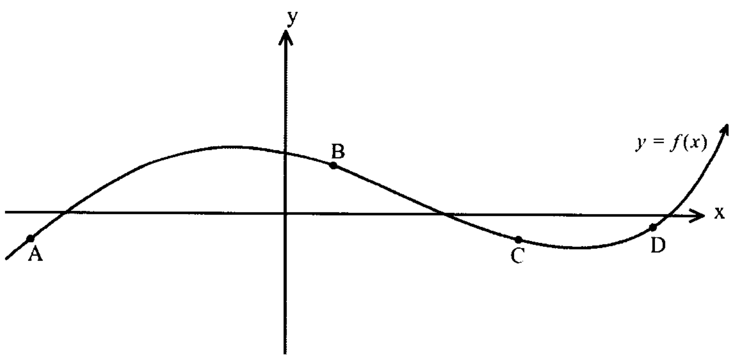
(A) $e$

(B) $e + 1$

(C) $2e$

(D) $e^2$

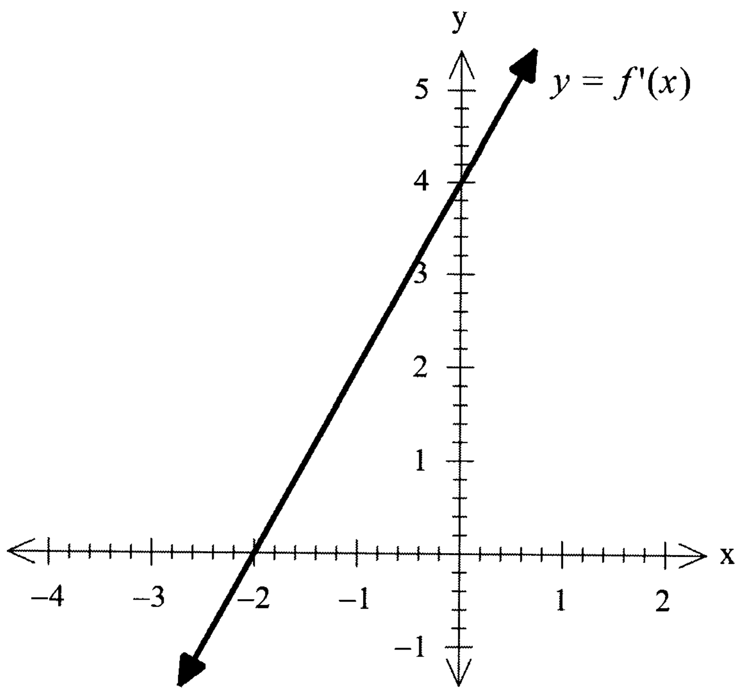
**9.** The diagram shows the points $A, B, C$ and $D$ on the graph $y = f(x)$




At which point is $f(x) < 0, f'(x) > 0$ and $f''(x) < 0$?

(A) A
(B) B
(C) C
(D) D

**10.** The graph of $y = f'(x)$ is given below.




The curve $y = f(x)$ has a minimum value of 6. What is the equation of the curve ?

(A) $y = x^2 - 4x + 2$

(B) $y = x^2 - 4x + 10$

(C) $y = x^2 + 4x + 2$

(D) $y = x^2 + 4x + 10$

**END OF SECTION I**

## Section II- Extended Response

Attempt Questions 11-16.

Allow about 75 minutes for this section.

## Question 11(12 Marks)

(a) Differentiate the following

(i) $y = \log_e(2x - 1)^4$ **2**

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

(ii) $y = \dfrac{x}{\sin^2 x}$ **2**

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

(iii) $y = 3^x$ **1**

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

**Question 11 continued on the next page**

b) Integrate the following

$$\int 3e^{5x+1} - \frac{1}{x^3}\,dx$$ **2**

c) Evaluate $\int_0^{\frac{\pi}{8}} \sec^2 2x dx$ **2**

**Question 11 continued on the next page**

d) Find the area bounded by $y = \ln x$ and the y-axis from $y = 0$ to $y = \ln 3$. **3**

**End of Question 11**

**Question 12 (7 Marks)**

(a)

(i) Differentiate $\cos 3x^3$ **2**

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

(ii) Hence find $\int x^2 \sin 3x^3 dx$ **2**

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

..........................................................................................................................................

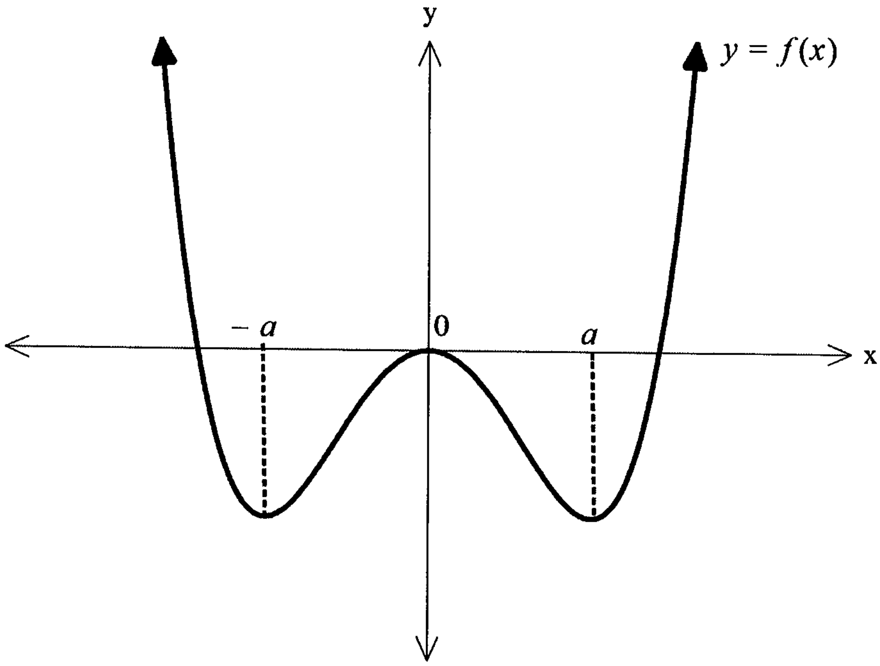
**Question 12 continued on the next page**

(b) Solve the equation given by $2\ln x = \ln(3 - 2x)$ **3**

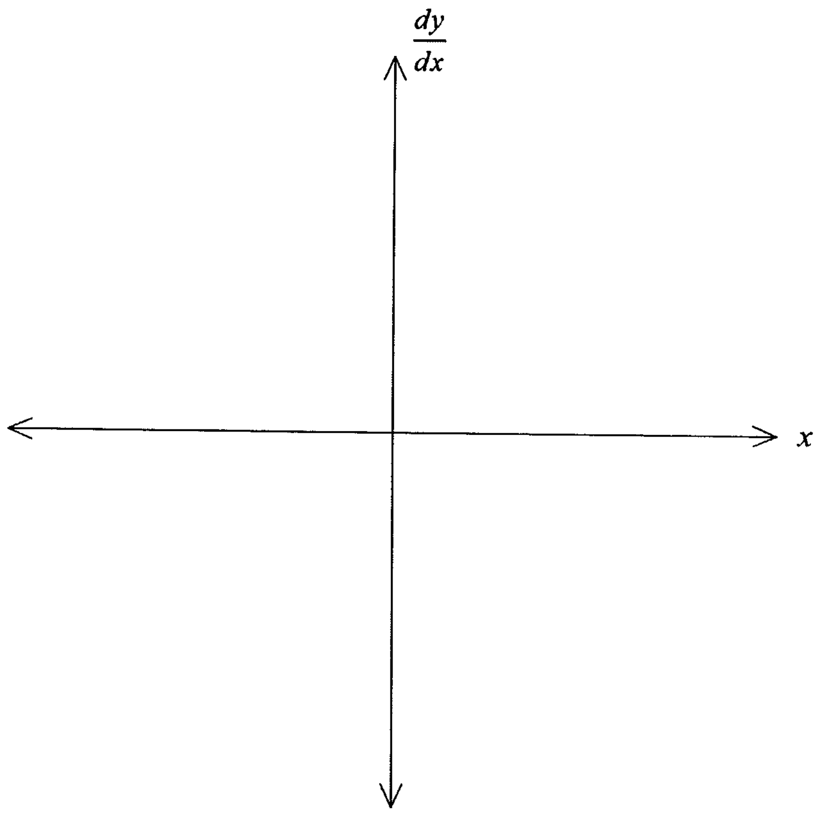
**End of Question 12**

**Question 13 (5 Marks)**

(a) Given the graph of $y = f(x)$ below **2**




Sketch a possible graph of $\frac{dy}{dx}$ on the axes provided.




**Question 13 continued on the next page**

(b) Water is flowing out of a pool at a rate given by $R = 4t - t^2$ cubic metres/minut **3**

If the volume of water in the pool is initially $10$ cubic metres, find the amount of the water in the pool after 4 minutes.

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

.......................................................................................................................................

**End of Question 13**

**Question 14 ( 8 marks)**

For the curve given by $f(x) = 2xe^{-x}$

(i) Show that $f'(x) = 2e^{-x}(1 - x)$. **1**

........................................................................................................................

........................................................................................................................

........................................................................................................................

........................................................................................................................

........................................................................................................................

........................................................................................................................

........................................................................................................................

........................................................................................................................

(ii) Hence or otherwise find any stationary point(s) and determine their nature. **2**

........................................................................................................................

........................................................................................................................

........................................................................................................................

........................................................................................................................

........................................................................................................................

........................................................................................................................

........................................................................................................................

........................................................................................................................

........................................................................................................................

........................................................................................................................

(iii) Determine $\lim_{x \to \infty} f(x)$. **1**

........................................................................................................................

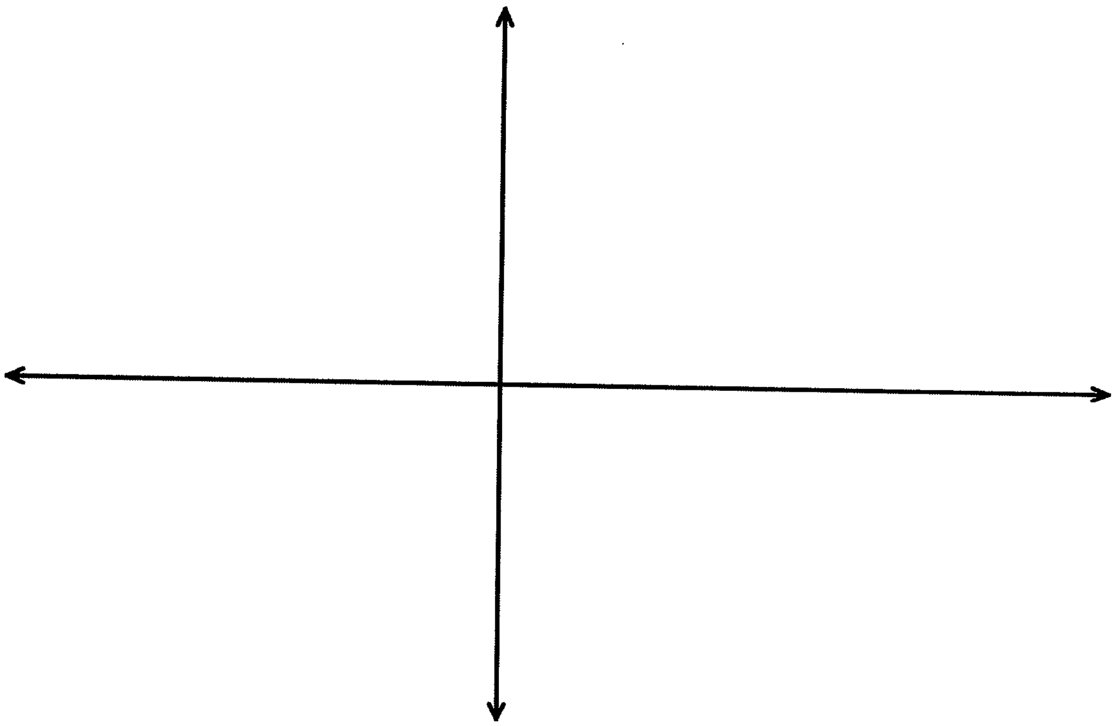
........................................................................................................................

........................................................................................................................

**Question 14 continued on the next page**

(iv) Find any possible point of inflection. **2**

(v) Hence sketch the curve. **2**



**End of Question 14**

**Question 15 (10 marks)**

The velocity of a particle is given by $v = 2\sin\left(t - \frac{\pi}{3}\right)$ metres per second.

Find:

(i) the equation of the displacement $x$ of the particle as a function of time if the particle is initially at the origin. **2**

(ii) when the particle comes to rest for first time. **1**

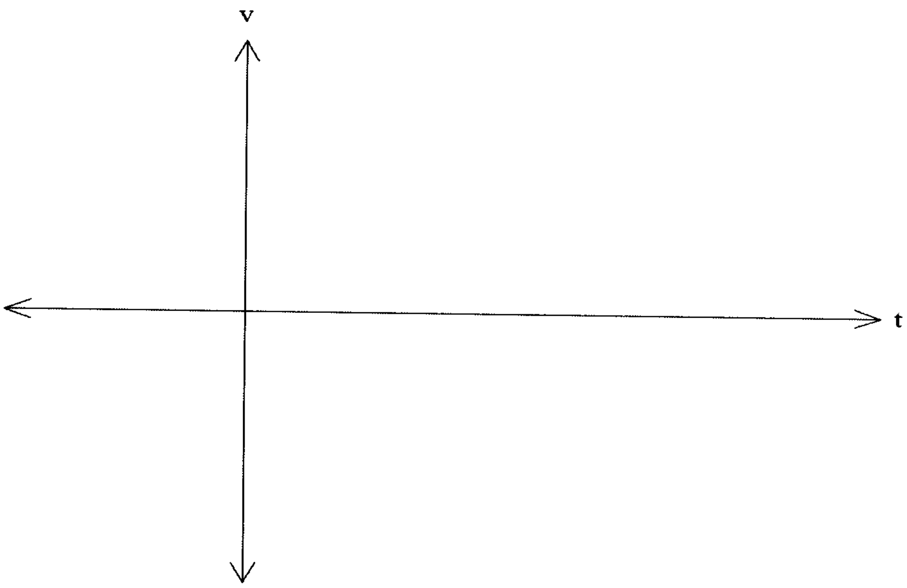
**Question 15 continued on the next page**

(iii) the exact initial velocity. **1**

(iv) the maximum speed of the particle. **1**

(v) Draw the graph of the velocity $v = 2\sin\left(t - \dfrac{\pi}{3}\right)$ for $0 \le t \le 2\pi$. **3**

Show all intercepts and stationary points.



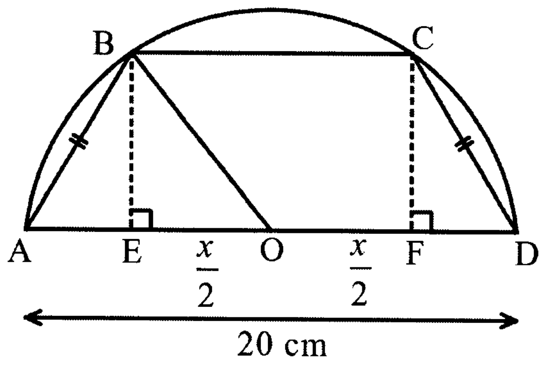

**Question 15 continued on the next page**

(vi) Using your graph above or otherwise, find the total distance travelled by the particle in the first $\pi$ seconds. **2**

**End of Question 15**

**Question 16 (7 Marks)**

An isosceles trapezium $ABCD$ is drawn with its vertices on a semicircle centre $O$ and diameter 20 cm. Perpendiculars $BE$ and $CF$ are drawn to meet the diameter $AD$ as shown in the diagram below.




(i) If $EO = OF = \frac{x}{2}$, show that $BE = \frac{1}{2}\sqrt{400 - x^2}$. **2**

.....................................................................................................................................

.....................................................................................................................................

.....................................................................................................................................

.....................................................................................................................................

.....................................................................................................................................

.....................................................................................................................................

.....................................................................................................................................

(ii) Show that the area of the trapezium $ABCD$ is given by

$A = \frac{1}{4}(x + 20)\sqrt{400 - x^2}$. **2**

.....................................................................................................................................

.....................................................................................................................................

.....................................................................................................................................

.....................................................................................................................................

.....................................................................................................................................

.....................................................................................................................................

.....................................................................................................................................

**Question 16 continued on the next page**

(iii) Hence find the length of $BC$ so that the area of the trapezium $ABCD$ is maximum. **3**

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

..................................................................................................................................

## End of Exam

**BAULKHAM HILLS HIGH SCHOOL**

**YEAR 12 – MATHEMATICS**

**NESA#**__________________

# Mathematics
# Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

**Sample:** $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

*correct*

A ~~●~~ B ~~●~~ C ○ D ○

**Start Here** →

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | A ○ | B ○ | C ○ | D ○ | **6.** | A ○ | B ○ | C ○ | D ○ |
| **2.** | A ○ | B ○ | C ○ | D ○ | **7.** | A ○ | B ○ | C ○ | D ○ |
| **3.** | A ○ | B ○ | C ○ | D ○ | **8.** | A ○ | B ○ | C ○ | D ○ |
| **4.** | A ○ | B ○ | C ○ | D ○ | **9.** | A ○ | B ○ | C ○ | D ○ |
| **5.** | A ○ | B ○ | C ○ | D ○ | **10.** | A ○ | B ○ | C ○ | D ○ |