



BAULKHAM HILLS HIGH SCHOOL
YEAR 12 ASSESSMENT TASK 1 2021
MATHEMATICS ADVANCED

NESA#: _____

Teacher: _____

Mathematics Advanced

**General
Instructions**

- Reading time- 5 minutes
- Working time- 1 hour
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 6-16, show relevant mathematical reasoning and/or calculations

**Total marks:
38**

Section I - 5 marks (pages 4-5)

- Attempt Questions 1-5
- Allow about 8 minutes for this section

Section II - 33 marks (pages 6-14)

- Attempt Questions 6-15
- Allow about 52 minutes for this section

Section I

5 marks

Attempt Questions 1-5

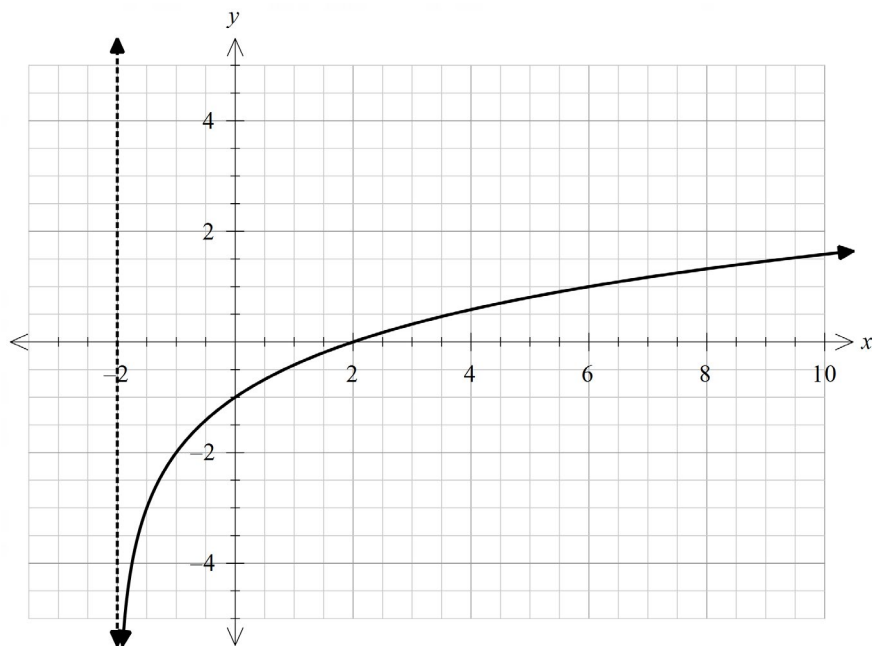
Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1-5.

1 How many terms are there in the geometric progression 3, 6, 12, 24, ..., 384?

- A) 7
- B) 8
- C) 9
- D) 10

2



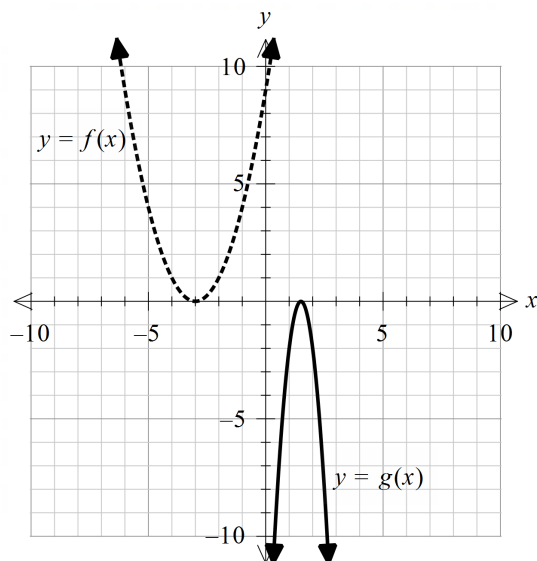
Which of the equations below represents the following graph?

- A) $y = \log_2(x - 2) - 2$
- B) $y = \log_2(x + 2) - 2$
- C) $y = -\log_2(x + 2) - 2$
- D) $y = -\log_2(x - 2) - 2$

3 Which of the following is the domain for $y = \sqrt{(x-1)(x-2)}$?

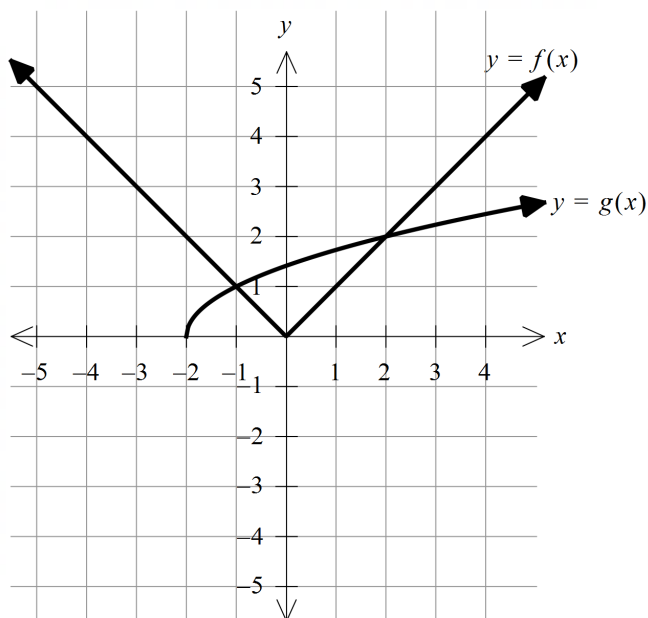
- A) All real x
- B) All real x , $x \neq 1$, $x \neq 2$
- C) $(-\infty, 1] \cup [2, \infty)$
- D) $(1, 2)$

- 4 The graph below shows the original function $y = f(x)$ and its transformed function $y = g(x)$.



Which of the following represents the transformation from $y = f(x)$ to $y = g(x)$?

- A) $g(x) = -2f(-2x)$
 - B) $g(x) = -2f(2x)$
 - C) $g(x) = 2f(-2x)$
 - D) $g(x) = 2f(2x)$
- 5 Consider the graphs of $y = f(x)$ and $y = g(x)$ given below.



Which of the following represents the solution to $f(x) > g(x)$?

- A) $[-1, 2]$
- B) $(-1, 2)$
- C) $(-\infty, -1) \cup (2, \infty)$
- D) $[-2, -1) \cup (2, \infty)$

End of Section I

Section II

34 marks

Attempt Questions 6-15

Allow about 52 minutes for this section

Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning.

Question 6 (4 marks)

Consider the sequence 2, 7, 12, 17, 22, ...

a) Find the 32nd term. 2

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b) Determine the first term of the sequence that exceeds 1500. 2

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Question 7 (2 marks)

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Find k given that $k, k+8$ and $9k$, in that order, are consecutive terms of a geometric sequence.

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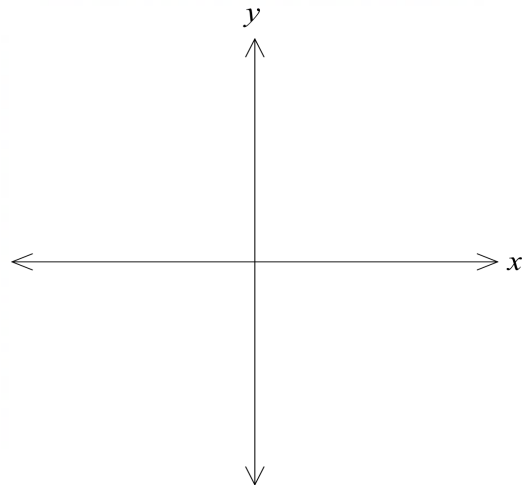
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Question 8 (3 marks)

Consider the function $f(x) = 2x^2 - 7x + 3$.

- a) Sketch the graph of $y = f(x)$, showing the vertex and any intercepts.

2



- b) Hence, or otherwise, solve $f(x) < 0$.

1

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End of Question 8

Question 9 (3 marks)

The n^{th} term of a geometric progression is given by the formula $T_n = 4(3^{-n})$.

- a) Find the first term and common ratio. **2**

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- b) Find the limiting sum of this geometric progression. **1**

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Question 10 (3 marks) **3**

$f(x) = ax^2 + bx + 2$ and $g(x) = cx - 4$. Find the values of a , b and c given that

$g(f(x)) = 12x^2 - 15x - 10$.

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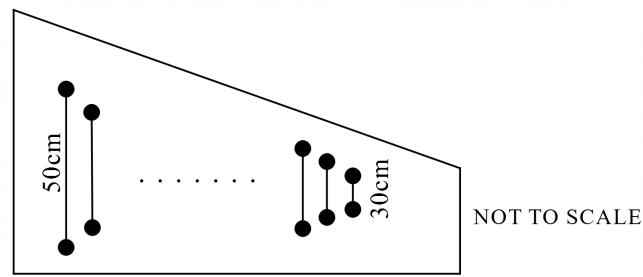
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Question 11 (2 marks)

A simple instrument has many strings, attached as shown in the diagram. The difference between the lengths of adjacent strings is a constant, so that the lengths of the strings are the terms of an arithmetic series. The shortest string is 30 cm long and the longest string is 50 cm. The sum of the lengths of all the strings is 1240 cm.



a) Find the number of strings. **1**

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b) Find the difference in length between adjacent strings. **1**

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End of Question 11

Question 12 (7 marks)

Consider the function $y = \frac{1-x}{x^2-4}$.

- a) State the zeroes of this function. **1**

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- b) Find the equations of the vertical and horizontal asymptotes. **3**

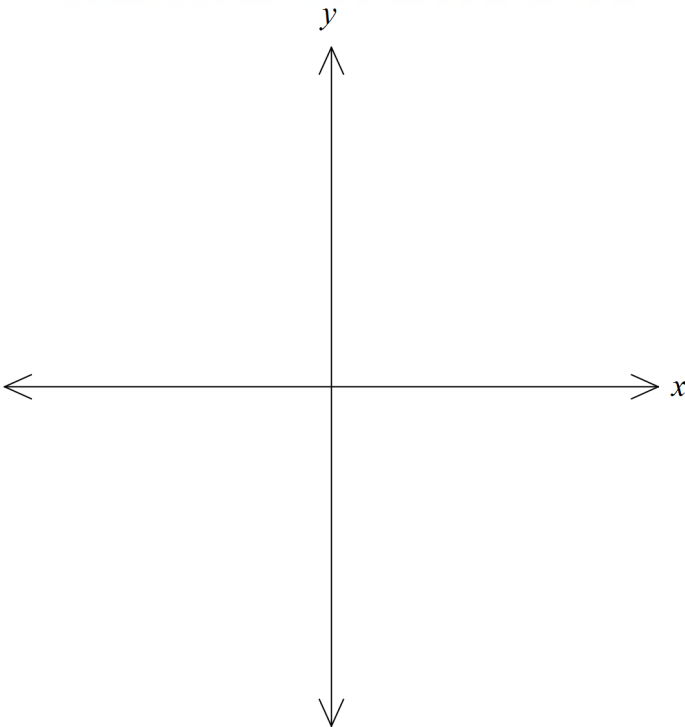
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- c) Using the above information, sketch the function $y = \frac{1-x}{x^2-4}$. Show all of the above features including intercepts. **3**



End of Question 12

Question 13 (2 marks)

2

Find a geometric series with common ratio $\frac{1}{w}$ that has a limiting sum $\frac{1}{1-w}$, where $|w| > 1$.

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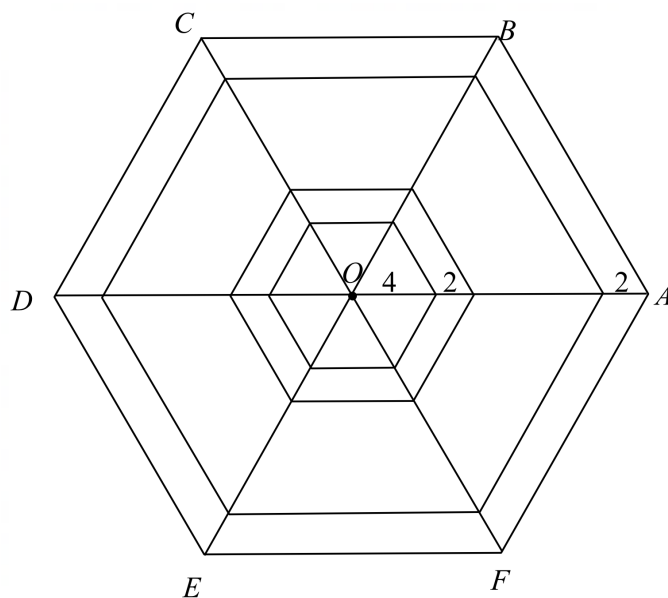
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End of Question 13

Question 14 (5 marks)

A particular spider's web consists of a series of regular hexagons with a common centre O , held together by rays through O , as in the figure below, where only some of the hexagons are shown.



Not to scale

The vertices of the smallest hexagon are 4cm from O , the vertices of the next hexagon are 2cm further away and they continue at 2cm intervals along the rays until the vertices of the last hexagon $ABCDEF$ are 60cm from O .

a) How many hexagons are there?

2

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Question 14 continues onto next page

b) What is the length, in cm, of the perimeter of the smallest hexagon?

1

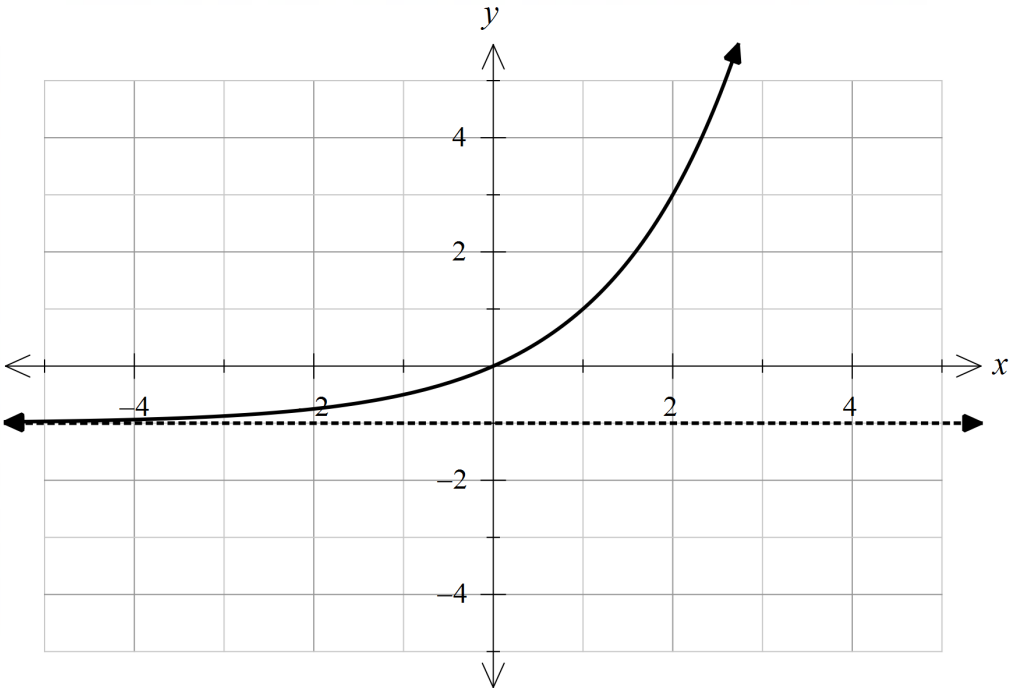
c) What is the total length of thread used by the spider in making this web (including the six rays from O)? 2

2

End of Question 14

Question 15 (2 marks)

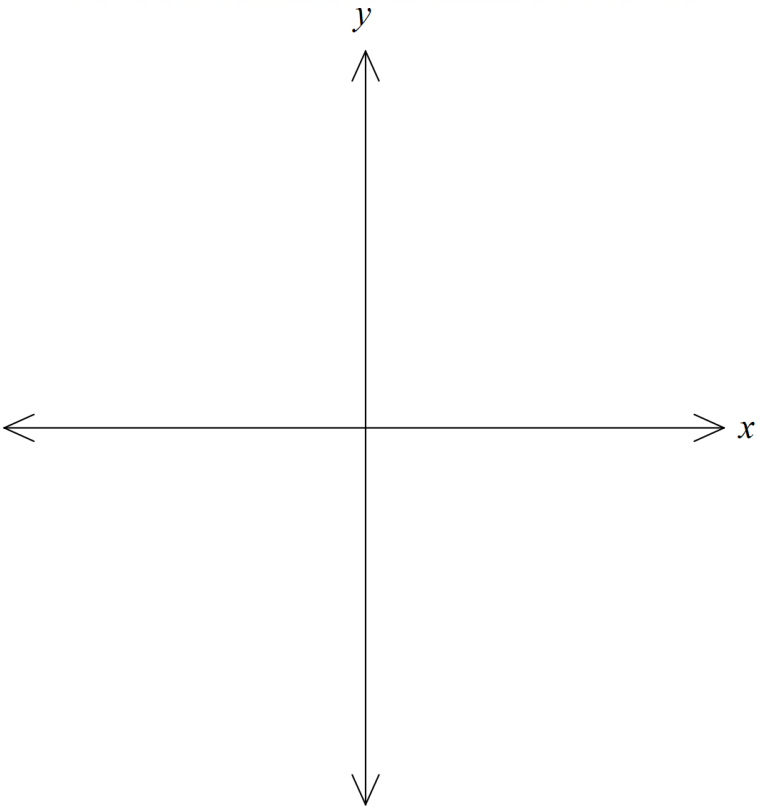
The graph below of $f(x) = 2^x - 1$ is drawn below.



A new function $g(x)$ is found by taking the graph of $y = -2f(x)$ and translating it up by 5 units.

Sketch the graph of $y = g(x)$ showing the y -intercept and the asymptote.

2



End of Exam

Baulkham Hills High School
Task 1 December Examination 2021

Marking Guideline- Yr 12 Mathematics Advanced

Section I (5 marks)

Award 1 marks to each correct answer.

Answers: 1.B 2.B 3.C 4. A 5. D

Question	Suggested solutions	Answer
1	$T_n = ar^{n-1}$ $384 = 3 \times 2^{n-1}$ $\ln 128 = (n-1) \ln 2$ $n = \frac{\ln 128}{\ln 2} + 1$ $n = 8$	B
2	The standard log graph has been shifted to the left by 2 units and down by 2 units. Therefore, it must be $y = \log_2(x+2) - 2$.	B
3	$y = \sqrt{(x-1)(x-2)}$ $(x-1)(x-2) \geq 0$ By testing some points, the domain is $(\infty, 1] \cap [2, -\infty)$	C
4	$f(x)$ has been reflected by the x -axis, therefore y has been replaced with $-y$. Options C and D are out. $f(x)$ has been reflected by the x -axis, therefore x has been replaced with $-x$. Therefore, it must be option A.	A
5	$f(x) > g(x)$ $f(x)$ is above $g(x)$ when: $\therefore [-2, -1) \cup (2, \infty)$	D

Section II (33 marks)

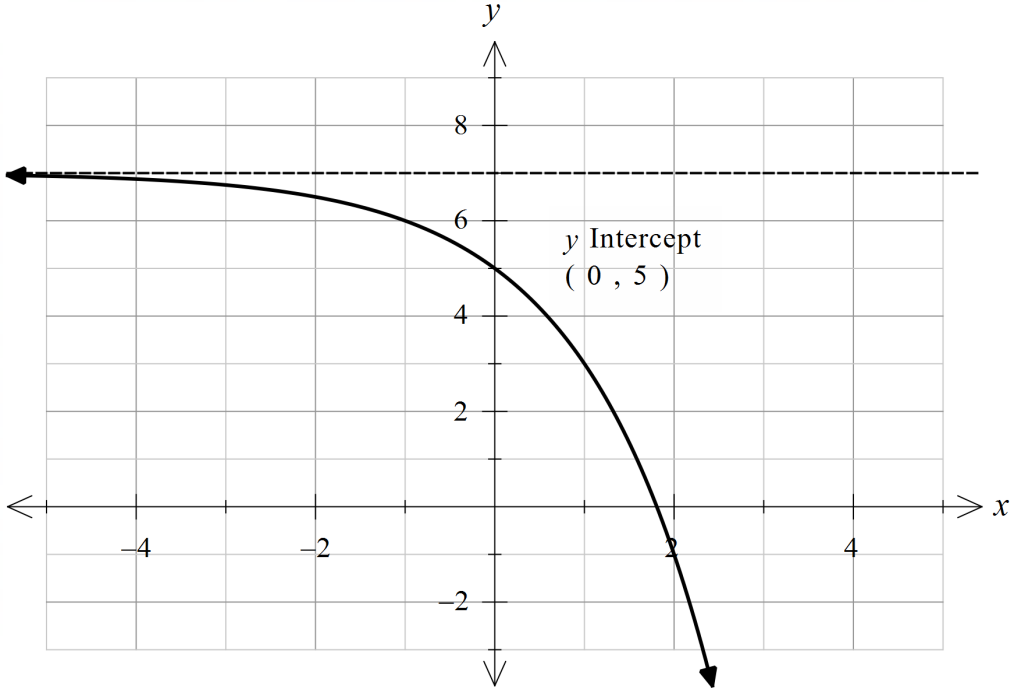
In all questions, award full marks for correct answers with necessary working.

Use the suggested solutions in conjunction with the marking criteria.

Question	Suggested Solutions	Marking criteria
6a	$T_n = a + (n-1)d$ $T_{32} = 2 + (32-1) \times 5$ $T_{32} = 157$	2- Correct solution 1- identifying the difference
6b	$T_n > 1500$ $1500 = 2 + (n-1) \times 5$ $n = 300.6$ $\therefore$ The 301 st term is 1502, which is the first term to exceed 1500.	2- Correct solution 1- identifying correctly the nth term
7	$k, k+8, 9k, \dots$ $\frac{k+8}{k} = \frac{9k}{k+8}$ $(k+8)^2 = 9k^2$ $k^2 + 16k + 64 = 9k^2$ $k^2 - 2k - 8 = 0$ $(k-4)(k+2) = 0$ $k = 4, -2$	2- correct solution 1- correctly forms a quadratic
8a	Graph of a parabola opening upwards. The y-axis is labeled 'y' and has tick marks from -5 to 10. The x-axis is labeled 'x' and has tick marks from -2 to 10. The parabola has a y-intercept at (0, 3) and x-intercepts at (3, 0) and (1/2, 0). The vertex is at (7/4, -25/8).	2- correct graph 1- correct shape with no intercepts or vertex
8b	$\frac{1}{2} < x < 3$	1- correct solution
9a	$T_n = 4(3^{-n})$ $4(3^{-1}), 4(3^{-2}), 4(3^{-3}), \dots$ $\frac{4}{3}, \frac{4}{3^2}, \frac{4}{3^3}, \dots$ $r = \frac{4}{3^2} \div \frac{4}{3}$ $r = \frac{1}{3}$	1- Correct solution

9b	$S = \frac{a}{1-r}$ $S = \frac{\left(\frac{4}{3}\right)}{1-\frac{1}{3}}$ $S = 2$	1- Correct solution
10	$g(f(x)) = g(ax^2 + bx + 2)$ $= c(ax^2 + bx + 2) - 4$ $= acx^2 + bcx + 2c - 4$ $acx^2 + bcx + 2c - 4 = 12x^2 - 15x - 10$ $2c - 4 = -10 \quad -15 = bc \quad 12 = ac$ $2c = -6 \quad -15 = b \times -3 \quad 12 = a \times -3$ $c = -3 \quad b = 5 \quad a = -4$	3- correct solution 2- correctly obtaining 2 of the pronumerals 1- correctly obtaining $g(f(x))$
11a	$S_n = \frac{n}{2}(a + l)$ $1240 = \frac{n}{2}(30 + 50)$ $1240 = 40n$ $n = 31$ $\therefore$ There are 31 strings	1- correct solution
11b	$t_n = a + (n-1)d$ $50 = 30 + (31-1)d$ $20 = 30d$ $d = \frac{2}{3}$ $\therefore$ Difference in length is $\frac{2}{3}$ cm.	1- correct solution
12a	$y = \frac{1-x}{(x-2)(x+2)}$ Zeroes: $x = 1$	1- Correct solution
12b	Vertical: $x = \pm 2$ Horizontal: $\lim_{x \rightarrow \infty} \frac{1-x}{x^2-4}$ $= \lim_{x \rightarrow \infty} \frac{\frac{1}{x^2} - \frac{1}{x}}{\frac{x^2}{x^2} - \frac{4}{x^2}}$ $= \lim_{x \rightarrow \infty} \frac{\frac{1}{x^2} - \frac{1}{x}}{1 - \frac{4}{x}} = 0$ $\therefore y = 0$ is the horizontal asymptote	3- correct solution 2- correctly find both vertical asymptotes or 1 vertical asymptote and 1 horizontal asymptote 1- correctly finds 1 asymptote

12c	y-intercept $-\frac{1}{4}$ x-intercept 1	3- correct graph with asymptotes shown and intercepts labelled 2- correct graph without the features 1- correct asymptotes shown or equivalent
13	$S = \frac{a}{1-r}$ $S = \frac{1}{1-w}, r = \frac{1}{w}$ $\frac{1}{1-w} = \frac{a}{1-\frac{1}{w}}$ $a = \frac{1-\frac{1}{w}}{1-w}$ $= \frac{\frac{w-1}{w}}{1-w}$ $= \frac{w-1}{w} \times \frac{1}{1-w}$ $= -\frac{1}{w}$ $\therefore a = -\frac{1}{w}, r = \frac{1}{w}$ The geometric series is $-\frac{1}{w} - \frac{1}{w^2} - \frac{1}{w^3} - \dots$	2- correct solution 1- correctly finds the first term or finds $a = \frac{1-\frac{1}{w}}{1-w}$
14a	The distances of the vertices from O constitute an arithmetic sequence. 4, (4 + 2), (4 + 2 + 2), ... 60 4, 6, 8, ..., 60 $a = 4, d = 2, T_n = 60, n = ?$ $T_n = a + (n-1)d$ $60 = 4 + (n-1) \times 2$ $n = 29$ $\therefore 29$ hexagons	2- correct solution 1- some progress towards solution

14b	Each regular hexagon can be divided into six equilateral triangles. The perimeter of the smallest regular hexagon is $6 \times 4 = 24\text{cm}$	1- correct solution
14c	Total length of thread is the sum of the hexagon perimeters plus the six rays. Sum of hexagon perimeters = $24 + (6 \times 6) + (6 \times 8) + \dots + (6 \times 60)$ $= 24 + 36 + 48 + \dots + 360$ $a = 24, d = 12, n = 29$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{29} = \frac{29}{2}[2(24) + (29-1) \times 12]$ $= 5568\text{cm}$ Length of 6 rays = 6×60 $= 360$ Total length = $5568 + 360$ $= 5928\text{cm}$	2- correct solution 1- correctly find the sum of the first 29 terms or correctly finds the length of 6 rays
15		2- correct graph 1- 1 mark for correct transformation of $y = -2f(x)$ or 1 mark for translating the graph up by 5 units Note: x intercept not needed.