



BAULKHAM HILLS HIGH SCHOOL

YEAR 11 TASK 1 2020

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour 30 minutes
- Write using black pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11-14
- Marks may be deducted for careless or badly arranged work

Total marks – 74

Exam consists of 17 pages.

This paper consists of TWO sections.

Section I – (10 marks) Pages (2-4) **Questions 1-10**

- Attempt Questions 1-10
- Answer on answer sheet provided
- Allow about 15 minutes for this section

Section II – (64 marks) Pages (6-17)

- Attempt Questions 11-14
- Allow about 1 hour and 15 minutes

Section I - 10 marks**Use the multiple choice answer sheet for questions 1-10**

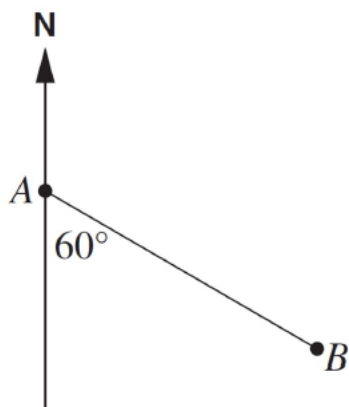
1. $\frac{16x^4}{4x^{-3}} =$

- A) $4x$
- B) $4x^7$
- C) $2x^7$
- D) $2x$

2. What is the slope of the line with equation $2y - 4x + 3 = 0$?

- A) -2
- B) $-\frac{1}{2}$
- C) $\frac{1}{2}$
- D) 2

3. What is the bearing of A from B?



- A) 060°
- B) 120°
- C) 300°
- D) 330°

4. Which of the following is an example of a many-to-one relation?

A) $x^2 + y^2 = 4$

B) $y = 2x^3$

C) $y = |x + 2|$

D) $y = 3^x$

5. The factorisation of $2x^2 - 9x + 4$ is:

A) $(2x - 1)(2x - 8)$

B) $(2x + 1)(x + 4)$

C) $(2x - 1)(x - 4)$

D) $(x - 1)(x - 8)$

6. Find the coordinates of the vertex for $y = 4x^2 + 10x - 7$.

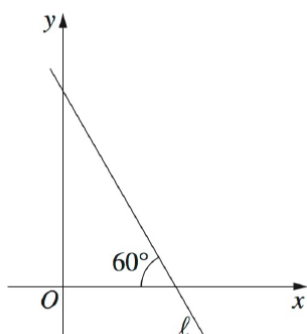
A) $(-\frac{5}{4}, -\frac{53}{4})$

B) $(\frac{5}{4}, \frac{47}{4})$

C) $(-\frac{5}{4}, -\frac{103}{4})$

D) $(-\frac{5}{2}, -7)$

7. The diagram shows the line ℓ .



What is the slope of the line ℓ ?

A) $-\sqrt{3}$

B) $-\frac{1}{\sqrt{3}}$

C) $\frac{1}{\sqrt{3}}$

D) $\sqrt{3}$

8. Expand and simplify $(3x - 2y)^2$.
- A) $3x^2 - 12xy - 2y^2$
 - B) $9x^2 - 12xy - 4y^2$
 - C) $3x^2 - 12xy + 2y^2$
 - D) $9x^2 - 12xy + 4y^2$
9. A monic polynomial $P(x)$ of degree 3 has zeros -2, 1 and 6. Which of the following is the correct equation of the polynomial?
- A) $P(x) = (x - 2)(x + 1)(x + 6)$
 - B) $P(x) = (x + 2)(x - 1)(x - 6)$
 - C) $P(x) = 3(x - 2)(x + 1)(x + 6)$
 - D) $P(x) = 3(x + 2)(x - 1)(x - 6)$
10. Bob and Mary each take turns rolling a pair of six-sided fair dice. The first person to throw a double wins the game. Bob rolls first. Find the probability that Mary wins the game on her second turn.
- A) $\frac{1}{6}$
 - B) $\frac{25}{216}$
 - C) $\frac{125}{1296}$
 - D) $\frac{5}{1296}$

Mathematics Advanced
Section II Answer Booklet

64 marks

Attempt Questions 11-14

Allow about 1 hour and 15 minutes for this section

Instructions:

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.

Please turn over

Question 11 (17 marks)**Marks**

a) Simplify the following:

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i)
$$\frac{5}{x+1} - \frac{3}{x-2}$$

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ii)
$$\frac{a^2 - 3ab + 2b^2}{ab - b^2}$$

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b) Solve simultaneously for x and y .**2**

$$9x - 2y = -1$$

$$7x - 4y = 9$$

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- c) By rationalising the denominator, find the values of a and b , such that

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$$\frac{\sqrt{2}+3}{\sqrt{2}-1} = a + \sqrt{b}$$

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- d) State whether the function $f(x) = \frac{x^3}{x^5 - x}$ is even, odd or neither, justify your answer.

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- e) Express $(x^{-1} + y^{-1})^{-1}$ without any negative indices and simplify.

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f) Solve $|2 - 3x| = 9$.

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g) The point $M(-2, -3)$ is the midpoint of the interval joining $A(5, 2)$ and $B(x, y)$. Find the coordinates of B.

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End of Question 11

Question 12 (16 marks)

a) Find the domain for $f(x) = \frac{1}{\sqrt{x^2 - 4}}$.

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b) If $f(x) = \begin{cases} 3x + 4 & x \geq 2 \\ -2x & x < 2 \end{cases}$

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evaluate $f(3) + f(2) - f(-4)$

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c) If $f(x) = \sqrt{x+4}$ and $h(x) = \frac{1}{x^2 - 1}$ where $x \geq -4$, find the simplified expression for $h(f(x))$.

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- d) Boyle's law is a gas law that states that the pressure, P , of a gas in a container is inversely proportional to the volume V of the container. At a given time, the volume of the container is 27.5m^3 and the pressure is 2.5 Pascals. Find the pressure if the volume is reduced to 20m^3 . 2

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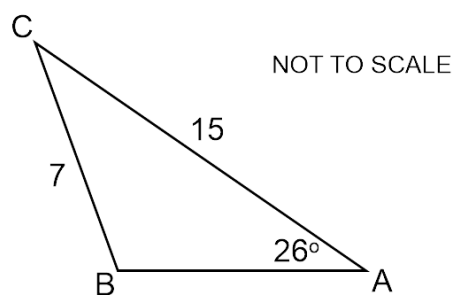
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- e) In triangle ABC, $AC=15$, $BC=7$ and $\angle A$ is 26° . Find $\angle B$, if it is obtuse, to the nearest degree. 3



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- f) Find the equation of the line that passes through the point $(-1, 2)$ and is perpendicular to $x + 2y + 2 = 0$.

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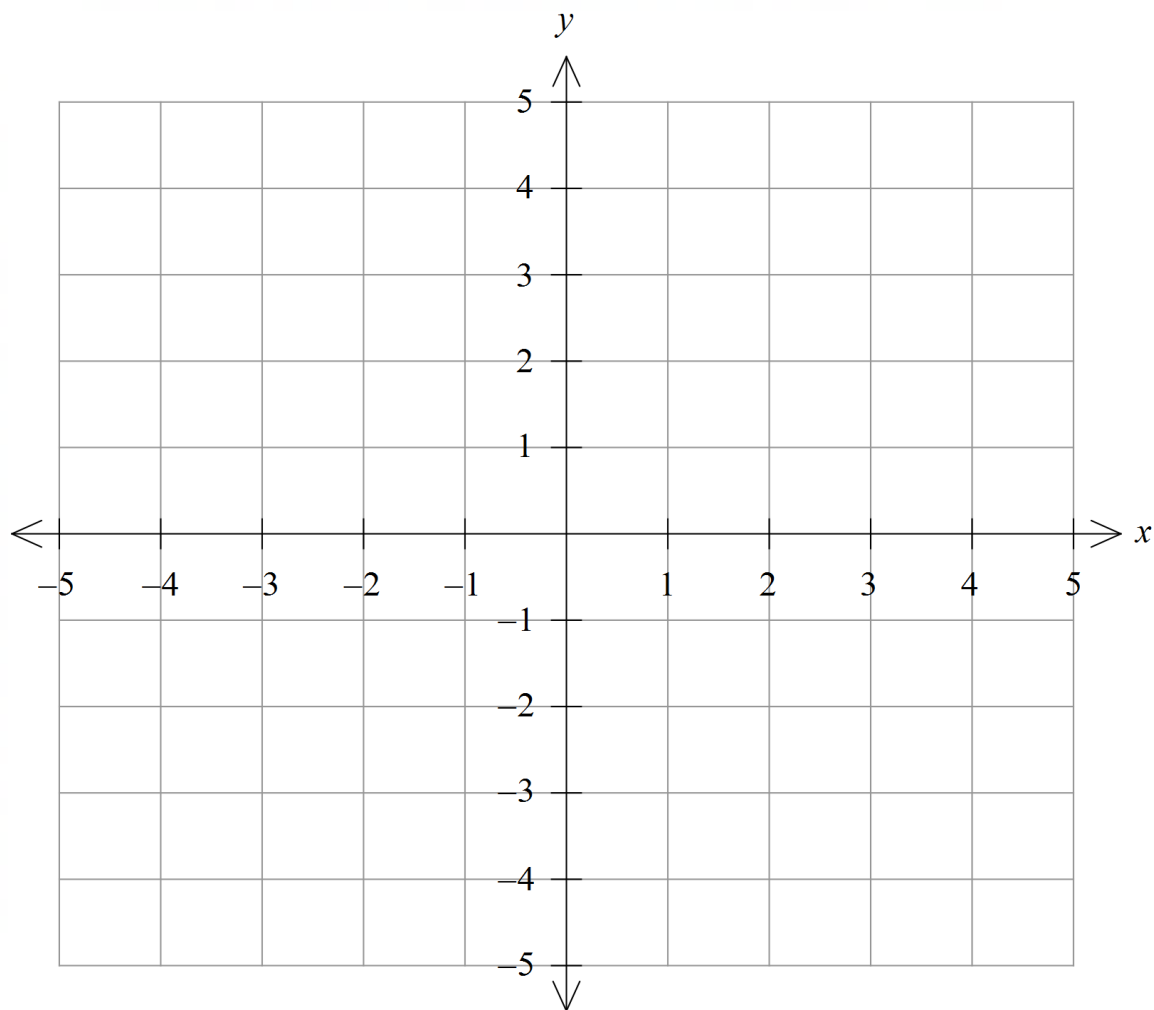
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- g) Sketch the graph of $y = (x + 2)^2(1 - x)$ showing all intercepts.

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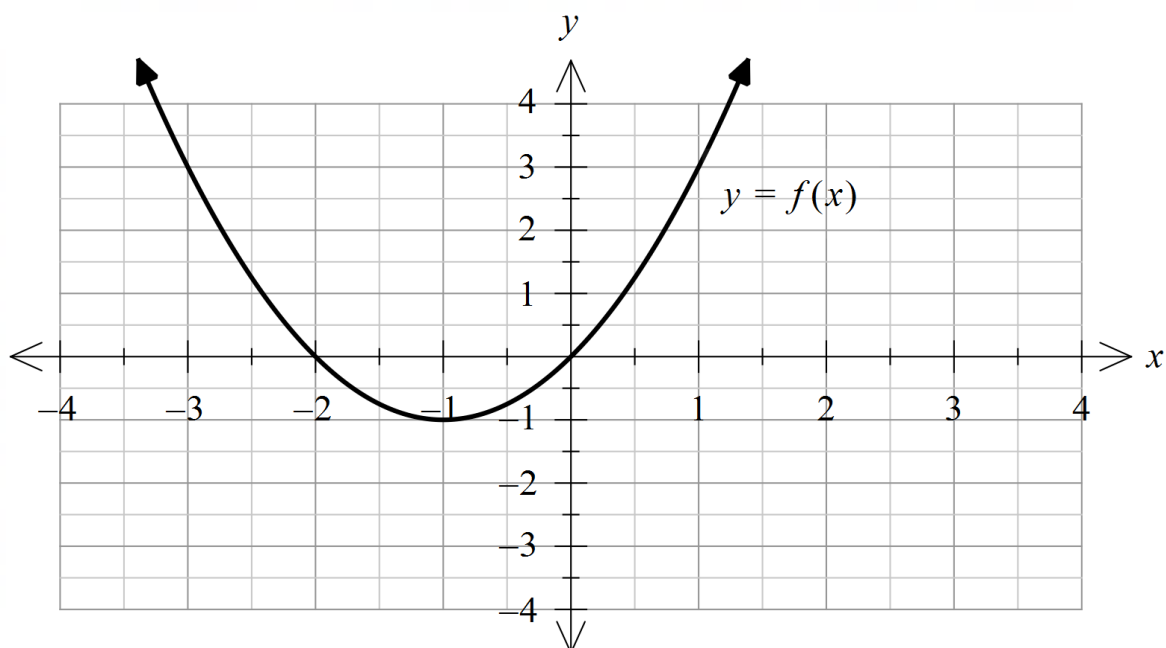
End of Question 12

Question 13 (16 marks)

Marks

- a) Draw $y = -f(-x)$ for the graph of $y = f(x)$ given below.

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- b) Find the value(s) of m in $mx^2 + 3x - 4 = 0$, such that there are no real solutions.

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c) Given that for two events A and B .

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i) $P(A) = 0.6$, $P(B) = 0.4$ and $P(A|B) = 0.8$, find $P(A \cup B)$.

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ii) Are events A and B independent? Justify your answer.

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d) Find the exact value of $\sin 210^\circ \times \tan 330^\circ - \tan 150^\circ \times \cos 30^\circ$, expressing your answer as a single fraction.

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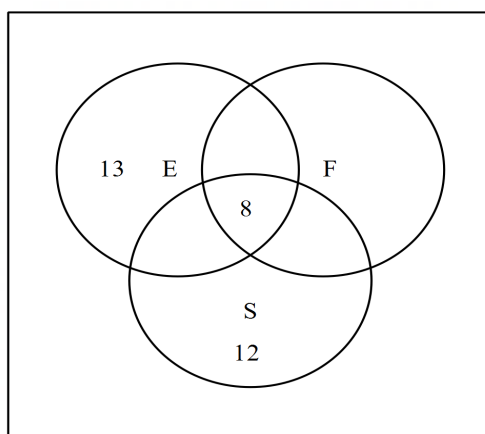
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e) 58 people took a trip to Europe and visited at least one of these countries. Of this group,

- 26 visited France
- 38 visited England
- 29 visited Spain
- 10 visited France and Spain
- 12 visited only Spain
- 13 visited only England
- 8 visited all 3 countries.

i) Use the above information to complete the Venn diagram.



ii) A person from the group of 58 was chosen at random. What was the probability that the person visited France only?

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iii) A person from the group of 58 was chosen at random. What was the probability that the person visited only one country?

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iv) Find the probability that a person visited England, given that they have visited Spain?

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End of Question 13

Question 14 (15 marks)

- a) Solve these two equations $a = b^2 - 5$ and $a = 2b^2 + 5b + 1$ simultaneously.

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- b) Prove $2 - \frac{\cot^2 x}{1 + \cot^2 x} = 1 + \sin^2 x$

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c) Simplify $\frac{a^{\frac{5}{4}} \times (ab)^{\frac{3}{4}}}{b^{-\frac{3}{4}} \times (a^2b)^{\frac{1}{2}}}$ 2

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d) Given the points $A(-3,1)$, $B(2,-2)$ and $P(x,y)$, and $PA = 2PB$:

i) use the distance formula to show that $3x^2 - 22x + 3y^2 + 18y + 22 = 0$ 3

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i) find the centre of the circle whose equation is $3x^2 - 22x + 3y^2 + 18y + 22 = 0$ 1

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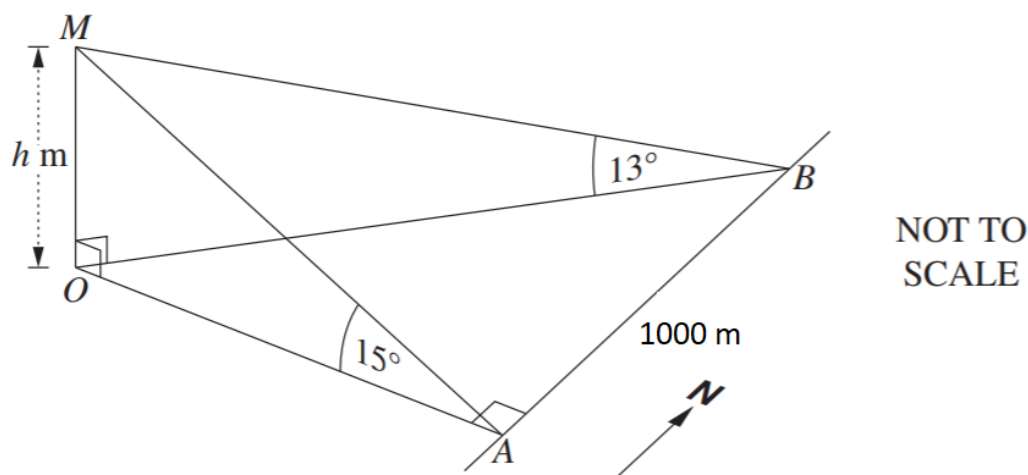
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- e) A person walks 1000 metres due north along a road from point A to point B. The point A is due east of a mountain OM, where M is the top of the mountain. The point O is directly below point M and is on the same horizontal plane as the road. The height of the mountain above point O is h metres.

From point A, the angle of elevation to the top of the mountain is 15° .

From point B, the angle of elevation to the top of the mountain is 13° .



- i) Show that $OA = \frac{h}{\tan 15^\circ}$.

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- ii) Hence find the value of h to the nearest metre.

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End of Exam

Baulkham Hills High School

Task 1 Examination 2020

Marking Guideline- Mathematics Advanced

Section I (10 marks)

Award 1 mark to each correct answer.

Answers: 1.B 2.D 3.C 4.C 5.C 6.A 7.A 8.D 9.B 10.C

Question	Suggested solution	Answer
1	$\frac{16x^4}{4x^{-3}} = 4x^7$	B
2	$2y = 4x - 3$ $y = 2x - \frac{3}{2}$ $m = 2$	D
3	Bearing of A from B: $360^\circ - 60^\circ = 300^\circ$	C
4	$y = x + 2 $	C
5	$2x^2 - x - 8x + 4$ $x(2x - 1) - 4(2x - 1)$ $(x - 4)(2x - 1)$	C
6	$x = -\frac{b}{2a}$ $= \frac{-10}{2(4)}$ $= \frac{-5}{4}$ $y = 4\left(-\frac{5}{4}\right)^2 + 10\left(-\frac{5}{4}\right) - 7$ $= \frac{-53}{4}$ $\left(-\frac{5}{4}, -\frac{53}{4}\right)$	A
7	$m = \tan \theta$ $= -\tan 60^\circ$ $= -\sqrt{3}$	A
8	$9x^2 - 12xy + 4y^2$	D
9	$P(x) = (x + 2)(x - 1)(x - 6)$	B
10	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{125}{1296}$	C

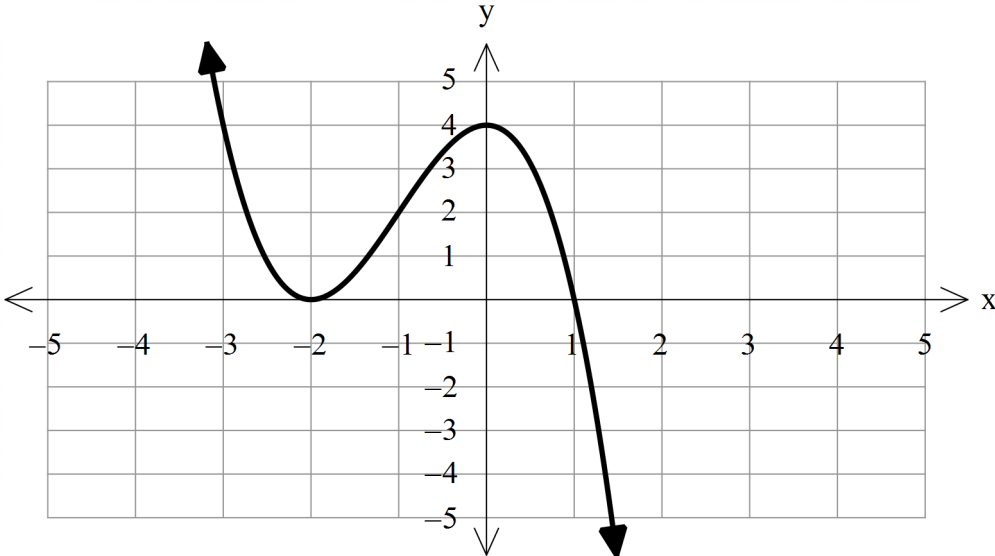
Section II (64 marks)

In all questions, award full marks for correct answers with necessary working.

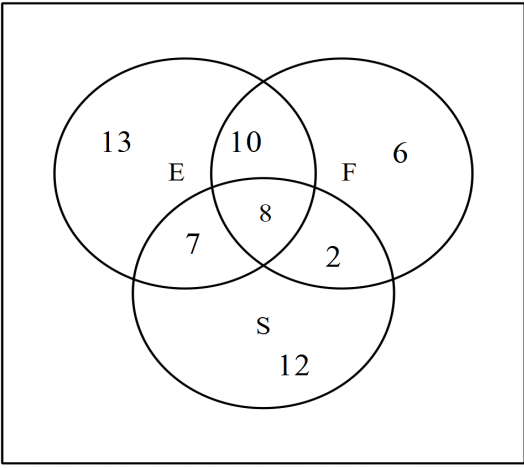
Use the suggested solutions in conjunction with the marking criteria.

Question	Suggested solutions	Marking criteria
11a) i)	$\frac{5}{x+1} - \frac{3}{x-2} = \frac{5(x-2) - 3(x+1)}{(x+1)(x-2)}$ $= \frac{5x - 10 - 3x - 3}{(x+1)(x-2)}$ $= \frac{2x - 13}{(x+1)(x-2)}$	2- correct solution 1- lowest common denominator
11a) ii)	$\frac{a^2 - 3ab + 2b^2}{ab - b^2} = \frac{(a-b)(a-2b)}{b(a-b)}$ $= \frac{(a-2b)}{b}$	2- correct solution 1- factorising correctly $a^2 - 3ab + 2b^2$
11b)	$(9x - 2y = -1) \times 2$ $7x - 4y = 9$ $18x - 4y = -2$ $7x - 4y = 9$ $\therefore x = -1$ $9(-1) - 2y = -1$ $\therefore y = -4$	2- correct value for x and y 1- one of the values (x or y)
11c)	$\frac{\sqrt{2} + 3}{\sqrt{2} - 1} = \frac{\sqrt{2} + 3}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$ $= \frac{2 + \sqrt{2} + 3\sqrt{2} + 3}{2 - 1}$ $= 5 + 4\sqrt{2}$ $= 5 + \sqrt{32}$ $\therefore a = 5, b = 32$	3- correctly establishes the value of a and b 2- getting the expression $5 + 4\sqrt{2}$ 1- multiply by the conjugate surd AND finding one of the values
11d)	$f(x) = \frac{x^3}{x^5 - x}$ $f(-x) = \frac{(-x)^3}{(-x)^5 - (-x)}$ $= \frac{-x^3}{-x^5 + x}$ $= \frac{-x^3}{-(x^5 - x)}$ $= \frac{x^3}{x^5 - x}$ $= f(x)$ $\therefore f(x) \text{ is even}$	2- correct solution 1- substituting $f(-x)$

11e)	$(x^{-1} + y^{-1})^{-1} = \left(\frac{1}{x} + \frac{1}{y}\right)^{-1}$ $= \left(\frac{x+y}{xy}\right)^{-1}$ $= \frac{xy}{x+y}$	2- correct solution 1- simplify one of the negative indices
11f)	$ 2 - 3x = 9$ $ 2 - 3x = 9$ $2 - 3x = \pm 9$ $x = -\frac{7}{3}, \frac{11}{3}$	2- both solutions 1- one solution
11g)	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $(-2, -3) = \left(\frac{5+x}{2}, \frac{2+y}{2}\right)$ $\frac{5+x}{2} = -2 \quad \frac{2+y}{2} = -3$ $x = -9 \quad y = -8$	2- value for x and y 1- value for x or y
12a)	Domain: $x < -2, x > 2$	2- correctly stating the domain 1- correctly stating $x^2 - 4 > 0$
12b)	$f(3) = 13$ $f(2) = 10$ $f(-4) = 8$ $f(3) + f(0) - f(-4) = 13 + 10 - 8$ $= 15$	2- correct solution 1- finding at least 2 of the function values
12c)	$h(f(x)) = h(\sqrt{x+4})$ $= \frac{1}{(\sqrt{x+4})^2 - 1}$ $= \frac{1}{x+4-1}$ $= \frac{1}{x+3}$	2- correct solution for $h(f(x))$ 1- correctly substituting $f(x)$ into $h(x)$

12d)	$V = \frac{k}{P}$ $27.5 = \frac{k}{2.5}$ $k = 68.75$ $V = \frac{68.75}{P}$ $V = \frac{68.75}{3.75}$ $V = 18.3\dot{\text{m}}^3$ $V = \frac{68.75}{P}$ $P = \frac{68.75}{20}$ $= 3.4375 \text{ Pascals}$	2- correct solution 1- find the value of k
12e)	$\frac{\sin \theta}{15} = \frac{\sin 26^\circ}{7}$ $\sin \theta = \frac{15 \sin 26^\circ}{7}$ $\theta = 69^\circ 57'$ Since it is obtuse: $180^\circ - 69^\circ 57' = 110^\circ$	3- correct solution 2- finding the acute angle for θ 1- use Sine rule correctly
12f)	$2y = -x - 2$ $y = \frac{-x}{2} - 1$ $m_1 = -\frac{1}{2}$ $m_2 = 2$ $y - y_1 = m(x - x_1)$ $y - 2 = 2(x + 1)$ $y = 2x + 4 \text{ or } 2x - y + 4 = 0$	2- correct solution 1- find the perpendicular gradient correctly or having the form $2x - y + k = 0$
12g)		3- correctly showing the x- and y-intercepts, having the correct shape (roots) and the correct direction 2- correct intercept and shape 1- correct x- and y-intercept or shape Note: Finding the maximum turning point is not needed.

13a)		2- correct vertex, and intercepts 1- correct intercepts
13b)	Since there are no solutions, $\Delta < 0$ $b^2 - 4ac < 0$ $(3)^2 - 4(m)(-4) < 0$ $16m < 0$ $m < -\frac{9}{16}$	2- correct solution 1- find the discriminant
13c)i)	$P(A B) = \frac{P(A \cap B)}{P(B)}$ $P(A \cap B) = P(B) \times P(A B)$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= P(A) + P(B) - P(B) \times P(A B)$ $= 0.6 + 0.4 - (0.4)(0.8)$ $P(A \cup B) = 0.68$	3- correct solution 2- substituting into the addition rule 1- finding $P(A \cap B)$
13c)ii)	$P(A) \times P(B) = P(A \cap B)$ $0.6 \times 0.4 = 0.24$ $\neq 0.32$ not independent	1- correct solution
13d)	$\sin 210^\circ \times \tan 330^\circ - \tan 150^\circ \times \cos 30^\circ$ $= \left(-\frac{1}{2} \times -\frac{1}{\sqrt{3}} \right) - \left(-\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} \right)$ $= \frac{1}{2\sqrt{3}} - \frac{1}{2}$ $= \frac{1}{2\sqrt{3}} + \frac{\sqrt{3}}{2\sqrt{3}}$ $= \frac{1+\sqrt{3}}{2\sqrt{3}} \text{ or } \frac{\sqrt{3}+3}{6}$	3- correct solution 2- all four exact values correct 1- 2 exact values correct

13e) i)		2- finding all values in the Venn diagram 1- finding at least any 3 of the values
13e) ii)	$P(\text{visiting France only}) = \frac{6}{58} = \frac{3}{29}$	1- correct solution
13e) iii)	$P(\text{visiting only one country}) = \frac{31}{58}$	1- correct solution
13e) iv)	$\frac{P(\text{England} \cap \text{Spain})}{P(\text{Spain})} = \frac{15}{29}$	1- correct solution
14a)	$b^2 - 5 = 2b^2 + 5b + 1$ $b^2 + 5b + 6 = 0$ $(b + 3)(b + 2) = 0$ $b = -3, -2$ $a = (-3)^2 - 5 \quad a = (-2)^2 - 5$ $a = 4 \quad a = -1$ $\begin{cases} a = 4, b = -3 \\ a = -1, b = -2 \end{cases}$	3- correct solutions 2- finding one of the points of intersections 1- attempt of using either substitution or elimination method
14b)	$2 - \frac{\cot^2 x}{1 + \cot^2 x}$ $= 2 - \frac{\left(\frac{1}{\tan^2 x}\right)}{\operatorname{cosec}^2 x}$ $= 2 - \frac{\sin^2 x}{\tan^2 x}$ $= 2 - \cos^2 x$ $= 2 - (1 - \sin^2 x)$ $= 1 + \sin^2 x$	2- correct proof 1- using at least 2 trig identities

14c)	$\frac{a^{\frac{5}{4}} \times (ab)^{\frac{3}{4}}}{b^{-\frac{3}{4}} \times (a^2b)^{\frac{1}{2}}}$ $= \frac{a^{\frac{5}{4}} \times a^{\frac{3}{4}} \times b^{\frac{3}{4}}}{b^{-\frac{3}{4}} \times a \times b^{\frac{1}{2}}}$ $= \frac{a^2 \times b^{\frac{3}{4}}}{a \times b^{-\frac{1}{4}}}$ $= ab$	2- correct solution 1- correct attempt of using index laws
14d) i)	$d_{PA} = \sqrt{(x+3)^2 + (y-1)^2}$ $d_{PB} = \sqrt{(x-2)^2 + (y+2)^2}$ $\sqrt{(x+3)^2 + (y-1)^2} = 2\sqrt{(x-2)^2 + (y+2)^2}$ $(x+3)^2 + (y-1)^2 = 4((x-2)^2 + (y+2)^2)$ $x^2 + 6x + 9 + y^2 - 2y + 1 = 4(x^2 - 4x + 4 + y^2 + 4y + 4)$ $3x^2 - 22x + 3y^2 + 18y + 22 = 0$	3- correct equation 2- expanding their expression 1- substitute into the distance formula or using the idea of squaring both sides
14d) ii)	$x^2 - \frac{22}{3}x + y^2 + 6y + \frac{22}{3} = 0$ $x^2 - \frac{22}{3}x + \frac{121}{9} + y^2 + 6y + 9 + \frac{22}{3} - \frac{121}{9} - 9 = 0$ $\left(x - \frac{11}{3}\right)^2 + (y+3)^2 = \frac{136}{9}$ Centre $\left(\frac{11}{3}, -3\right)$	1- correctly finding the centre of the circle
14e)i)	In $\triangle MOA$ $\tan 15^\circ = \frac{h}{OA}$ $OA = \frac{h}{\tan 15^\circ}$	1- correct solution
14e)ii)	$\tan 13^\circ = \frac{h}{OB}$ $OB = h \cot 13^\circ$ In $\triangle AOB$ $OA^2 + AB^2 = OB^2$ $OB^2 - OA^2 = AB^2$ $h^2 \cot^2 13^\circ - h^2 \cot^2 15^\circ = 1000^2$ $h^2 (\cot^2 13^\circ - \cot^2 15^\circ) = 1000^2$ $h = 455 \text{ metres}$	3- correct solution 2- using Pythagoras theorem 1- find the equation for OB