

Baulkham Hills High School

Year 11 Task 1 - 2021

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 1 hour 30 minutes
- Write using black or blue pen
- Calculators approved by NESA may be used.
- In questions 11-14 show relevant mathematical reasoning and/or calculations.

Total Marks – 73

Section I - 10 marks

- Attempt questions 1 – 10
- Allow about 15 minutes for this section

Section II - 63 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 15 minutes for this section.

SECTION I (10 Marks)

Shade the correct option on the provided answer sheet for the multiple-choice questions.

1. What is the product of 3.2×10^6 and 4×10^2 in scientific notation?

- A) 12×10^8
- B) 1.28×10^9
- C) 12.8×10^8
- D) 8×10^7

2. $\frac{1}{ab} + \frac{a}{bc}$ is equal to

- A) $\frac{1+a}{abc}$
- B) $\frac{c+a^2}{abc}$
- C) $\frac{1+a}{ab^2c}$
- D) $\frac{c+a^2}{ab^2c}$

3. What are the factors of $4 - (x+1)^2$?

- A) $(3-x)$ and $(5+x)$
- B) $(2-x)$ and $(2+x)$
- C) $(1+x)$ and $(3-x)$
- D) $(1-x)$ and $(3+x)$

4. The function $f(x) = x^3$ is transformed to $g(x) = (x-2)^3 + 5$ by a horizontal translation followed by a vertical translation.

Which of the following shows the directions of translations?

- A) 2 units left and 5 units up
- B) 2 units right and 5 units up
- C) 2 units left and 5 units down
- D) 2 units right and 5 units down

5. What type of relation is $xy = 36$?

- A) one-to-one
- B) many-to-one
- C) many-to-many
- D) one-to-many

6. What are the solutions of $2x^2 - 5x - 1 = 0$?

- A) $x = \frac{-5 \pm \sqrt{17}}{4}$
- B) $x = \frac{5 \pm \sqrt{17}}{4}$
- C) $x = \frac{-5 \pm \sqrt{33}}{4}$
- D) $x = \frac{5 \pm \sqrt{33}}{4}$

7. Which of the following statements is correct about the function $f(t) = t^2 - 9$?

- A) Domain is $t \geq 9$
- B) Range is $f(t) \leq -9$
- C) Range is $f(t) \geq -9$
- D) Domain is $t > 0$

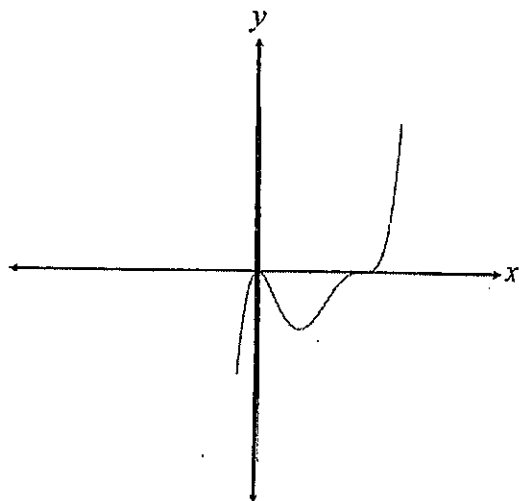
8. A bag contains 4 red marbles and 6 blue marbles. Three marbles are selected at random one at a time without replacement.

What is the probability that at least one of the marbles selected is red?

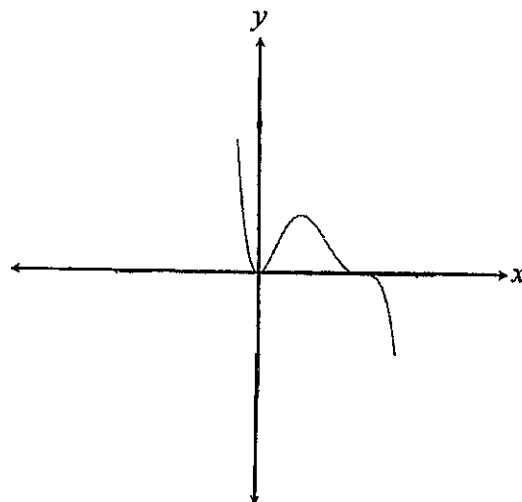
- A) $\frac{1}{6}$
- B) $\frac{1}{2}$
- C) $\frac{5}{6}$
- D) $\frac{29}{30}$

9. Which of the following is the graph of $y = x^2(2-x)^3$?

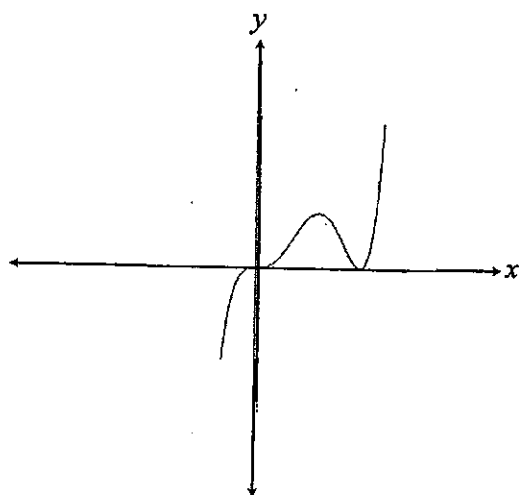
A)



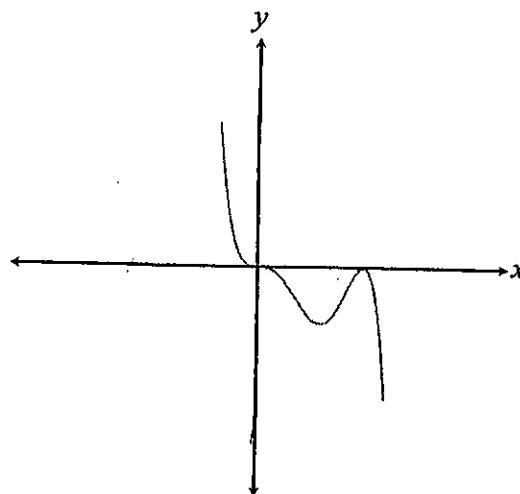
B)



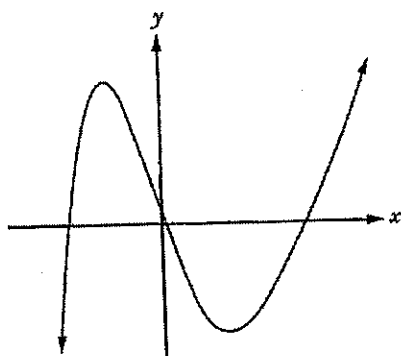
C)



D)

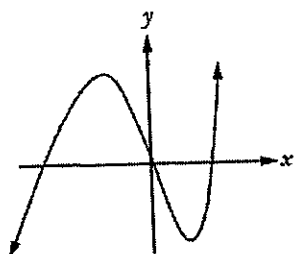


10. The graph of $y = f(x)$ is given below.

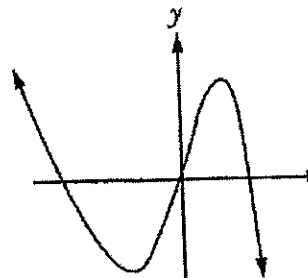


Which of the following is the graph of $y = -f(-x)$?

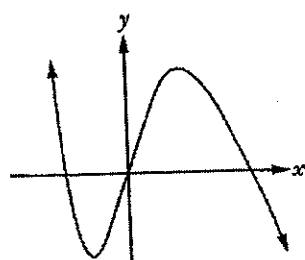
A)



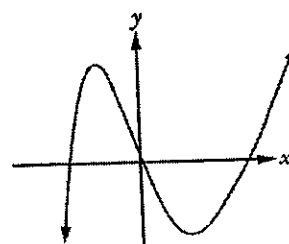
B)



C)



D)



End of Section I

Instructions

- Answer Questions 1 – 10 in your answer sheet on **page 3** of this booklet. Shade the option that corresponds to your answer.
- Write your answers in the space provided in this booklet for Questions 11 - 14. Show relevant mathematical reasoning and/or calculations.
- Write using black or blue non-erasable pen. (Black is recommended)
- You may use the extra writing pages if you need more space. These pages are at the end of this Booklet.
 - (a) At the end of appropriate answer space, write “CONTINUED”
 - (b) On the extra page(s) clearly write the question number and part of the answer being written
- At the conclusion of the exam, return the multiple-choice question paper and this booklet to the exam supervisor.

SECTION II

Question 11 (17 Marks)

Marks

(a) Simplify the following:

(i) $\frac{5}{x-2} - \frac{3}{x+1}$

2

(ii) $\frac{y^2 - yx - 6x^2}{yx - 3x^2}$

2

(b) Solve simultaneously for x and y .

$$2x + 3y = 14$$

$$4x - 5y = 17$$

2

(c) Rationalise the denominator: $\frac{\sqrt{2}-1}{2\sqrt{2}-1}$.

2

(d) Express $\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right)^2$ with positive indices and simplify as a single fraction.

2

(e) Solve $|2x-9|=13$.

2

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(f) Find the value of k if $f(-3)=0$ and $f(x)=x^3+9x^2+kx+27$

2

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(g) A function is defined as $f(x)=\begin{cases} x^2, & x \leq 1 \\ 2-x, & x > 1 \end{cases}$.

(i) Sketch the graph of $f(x)$.

2

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(ii) What is the range of $f(x)$?

1

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End of Question 11

Question 12 (15 Marks)

(a) If $f(x) = \begin{cases} \sqrt{x-1}, & x \geq 1 \\ x-1, & x < 1 \end{cases}$, find $f(1) + f(10) + f(-10)$.

2

(b) If $f(x) = x^2 + x - 3$ and $h(x) = |x|$. Given that $x < 0$ write a simplified expression for $f(h(x))$ without any use of absolute values.

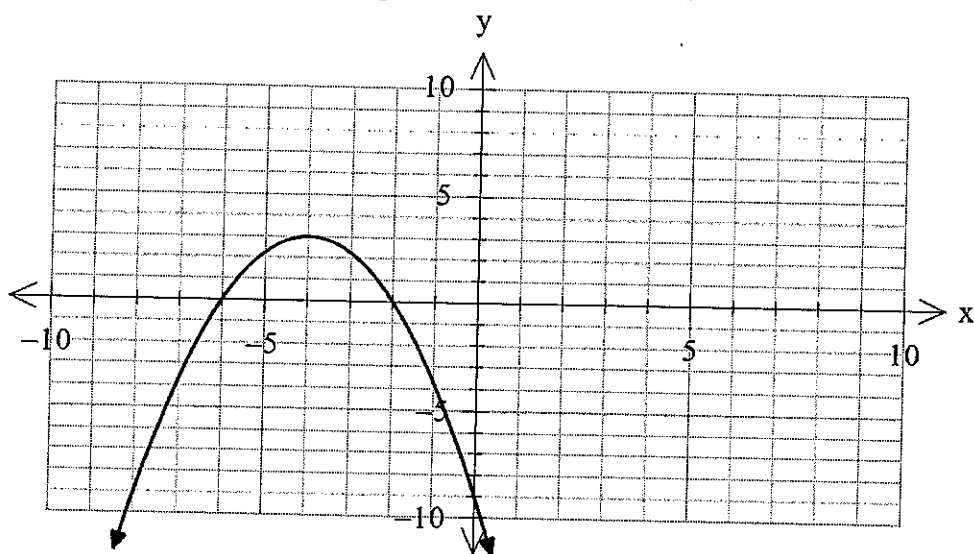
2

(c) For what value of m , the roots of $x^2 + 2x + m - 1 = 0$, are real?

2

(d) Find, in factored form, the equation of the parabola sketched below.

2



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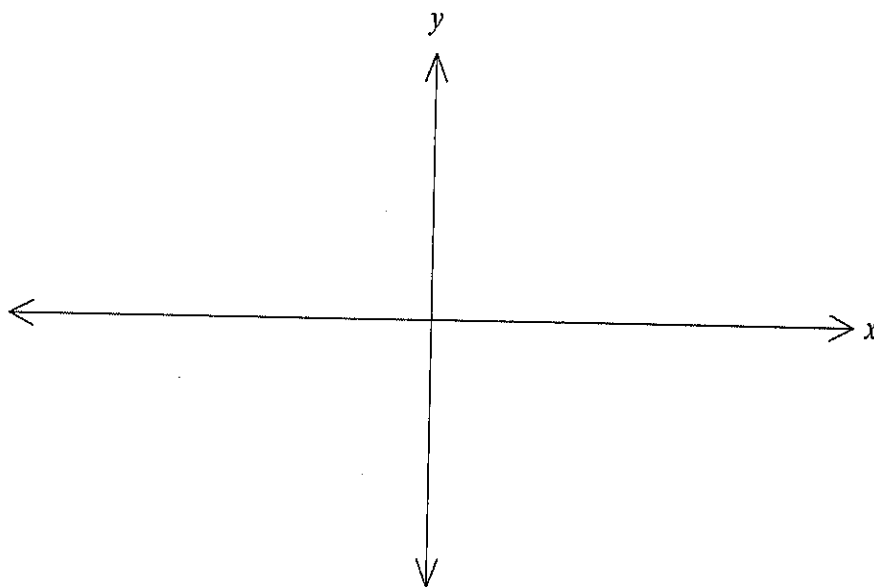
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(e) Sketch the graph of $y = x(1-x)(x+2)^2$.

3



f) Container A holds 3 red ball and 1 green ball. Container B holds 3 red balls and 2 green balls. A container is selected at random and from it two balls are selected without replacement.

i) Draw a probability tree diagram to show the sample space of this three-stage process.

2

ii) What is the probability that both balls drawn are green?

1

iii) What is the probability that both balls drawn are different colours?

1

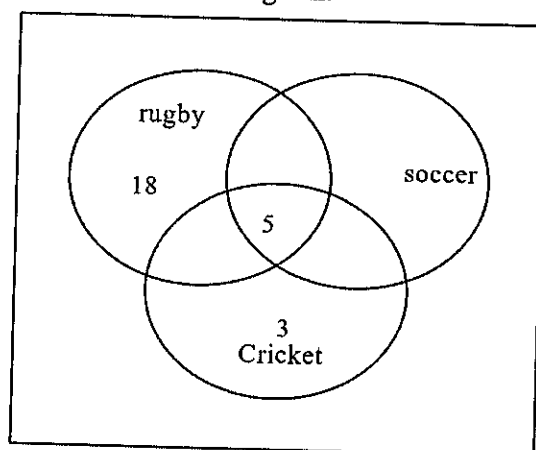
End of Question 12

Question 13 (12 Marks)

a) A group of 60 students was invited to try out for three sports: rugby, soccer and cricket.

Rugby	32
Soccer	29
Cricket	15
Rugby and soccer	11
Soccer and cricket	9
Rugby and cricket	8
All three sports	5

i) Use the above information to complete the Venn diagram.



ii) Find the probability that a student chosen at random tried out for at least two sports.

1

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iii) Find the probability that a student tried out for rugby, given that they have tried out for cricket.

1

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iv) Two students are chosen at random, one after the other, what is the probability that the first student tried out for all three sports and the second student tried out for at least rugby and soccer?

1

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1

- b) Given that for two events A and B, determine whether the events are independent.
 $P(A) = 0.3$, $P(B) = 0.7$ and $P(A \cap B) = 0.21$.

2

- c) Find the domain of $f(x) = \frac{\sqrt{5-x}}{x+1}$.

2

- d) Classify the function, $f(x) = \frac{x^3}{x+1}$ as even, odd or neither. Justify your answer.

2

- e) If $f(2x) = \frac{x}{3x-1}$, find $f(x)$.

End of question 13

Question 14 (19 Marks)

a) ABC Ltd manufactures aluminium rods. The price of a rod is p dollars and the quantity sold in a year is q rods. The company uses the equation $pq = k$ to model their sales (in dollars) where k is a constant.

i) Last year the price of a rod was \$6 and the quantity sold was 40000. Find the value of k .

1

(ii) If the company raise the price to \$8, how many rods can the company expect to sell?

1

(iii) Sketch the graph of the sales model with p on the horizontal axis.

2

(iv) Under this model, what will happen to the sales if the price of each rod is doubled?

1

2

b) Let $f(x) = \frac{x}{x^2 - 1}$. Express $f(x) + f\left(\frac{1}{x}\right)$ in its simplest form.

c) The circle $x^2 - 6x + y^2 + 4y - 3 = 0$ is reflected in the y -axis.

1

i) What is the equation of the reflected circle?

2

ii) State the radius and the coordinates of the centre of the reflected circle.

d) (i) Show that $\frac{1}{\sqrt{n} + \sqrt{n+1}}$ is equivalent to $\sqrt{n+1} - \sqrt{n}$.

2

(ii) Hence, simplify

2

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{99}+\sqrt{100}}.$$

e) A burger truck normally sells 60 burgers a day for \$10 per burger. The owner is advised that they should increase the price of the burger. However, for every 50 cents extra that the burger costs, the number of daily sales will drop by 2. The owner decides to increase the price in 50-cent increments. Let n be the number of 50-cent price increases.

i) Let S be the total daily sales in dollars. Show that S is given by $S = (30 - n)(20 + n)$.

2

ii) Hence, find the price per burger that the owner should charge in order to maximise the total daily sales. 2

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(iii) State the maximum total daily sales. 1

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End of Exam

Section II extra writing space

If you use this space clearly indicate which question you are answering.

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-17-

Section II extra writing space

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Section II extra writing space
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Baulkham Hills High School
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Marking Guideline - Mathematics

Section I (10 marks)

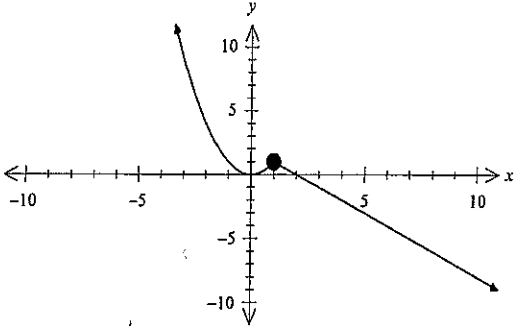
Award 1 mark to each correct answer.

Answers: 1. B 2. B 3. D 4. B 5. A 6. D 7. C 8. C 9. B 10. A

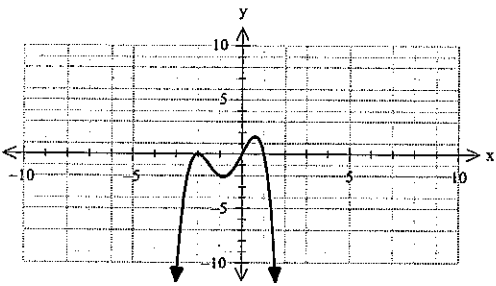
Section II (63 marks)

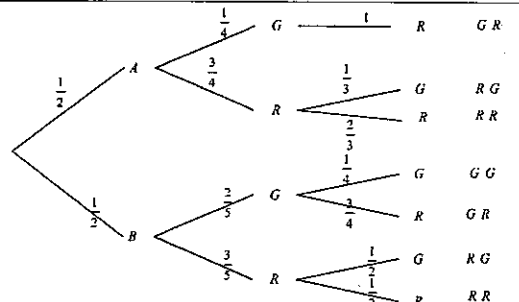
In all questions, award FULL marks for correct answers with necessary working.

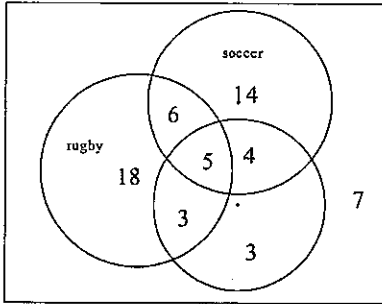
Question	Suggested solution	Mark allocation
11(a)(i)	$\frac{5(x+1)-3(x-2)}{(x-2)(x+1)}$ $= \frac{2x+11}{(x-2)(x+1)}$	• correct answer - 2 • correct denominator - 1
(ii)	$\frac{(y-3x)(2x+y)}{x(y-3x)}$ $= \frac{y+2x}{x}$	• Correct answer - 2 • correct factorisation of the numerator - 1
11(b)	Answer: $x = 5\frac{1}{2}, y = 1$	• Correct answer - 2 • incorrect answer but correct approach to the solution - 1
11(c)	$= \frac{\sqrt{2}-1}{2\sqrt{2}-1} \times \frac{2\sqrt{2}+1}{2\sqrt{2}+1}$ $= \frac{4+\sqrt{2}-2\sqrt{2}-1}{7}$ $= \frac{3-\sqrt{2}}{7}$	• Correct answer - 2 • attempt to rationalise the denominator -1

Question	Suggested solution	Mark allocation
11(d)	$= \left(a^{\frac{1}{2}} + \frac{1}{b^{\frac{1}{2}}} \right)^2$ $= a + 2 \left(\frac{a}{b} \right)^{\frac{1}{2}} + \frac{1}{b}$ $= \frac{ab + 2(ab)^{\frac{1}{2}} + 1}{b}$	• Correct answer - 2 • Expression with positive indices -1
11(e)	$2x - 9 = 13$ $x = 11$ $-(2x - 9) = 13$ $x = -2$	• Correct answer - 2 • Correct equations to solve for $x - 1$
11(f)	$f(-3) = (-3)^3 + 9(-3)^2 + k(-3) + 27$ $0 = -27 + 81 - 3k + 27$ $k = 27$	• Correct answer - 2 • Correct equation to find $k - 1$
11(g) (i)		• Correct answer - 2 • Correct x and y intercepts -1
11(g)(ii)	All real values of y , $y \in \mathbb{R}$ or all real values of y in $(-\infty, +\infty)$	1
TOTAL		17

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Marking Guideline - Mathematics

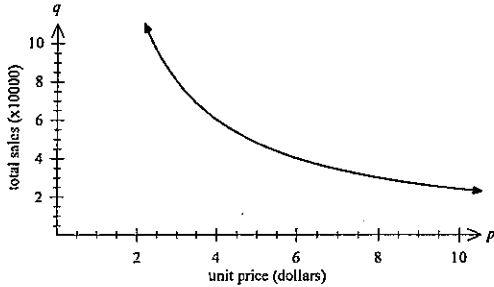
Question	Suggested solution	Mark allocation
12(a)	$f(1) = 0; f(-10) = -11; f(10) = 3$ $f(1) + f(10) + f(-10) = -8$	- Correct answer - 2 - Evaluating $f(1)$ - 1
(b)	$f(h(x)) = x ^2 + x - 3$ $= x^2 + x - 3$ when $x < 0, x = -x$. $= x^2 - x - 3$	- Correct answer - 2 - Correct substitution of $h(x)$ into $f(x)$ - 1
(c)	$\Delta = 4 - 4(m-1)$ For real roots $\Delta \geq 0$. $4 - 4(m-1) \geq 0$ $\therefore m \leq 2$	- Correct answer - 2 - Identified $\Delta \geq 0$. - 1
(d)	Possible equation is $y = a(x+6)(x+2)$ where a is a constant. Passes through $(0, -9)$. $-9 = a(0+6)(0+2)$ $a = -\frac{3}{4}$ $\therefore y = -\frac{3}{4}(x+6)(x+2)$	- Correct equation - 2 - Generic equation in factored form - 1
(e)	Answer: 	- Correct graph - 3 - Correct x-intercepts and y-intercepts - 2 - Correct shape - 1

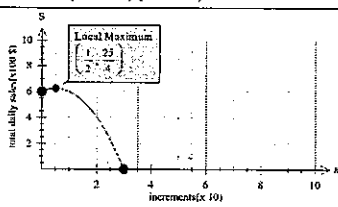
Question	Suggested solution	Mark allocation
12(f)(i)		- Correct probability tree - 2 - Correct Probability tree for stages one and two OR all 7 outcomes - 1
(f)(ii)	$P(GG) = \frac{1}{20}$	1
(f)(iii)	$P(RR \text{ from A or B}) = \frac{3}{20} + \frac{1}{4}$ $P(\text{same colour}) = P(GG) + P(RR) = \frac{1}{20} + \frac{3}{20} + \frac{1}{4} = \frac{9}{20}$ $P(\text{different colour}) = 1 - \frac{9}{20} = \frac{11}{20}$	1
TOTAL		15

Question	Suggested solution	Mark allocation
13(a)(i)		- Correct answer - 2 - Correct ALL overlapping numbers - 1
(ii)	$P(\text{tried out at least 2 sports})$ $= P(\text{tried out 2 sports}) + P(\text{tried out 3 sports})$ $= \frac{5}{60} + \frac{13}{60}$ $= \frac{3}{10}$	1

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Question	Suggested solution	Mark allocation
(iii)	$P(\text{tried out rugby} \text{tried out cricket})$ $= \frac{\frac{8}{60}}{\frac{15}{60}}$ $= \frac{8}{15}$	1
(iv)	$P(\text{1st student tried out all 3 sports}) = \frac{5}{60}$ $P(\text{2nd student tried out at least rugby and soccer}) = \frac{10}{59}$ $P(\text{choosing 1st and the 2nd student}) = \frac{5}{60} \times \frac{10}{59} = \frac{5}{354}$	1
13 (b)	$P(A \cap B) = 0.21$ $P(A) \times P(B) = 0.7 \times 0.3 = 0.21$ $P(A \cap B) = P(A) \times P(B)$ $\therefore A$ and B are independent.	1
13 (c)	For $f(x)$ to be defined $x+1 \neq 0$ and $5-x \geq 0$ Domain: All real values of x such that $x \leq 5$ and $x \neq -1$.	- Correct answer - 2 $x \leq 5$ or $x \neq -1$ - 1
13(d)	$f(-x) = \frac{(-x)^3}{(-x)+1}$ $= \frac{-x^3}{-x+1} = -\frac{x^3}{x-1}$ $\neq -f(x)$ Also $\neq f(x)$ Hence $f(x)$ is neither odd nor even.	- Correct answer - 2 - Test only -1
13(e)	$f(2x) = \frac{2x}{2\left[\frac{3}{2}(2x)-1\right]}$ Hence, $f(x) = \frac{x}{3x-2}$	- Correct answer - 2 - Attempt of manipulating $f(2x)$ towards $f(x) - 1$
TOTAL		12

Question	Suggested solution	Mark allocation
14(a)(i)	$pq = k$ $6 \times 40000 = k \therefore k = 240000$	1
(ii)	$8q = 240000$ $q = 30000$	Correct numerical expression -1
(iii)		- Scale & labelled axes - 1 - Correct shape -1
(iv)	$pq = k \therefore q = \frac{k}{p}$ when the price is doubled, $(2p)q = k$ $q = \frac{1}{2} \times \frac{k}{p}$ Hence sales is halved.	1
14(b)	$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}}{\frac{1}{x^2}-1}$ $= \frac{x}{1-x^2}$ $f(x) + f\left(\frac{1}{x}\right) = \frac{x}{x^2-1} + \frac{x}{1-x^2}$ $= 0$	- Correct answer - 2 - Found $f\left(\frac{1}{x}\right) - 1$
14(c) (i)	$(-x)^2 - 6(-x) + y^2 + 4y - 3 = 0$ $x^2 + 6x + y^2 + 4y - 3 = 0$ $(x+3)^2 + (y+2)^2 - 13 - 3 = 0$ $(x+3)^2 + (y+2)^2 = 16$	1
14(c) (ii)	Centre: $(-3, -2)$ Radius: 4 units	1 1

Question	Suggested solution	Mark allocation
14(d) (i)	$\frac{1}{\sqrt{n} + \sqrt{n+1}} = \frac{1}{\sqrt{n} + \sqrt{n+1}} \times \frac{\sqrt{n} - \sqrt{n+1}}{\sqrt{n} - \sqrt{n+1}}$ $= \frac{\sqrt{n} - \sqrt{n+1}}{n - (n+1)}$ $= -\sqrt{n} + \sqrt{n+1}$ $= \sqrt{n+1} - \sqrt{n}$	• Correct proof - 2 • Attempt to rationalise - 1
(ii)	$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{99}+\sqrt{100}}$ $= (\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) + \dots + (\sqrt{100} - \sqrt{99})$ $= -\sqrt{1} + \sqrt{100}$ $= 9$	• Correct answer - 2 • Correct use of part (i) - 1
14(e)(i)	For n increments, number of burgers sold will be $(60-2n)$. The new price of a burger will be $(10 + \frac{n}{2})$ dollars. $\therefore S = (60-2n)(10 + \frac{n}{2})$ $= (30-n)(20+n)$	• Correct answer - 2 • Found $(60-2n)$ or $(10 + \frac{n}{2})$ - 1
14(e)(ii)	 Price of a burger to maximise sales = $10 + \frac{1}{2} \times 10 \times 0.5$ $= \$12.50$	• Correct answer - 2 • Found $n=5$ - 1
14(e)(iii)	when $n = 5$ $s = (30-5)(20+5)$ $= 625$ \$625	1
TOTAL		19