$\therefore\ 3\ln x\Big|_1^2$

$= 3(\ln 2 - \ln 1)$

$= 3\ln 2$

$= \log_e 8 \longrightarrow$ $\boxed{B}$

$\therefore\ 4^m = 27$

$\log_e 4^m = \log_e 3^3$

$m = \dfrac{3\log_e 3}{2\log_e 2}$ $\boxed{D}$

$\therefore\ 6\left(e^{3x+2}\right)^5 \times 3$

$= 18\left(e^{3x+2}\right)^5$ $\boxed{A}$

$y' = \frac{1}{3}x^{-2} \times -1$

$= -\dfrac{1}{3x^2}$ $\boxed{C}$

$\therefore\ y' = 3x^2 - 12x + 12 = 0$

$(x-2)^2 = 0$

$\therefore x = 2$

$y = 10$

$(2, 10)$ $\boxed{C}$

6. $\dfrac{2e^x - 2xe^x}{(e^x)^2}$

$= \dfrac{2(1-2x)}{e^x}$ $\boxed{A}$

7. $\dfrac{1}{2\sqrt{2}}\sec^2(\sqrt{x}-2)$ $\boxed{C}$

8. $y' = -\sin x\, e^{\cos x}$ $\boxed{B}$

9. $-\frac{1}{2} \times \dfrac{(2-x^2)^{-3}}{-3} + c$

$\dfrac{1}{6(2-x^2)^3} + c$ $\boxed{B}$

10. $\boxed{D}$

Section II

Q11

a) Find the value of $\int_0^1 e^{\frac{-x}{2}}\,dx$ in exact form 2

$\int_0^1 e^{-\frac{x}{2}}\,dx$

$= -\frac{1}{\frac{1}{2}}\left[e^{-\frac{x}{2}}\right]_0^1$

$= -2\left[e^{-\frac{x}{2}}\right]_0^1$ 1 mark for integration

$= -2\left[e^{-\frac{1}{2}} - e^{0}\right]$

$= -2\left[e^{-\frac{1}{2}} - 1\right]$

$= -2e^{-\frac{1}{2}} + 2$

$= \cancel{2}\ -2(e^{\frac{1}{2}} - 1)$ 1 mark for Correct Ans.

b) $\int_{\frac{\pi}{15}}^{\frac{\pi}{2}} 15\cos(4x)\,dx$ 2

$= \frac{15}{4}\left[\sin 4x\right]_{\frac{\pi}{15}}^{\frac{\pi}{2}}$

$= \frac{15}{4}\left[\sin 2\pi - \sin\frac{4\pi}{15}\right]$

1mark for Integrali

$= \frac{15}{4}\left[0 - \sin\frac{4\pi}{15}\right]$

$= -\frac{15}{4}\sin\frac{4\pi}{15}$ 1mark for Correct ANS

c) The function $f(x)$ is defined by the rule $f(x) = x^2(6-x)$ in the domain $-1 \le x \le 6$. 3

i) Find the stationary points of $f(x)$ and determine their nature.

$f'(x) = 6x^2 - x^3$

$= 12x - 3x^2$

$f'(x) = 0$, (stationary point)

$12x - 3x^2 = 0$

$3x(4-x) = 0$ 1 mark for diff.

$x = 0, x = 4$

at $x = 0$, $y = 0^2(6-0) = 0 \rightarrow (0,0)$

$x = 4$, $y = 4^2(6-4) = 32 \rightarrow (4,32)$

$\therefore (0,0)$ and $(4,32)$ stationary points.

TESTING STATIONARY POINTS

$f''(x) = 12 - 6x$

at $x = 0$, $f''(0) = 12 > 0$

$\therefore (0,0)$ minimum.

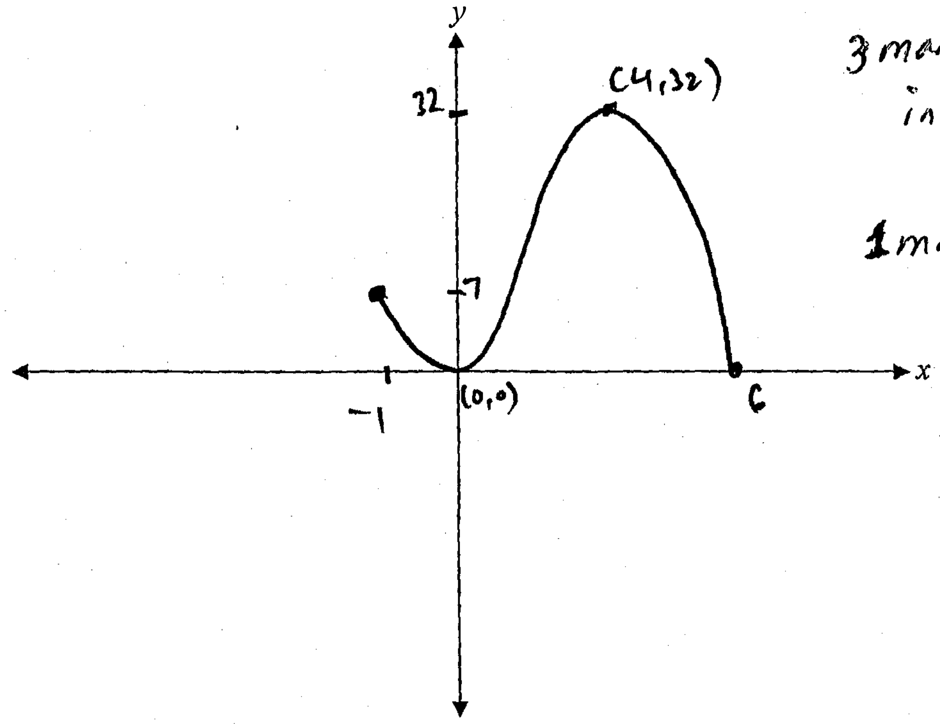
at $x = 4$, $f''(4) = 12 - 24 = -12 < 0$.

$\therefore (4,32)$ maximum.

1 mark for min, max.

1 mark for stationary points coordinates

ii) Sketch the graph of $y = f(x)$. Showing all intercept and stationary points. 3



3 mark Correct graph, include correct labels.

1 mark graphs No lable.

iii) State the range of $f(x)$. 1

at Domain $-1 \le x \le 6$

Range $\therefore 0 \le y \le 32$. 1 mark Correct ANS

Q12 a) Find $\int \frac{dx}{(3x-1)^2}$ 2

$\int (3x-1)^{-2}dx$

$\frac{1}{3}\left[\frac{(3x-1)^{-2+1}}{-2+1}\right]+C$ 2 marks for Correct ANs Include Simplifying

$= -\frac{1}{3}(3x-1)^{-1}+C$

$= \frac{-1}{3(3x-1)}+C$ ✓ 1 mark for just integration

b) Find $\int \frac{3x}{x^2+1}dx$ 2

$= \frac{3}{2}\left[\ln(x^2+1)\right]+C$ 2 marks Correct ANs

1 mark for: $\ln(x^2+1)$.

c) Find $\int \frac{x^3-x^2+1}{x^2}dx$ 2

$= \int \frac{x^3-x^2+1}{x^2}dx$

$= \int \left(x-1+\frac{1}{x^2}\right)dx$

$= \int (x-1+x^{-2})dx$

$= \frac{x^2}{2}-x+\frac{x^{-2+1}}{-2+1}+C$ ⟵ 1 mark

$= \frac{x^2}{2}-x-\frac{1}{x}+C$ 2 marks Correct ANs

d) Find $\int_3^5 \frac{e^x}{e^x - 7}\,dx$ 3

State your answer as an exact value in simple form.

$= \left[\ln(e^x - 7)\right]_3^5$ ①

$= \ln(e^5 - 7) - \ln(e^3 - 7) = \ln\left[\frac{e^5 - 7}{e^3 - 7}\right]$ ① ①

$=$

$= 2.38$

① identify ln

② integrate correctly

① substitute & simplify

Q13

a) Differentiate the following $y = \ln\frac{x(x^2-1)}{x+1} = \ln\left[\frac{x(x-1)\cancel{(x+1)}}{\cancel{(x+1)}}\right] = \ln[(x)(x-1)]$ 2

$\frac{3x^2-1}{x^3-x} - \frac{1}{x+1}$ $\quad \frac{2x-1}{x(x-1)}$

$y = \ln[(x)(x-1)]$

$y = \ln x + \ln(x-1)$ $\quad y = (\ln x + \ln(x^2-1)) - \ln(x+1)$

$\left.\frac{dy}{dx} = \frac{1}{x} + \frac{1}{x-1}\right\}$ full marks ← 1 mark

2 $= \frac{x-1+x}{x(x-1)} = \frac{2x-1}{x(x-1)} = \frac{2x-1}{x^2-x}$ ← 2 marks Correct ANS $\frac{2x^3+3x^2-1}{x(x+1)^2(x-1)}$

b) Find the equation of the tangent to the curve $y = x\cos x$ at the point $(\frac{\pi}{3}, \frac{\pi}{6})$ 3

$\frac{dy}{dx} = x(-\sin x) + \cos x$ ← 1 mark for diff

$\frac{dy}{dx} = -x\sin x + \cos x$

at $x = \frac{\pi}{3}$, $M_1 = \frac{dy}{dx} = -\frac{\pi}{3}\sin\frac{\pi}{3} + \cos\frac{\pi}{3} = -\frac{\sqrt{3}}{2}\cdot\frac{\pi}{3} + \frac{1}{2}$

$= -\frac{\sqrt{3}\pi}{6} + \frac{1}{2} = -0.40689$

$= -\frac{\sqrt{3}\pi + 3}{6}$ ← 1 mark for gradient

Equation of tangent

$y - \frac{\pi}{6} = \left(\frac{-\sqrt{3}\pi + 3}{6}\right)\left(x - \frac{\pi}{3}\right)$ ← 1 mark for correct equation

OR $(\sqrt{3}\pi - 3)x + 6y - \frac{\sqrt{3}\pi^2}{3} = 0$ OR

$y = \frac{1}{18}(-3\sqrt{3}\pi x + 9x + \sqrt{3}\pi^2)$

Q14

a) Use two applications of the Trapezoidal rule to find an approximation to $\int_2^4 \ln x\,dx$ 3

Correct to 3 significant figures. $f(x) = \ln x$

$h = \frac{4-2}{2} = 1$

$f(2) = \ln 2 = 0.693$

$f(3) = \ln 3 = 1.097$

$f(4) = \ln 4 = 1.386$

$\int_2^4 \ln x\,dx \doteq \frac{h}{2}\left[f(2) + 2f(3) + f(4)\right]$

$= \frac{1}{2}\left[0.693 + 2(1.097) + 1.386\right]$

$= 2.1365$

$= 2.14$ 3 sig fig

$\boxed{3}$ marks Correct ANS.

$\boxed{2}$ marks for Correct (3 sig) Integration

$\boxed{1}$ mark for correct Applying Tapezoidal Rule

b) Find the $\lim_{x \to 0} \frac{\sin 4x}{x}$ 2

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$4 \lim_{x \to 0} \frac{\sin 4x}{4x}$ ⟵ $\boxed{1}$ mark

$= 4 \times 1 = 4.$

2 mark Correct Ans

Q15 a) Prove that $\tan\theta(\sec\theta - \sin\theta\tan\theta) = \sin\theta$ 2

LHS $= \frac{\sin\theta}{\cos\theta}\left[\frac{1}{\cos\theta} - \frac{\sin\theta\sin\theta}{\cos\theta}\right]$

$= \frac{\sin\theta}{\cos\theta}\left[\frac{1-\sin^2\theta}{\cos\theta}\right]$ ⟵ $\boxed{1}$ mark.

$= \frac{\sin\theta}{\cos\theta}\left[\frac{\cos^2\theta}{\cos\theta}\right]$

$= \frac{\sin\theta\cos^2\theta}{\cos^2\theta}$

$= \sin\theta$ · RHS.

$\boxed{2}$ mark Correct proof

b) Hence: Solve $2\tan\theta(\sec\theta - \sin\theta\tan\theta) = 1$ for $0 \le \theta \le 2\pi$. 2

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$ ← $\boxed{1}$ mark

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $(0 \le \theta \le 2\pi)$ ← $\boxed{1}$ mark

Q16 Find $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{2+\sin x}\,dx$. State your answer as an exact value in simplest form. 2

$= \ln\left[2+\sin x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$= \ln\left[2+\sin\frac{\pi}{2}\right] - \ln\left[2+\sin\frac{-\pi}{2}\right]$

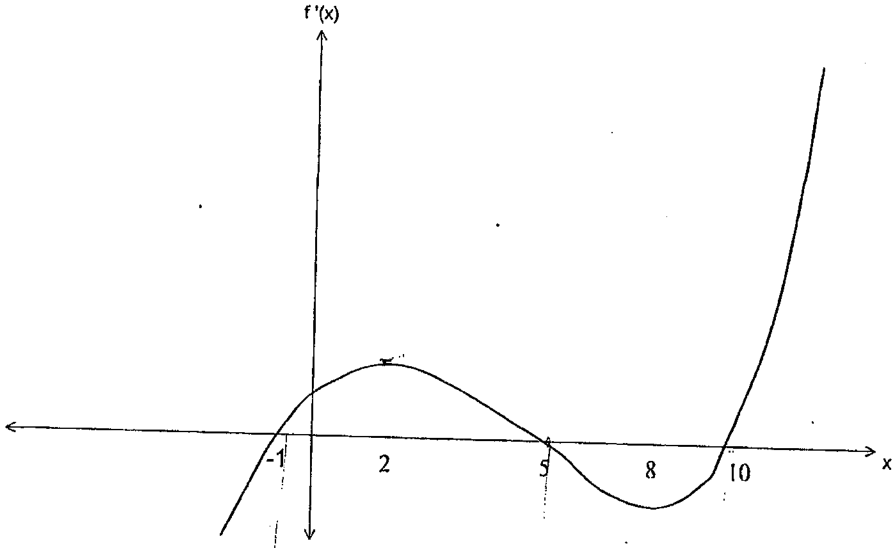
$= \ln 3 - \ln 1$

$= \ln 3$

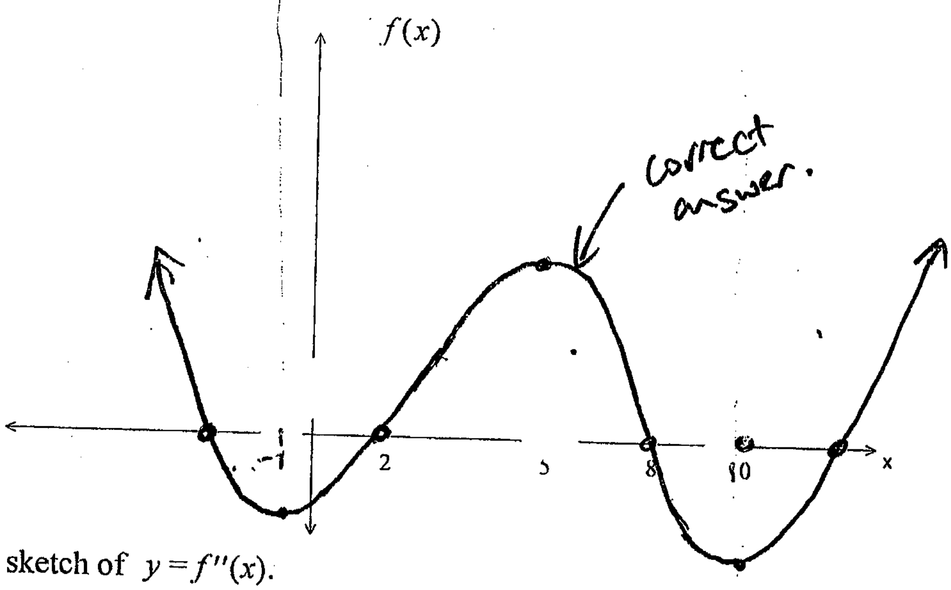
$\boxed{2}$ marks Correct ANS

$\boxed{1}$ mark for integration

Q17 The graph of $y = f'(x)$ is shown below.



(i) Draw a sketch of $y = f(x)$. 2

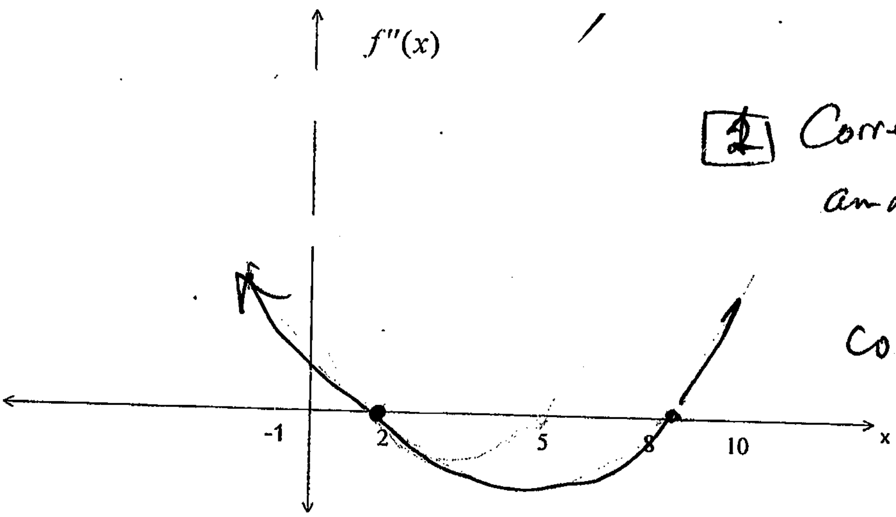


correct answer.

$\boxed{2}$ marks correct x ints

$\boxed{1}$ correct shape.

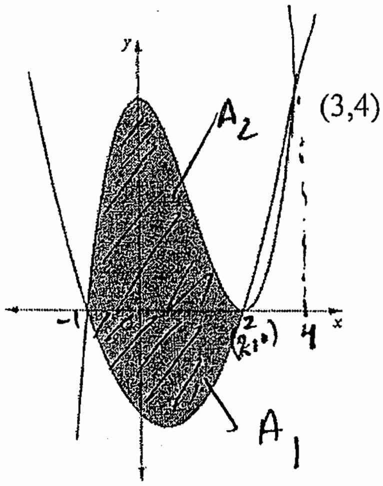
(ii) Draw a sketch of $y = f''(x)$. 2



$\boxed{2}$ Correct shape and concavity

correct x int

Q19 The graphs of $f(x) = x^3 - 3x^2 + 4$ and $g(x) = x^2 - x - 2$ are drawn on the diagram.



a) One of the points of intersection is (3, 4). The other points of intersection lies on the x-axis. 2

Find the coordinates of the other two points of intersection.

$y = x^3 - 3x^2 + 4$ ①

$y = (x^2 - x - 2) = 0$

$(x+1)(x-2) = 0$ ← $\boxed{1}$ mark

$x = -1, \ x = 2$

at $x = -1, y = 0$ $(-1, 0)$

$x = 2, y = 0$ $(2, 0)$ ← $\boxed{1}$ mark

b) Calculate the shaded area of the region. 5

$\int_{-1}^{2} x^3 - 3x^2 + 4 - (x^2 - x - 2) \, dx$

$= \int_{-1}^{2} x^3 - 4x^2 + x + 6 \, dx.$

$\left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x \right]_{-1}^{2}$

$\frac{2^4}{4} - \frac{4 \times 2^3}{3} + \frac{2^2}{2} + 6 \times 2 - \left[\frac{(-1)^4}{4} - \frac{4 \times (-1)^3}{3} + \frac{(-1)^2}{2} + 6(-1) \right.$

$\frac{16}{4} - \frac{32}{3} + \frac{4}{2} + 12 - \left[\frac{1}{4} + \frac{4}{3} + \frac{1}{2} - 6 \right.$

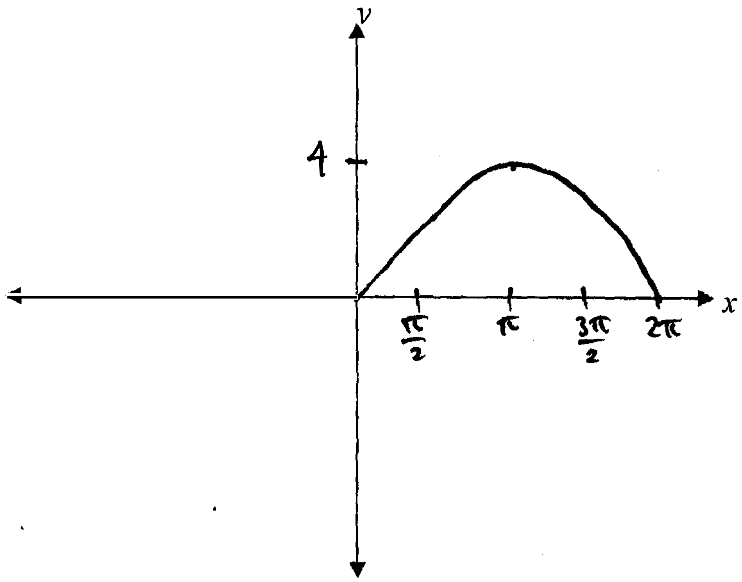
$\frac{4 \times 22}{4 \times 3} + \frac{47}{12}$ $\qquad \frac{3 + 16 + \cancel{6}}{12} - \frac{72}{12}$

$= \frac{45}{4} u^2$ $\qquad \frac{41}{12}$

Q19 2

Consider the curve $y = 4\sin\left(\frac{x}{2}\right), 0 \le x \le 2\pi$.

a) Sketch the curve.



2

b) Find the area bounded by the curve and the x-axis.

$A = \int_0^{2\pi} \left(4\sin\frac{x}{2}\right) dx$

$= -4 \times \frac{1}{\frac{1}{2}} \left[\cos\left(\frac{x}{2}\right)\right]_0^{2\pi}$

$= -8\left[\cos\pi - \cos 0\right]$

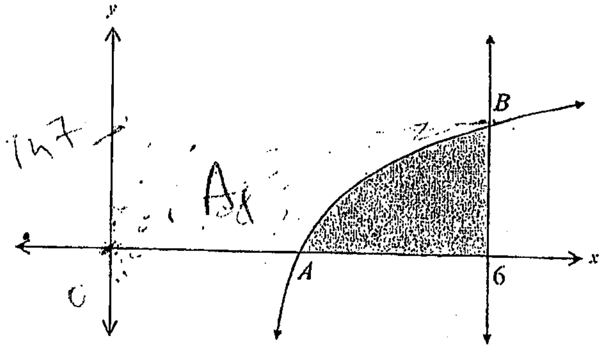
$= -8[-1-1]$

$= -8(-2)$

$= 16 \text{ unit}^2$

Q20 The diagram shows a shaded region which is bounded by the curve $y = \ln(2x-5)$, the x axis and the line $x = 6$.

The curve $y = \ln(2x-5)$ intersects the x axis at A and the line $x = 6$ at B.



Req area = Ar. of rect. − Area

$2\int_0^{\ln 7} \frac{e^y+5}{2}$ ①

a) Show that the coordinates of points A and B are $(3, 0)$ and $(6, \ln 7)$ respectively. 2

$y = \ln(2x-5)$

at $y = 0$.

$0 = \ln(2x-5)$

$\ln 1 = \ln(2x-5)$

$1 = 2x-5$

$2x = 6 \rightarrow x = 3. \rightarrow (3, 0)$

$\boxed{1}$ mark for A

at $x = 6$

$y = \ln(2x-5)$

$= \ln[2(6)-5]$

$= \ln(12-5)$

$= \ln 7$

$\therefore (6, \ln 7)$.

$\boxed{1}$ mark for B

b) Show that if $y = \ln(2x-5)$, then $x = \dfrac{e^y+5}{2}$. 2

$e^y = e^{\ln(2x-5)}$

$e^y = 2x-5$ ← 1 mark

$2x = e^y+5$

$x = \dfrac{e^y+5}{2}$ ← 1 mark

$\int_3^6 y\,dx$.

Area of Rect = $6 \ln 7$

$$A_1 = \int_0^{\ln 7} \frac{e^y + 5}{2}\, dy$$

$$= \left. \frac{e^y + 5y}{2} \right]_0^{\ln 7}$$

$$\frac{1}{2}\left[e^{\ln 7} + 5 \ln 7 - e^0\right]$$

$$\frac{1}{2}\left[6 + 5 \ln 7\right]$$

$$\therefore \text{Req } A = 6 \ln 7 - 3 - \tfrac{5}{2} \ln 7$$

$$= \frac{7}{2} \ln 7 - 3$$

1

$3\frac{1}{4}$ $3\frac{1}{5}$ 13 (12

Q21 a) Write the derivative of $log_e(\tan x), \tan x > 0$. 1

$= \frac{1}{\tan} \times \sec^2 x$

$= \operatorname{cosec} x \cdot \sec x$ 1 mark

b) Hence find $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \operatorname{cosec} x \sec x \, dx$ 3

Leave your answer as an exact value in simplest form.

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \operatorname{cosec} x \sec x \, dx$

$= \left[\boxed{\log_e(\tan x)} \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ ← $\boxed{1}$ mark

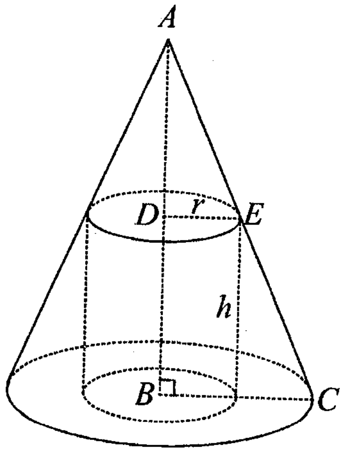
$= \log_e\left(\tan\frac{\pi}{3}\right) - \log_e\left(\tan\frac{\pi}{4}\right)$

$= \log_e(\sqrt{3}) - \log_e(1)$ ← $\boxed{1}$ mark

$= 0.55$ ← $\boxed{1}$ mark

Q22 The radius of the base of a cone is 9 cm and its height is 18 cm.

A cylinder of radius r and height h is inscribed in the cone as sown below.



a) Show that $h = 18 - 2r$. 3

$\angle ADE = \angle ABC = 90^\circ$

$\angle AED = ACB$ Corresponding in parallel lines.

$\therefore \Delta ABE /// \Delta ABC$

$AB = 18$

~~$AD = 18 - h$~~

$\frac{AD}{AB} = \frac{DE}{BC}$

$AD = 18 - h.$

$\frac{18-h}{18} = \frac{r}{9}.$

$h = 18 - 2r.$

b) Show that the volumes of the cylinder of maximum volume and the cone, are in the ratio 4:9. 3

$V = \pi r^2 h.$

$= \pi r^2 (18 - 2r).$

$= 18\pi r^2 - 2\pi r^3$

1 mark

$V' = 36\pi r - 6\pi r^2$

$= 6\pi r(6 - r)$

$V' = 0$

1 mark

$6\pi r(6-r) = 0$

$r = 0$, or 6.

But r can't be 0.

$\therefore r = 6$

1 mark for select $r=6$ as max.

END OF EXAMINATION

BAULKHAM HILLS HIGH SCHOOL

YEAR 12 TASK 2

2021

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used.
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- A reference sheet is provided

Total marks – 76

This paper consists of TWO sections.

Section I – (10 marks) Pages (2-5)

- Attempt Questions 1-10
- Answer on answer sheet provided
- Allow about 15 minutes for this section

Section II – (66 marks) Pages (7-21)

- Attempt Questions 11-22
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

---

1. What is the value of $\int_1^2 \frac{3}{x}\,dx$?

(A) $2\log_e 3$

(B) $\log_e 8$

(C) $2\log_e 3^2$

(D) $3\log_e 3$

2. What is the solution of $4^m = 27$?

(A) $m = \frac{3\log_e 3}{4}$

(B) $m = \log_e\left(\frac{27}{4}\right)$

(C) $m = \frac{27}{\log_e 4}$

(D) $m = \frac{3\log_e 3}{2\log_e 2}$

3. What is the derivative of $(e^{3x+2})^6$?

(A) $18\,(e^{3x+2})^6$

(B) $6(e^{3x+2})^5$

(C) $18(e^{3x+2})^7$

(D) $6(e^{3x+2})^6$

4. What is the derivative of $f(x) = \dfrac{1}{3x}$?

(A) $-\dfrac{3}{x^2}$

(B) $-\dfrac{1}{(3x)^2}$

(C) $-\dfrac{1}{3x^2}$

(D) $\dfrac{3}{x^2}$

5. What is the stationary point on the curve $y = x^3 - 6x^2 + 12x + 2$?

(A) $(-2, -54)$

(B) $(2,0)$

(C) $(2,10)$

(D) $(-2,0)$

6. What is the derivative of $\dfrac{2x}{e^x}$?

(A) $\dfrac{2-2x}{e^x}$

(B) $\dfrac{e^{-x} - xe^{-x}}{x^2}$

(C) $e^{-x}(2+2x)$

(D) $\dfrac{2+2x}{e^x}$

7. If $f(x) = \tan(\sqrt{x} - 2)$, what is $f'(x)$?

(A) $2\sec^2(\sqrt{x} - 2)$

(B) $\sec^2(\sqrt{x} - 2)$

(C) $\frac{1}{2\sqrt{x}}\sec^2(\sqrt{x} - 2)$

(D) $\frac{1}{2}\sec^2(\sqrt{x} - 2)$

8. What is the derivative of $f(x) = e^{\cos x}$

(A) $\cos x e^{\cos x}$

(B) $-\sin x e^{\cos x}$

(C) $e^{-\sin x}$

(D) $-cosxe^{-\sin x}$

9. Which of the following is an expression for $\int \frac{x}{(2-x^2)^4} dx$?

(A) $\frac{1}{2(2-x^2)^3} + C$

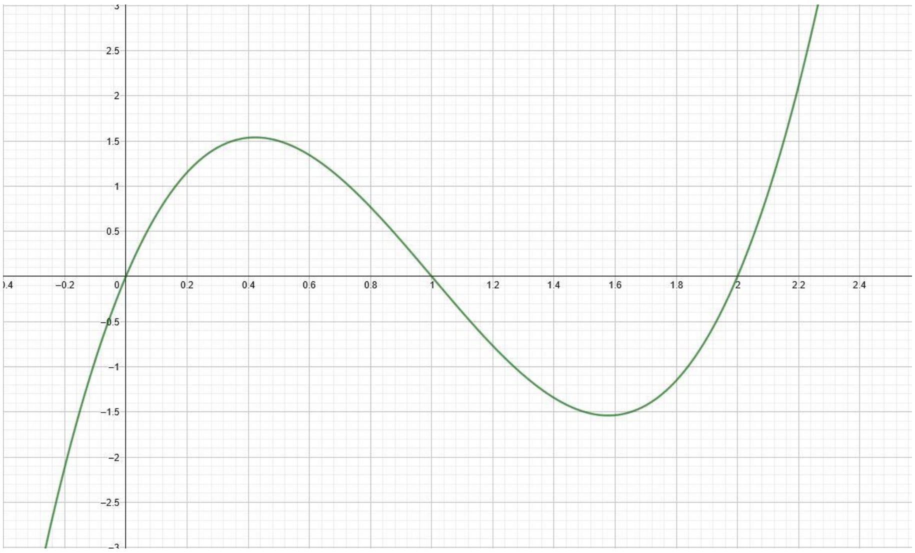
(B) $\frac{1}{6(2-x^2)^3} + C$

(C) $\frac{1}{4(2-x^2)^4} + C$

(D) $\frac{-1}{3(2-x^2)^3} + C$

10. The diagram shows the cubic $y = 4x(x-1)(x-2)$.

Which expression gives the area bounded by the cubic and the x-axis?



(A) $\int_0^2 (4x(x-1)(x-2))dx$

(B) $\left|\int_2^0 (4x(x-1)(x-2))\,dx\right|$

(C) $\left|\int_0^1 (4x(x-1)(x-2))\,dx\right| + \int_1^2 (4x(x-1)(x-2))\,dx$

(D) $\int_0^1 (4x(x-1)(x-2))\,dx + \left|\int_1^2 (4x(x-1)(x-2))\,dx\right|$

End of Section I

This page is intentionally blank

BAULKHAM HILLS HIGH SCHOOL
YEAR 12 TASK 2, 2021
MATHEMATICS ADVANCED

FINAL MARK

NESA#: ____________________

Teacher: ____________________

Question	Curve-sketching using Calculus	Integration	Exponentials & Logarithmic functions	Trigonometric Functions	Total
MC	4, 5 /2	9, 10 /2	1, 2, 3, 6 /4	7, 8 /2	/10
11	c /7		a /2	b /2	/11
12		a, b, c /6	d /3		/9
13			a /2	b /3	/5
14		a /3		b /2	/5
15				a /4	/4
16				/2	/2
17	/4				/4
18		a, b /5			/5
19				/4	/4
20			a, b, c /7		/7
21				a, b /4	/4
22	a, b /6				/6
Total	**/19**	**/16**	**/18**	**/23**	**/76**

BAULKHAM HILLS HIGH SCHOOL

YEAR 12 TASK 2 - 2021

MATHEMATICS ADVANCED

NESA#: ______________________________

Teacher: ______________________________

Mathematics
Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

Start Here →

1.	A ○	B ○	C ○	D ○	**6.**	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○	**7.**	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○	**8.**	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○	**9.**	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○	**10.**	A ○	B ○	C ○	D ○

Section II Answer the questions in the spaces provided. Extra writing paper is available if required.
Your responses should include relevant mathematical reasoning and/or calculations.

Q11 a) Find the value of $\int_0^1 e^{\frac{-x}{2}}\,dx$ in exact form. 2

....

....

....

....

....

....

....

....

....

....

b) Find the value of $\int_{\frac{\pi}{15}}^{\frac{\pi}{2}} 15\cos(4x)\,dx$. 2

....

....

....

....

....

....

....

....

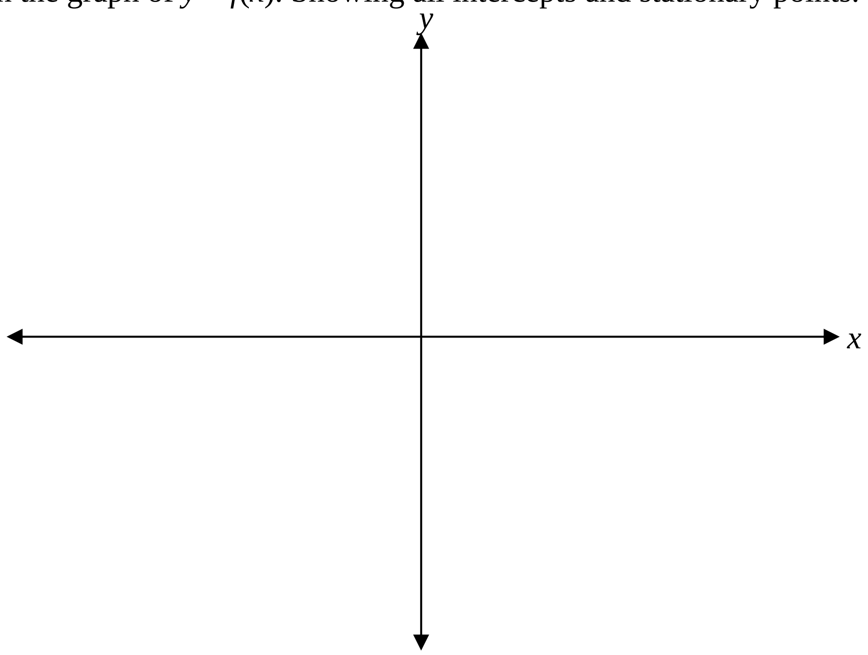
....

....

c) The function $f(x)$ is defined by the rule $f(x) = x^2(6 - x)$ in the domain $-1 \leq x \leq 6$.

i) Find the stationary points of $f(x)$ and determine their nature. 3

ii) Sketch the graph of $y = f(x)$. Showing all intercepts and stationary points. 3



iii) State the range of $f(x)$. 1

Q12 a) Find $\int \frac{dx}{(3x-1)^2}$. 2

....

....

....

....

....

b) Find $\int \frac{3x}{x^2+1}\,dx$. 2

....

....

c) Find $\int \frac{x^3-x^2+1}{x^2}\,dx$. 2

....

....

....

....

....

....

....

....

....

d) Find $\int_3^5 \frac{e^x}{e^x - 7}\,dx$. State your answer as an exact value in simplest form. 3

....

....

....

....

....

....

Q13 a) Differentiate $y = \ln \frac{x\left(x^2-1\right)}{x+1}$. 2

....

....

....

....

....

....

....

....

b) Find the equation of the tangent to the curve $y = x\cos x$ at the point $\left(\frac{\pi}{3}, \frac{\pi}{6}\right)$. 3

....

....

....

....

....

....

Q14 a) Use two applications of the trapezoidal rule to find an approximation to $\int_2^4 \ln x \, dx$. 3

Give your answer correct to 3 significant figures.

....

....

....

....

....

....

....

....

....

....

b) Find $\lim_{x \to 0} \frac{\sin 4x}{x}$. Show all working. 2

....

....

....

....

Q15 a) Prove that $\tan\theta(\sec\theta - \sin\theta\tan\theta) = \sin\theta$. 2

....

....

....

....

....

....

....

b) Hence: Solve $2\tan\theta(\sec\theta - \sin\theta\tan\theta) = 1$ for $0 \le \theta \le 2\pi$. 2

....

....

....

....

Q16 Find $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{2+\sin x}\,dx$. State your answer as an exact value in simplest form. 2

....

....

....

....

....

....

....

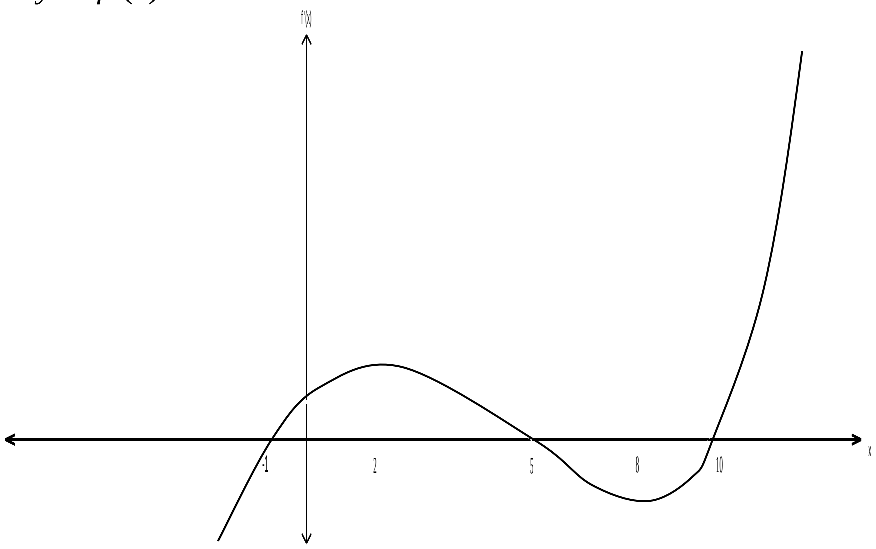
....

....

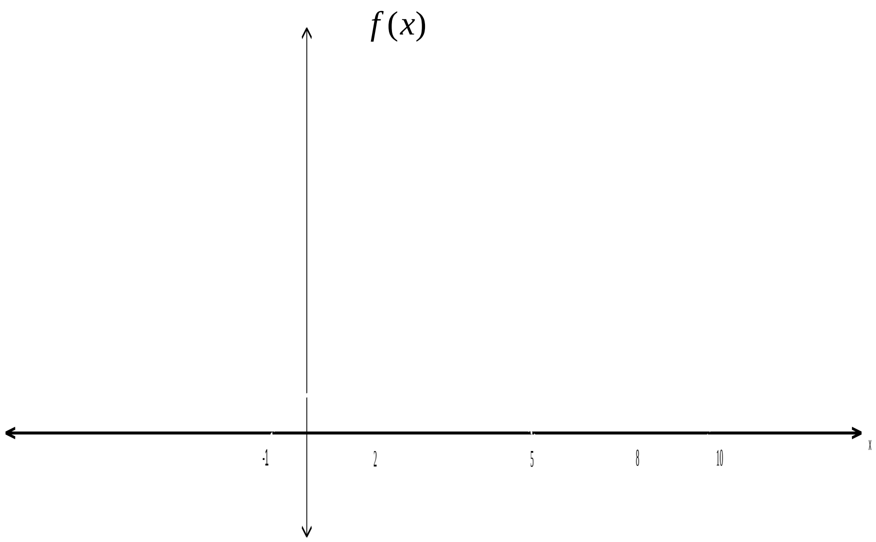
....

....

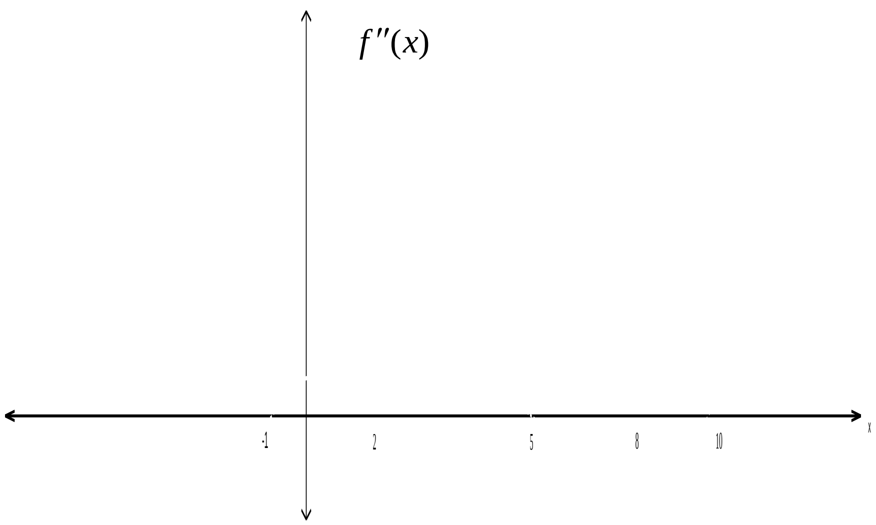
Q17 The graph of $y = f'(x)$ is shown below.



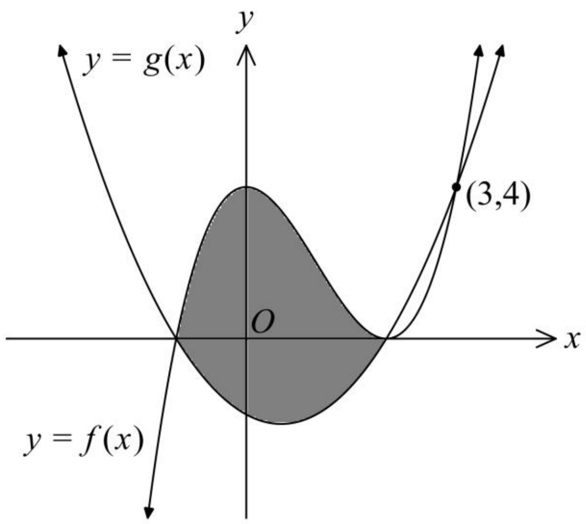
(i) Draw a sketch of $y = f(x)$. 2



(ii) Draw a sketch of $y = f''(x)$. 2



Q18 The graphs of $f(x) = x^3 - 3x^2 + 4$ and $g(x) = x^2 - x - 2$ are drawn on the diagram.



a) One of the points of intersection is (3, 4). The other points of intersection are on the x-axis. Find the coordinates of the other two points of intersection. 2

....

....

....

....

b) Calculate the shaded area of the region. 3

....

....

....

....

....

....

....

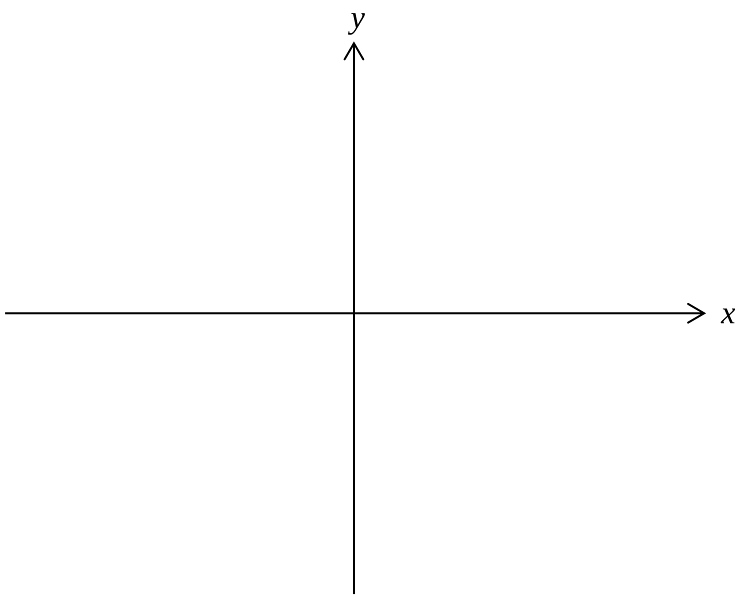
....

....

....

Q19 Consider the curve $y = 4\sin\left(\dfrac{x}{2}\right), 0 \leq x \leq 2\pi$.

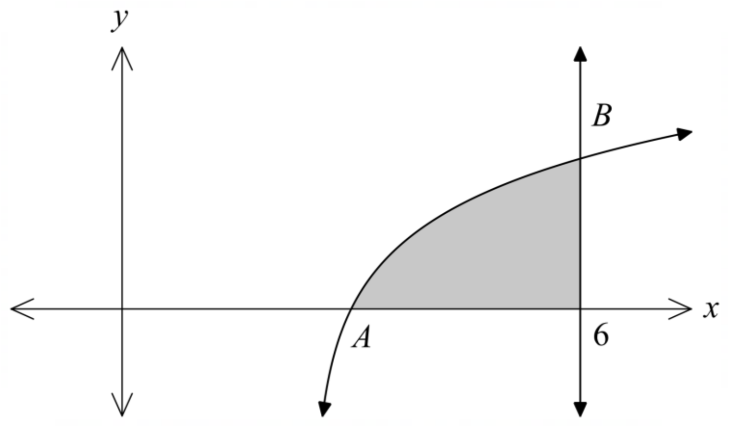
a) Sketch the curve. 2



b) Find the area bounded by the curve and the x-axis. 2

Q20 The diagram shows a shaded region which is bounded by the curve $y = \ln(2x - 5)$, the x-axis and the line $x = 6$.

The curve $y = \ln(2x - 5)$ intersects the x axis at A and the line $x = 6$ at B.



a) Show that the coordinates of points A and B are (3, 0) and (6, ln7) respectively. 2

....

....

....

....

....

....

....

....

....

....

....

....

....

b) Show that if $y = \ln(2x-5)$, then $x = \dfrac{e^{y} + 5}{2}$. 2

....

....

....

....

....

....

....

....

c) Hence find the exact area of the shaded region. 3

....

....

....

....

....

....

....

....

Q21 a) Write the derivative of $\log_e(\tan x)$, where $\tan x > 0$. 1

....

....

....

....

....

....

....

b) Hence find $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \operatorname{cosec} x \sec x \, dx$. 3

Leave your answer as an exact value in simplest form.

....

....

....

....

....

....

....

....

....

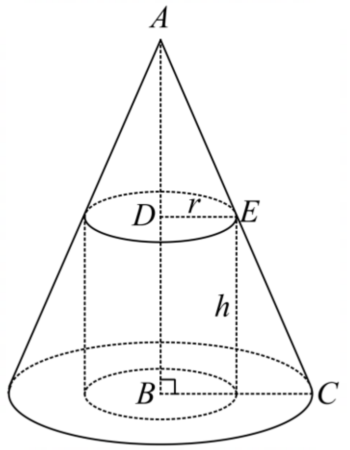
....

....

....

Q22 The radius of the base of a cone is 9 cm and its height is 18 cm.

A cylinder of radius r and height h is inscribed in the cone as sown below.



a) Show that $h = 18 - 2r$. 3

....

....

....

....

....

....

....

b) Show that the volumes of the cylinder of maximum volume and the cone, 3

are in the ratio 4:9.

....

....

....

....

....

....

....

....

END OF EXAMINATION