BAULKHAM HILLS HIGH SCHOOL
YEAR 12 ASSESSMENT TASK 1 2022
MATHEMATICS ADVANCED

NESA#: ____________________________

Teacher: ___________________________

Mathematics Advanced

General Instructions	• Reading time- 5 minutes • Working time- 1 hour • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • In Questions 6-15, show relevant mathematical reasoning and/or calculations
Total marks: 41	**Section I - 5 marks** (pages 4-5) • Attempt Questions 1-5 • Allow about 8 minutes for this section **Section II - 36 marks** (pages 6-14) • Attempt Questions 6-15 • Allow about 52 minutes for this section

Section I

5 marks
Attempt Questions 1-5
Allow about 8 minutes for this section
Use the multiple-choice answer sheet for Questions 1-5.

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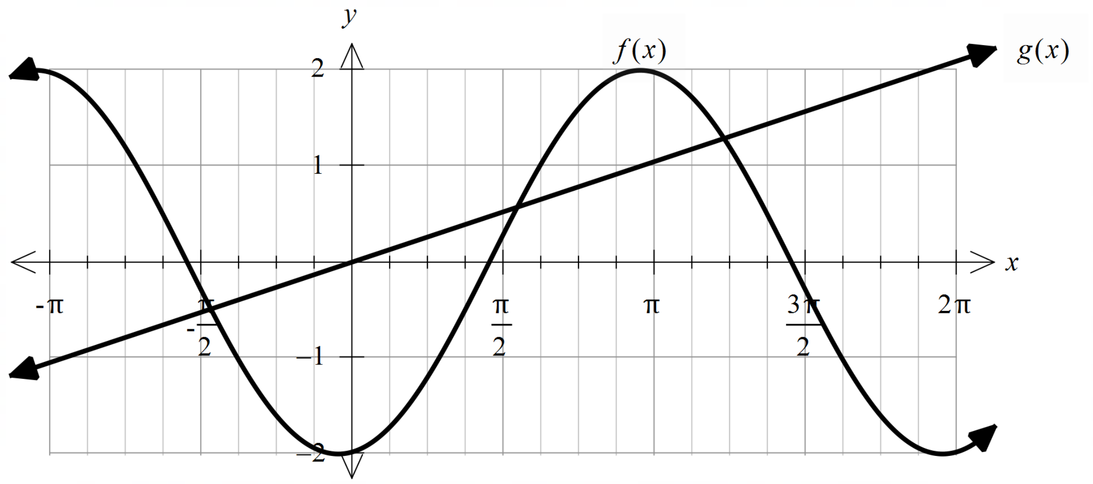
1 The graphs of $f(x)$ and $g(x)$ are given below.



How many solutions are there to the equation $f(x) = g(x)$ in the domain $-\pi < x < \pi$?

A. 1
B. 2
C. 3
D. 4

2 Which of the following is the period of the graph of $y = \tan 4x$?

A. $\frac{\pi}{4}$
B. $\frac{\pi}{2}$
C. π
D. 4π

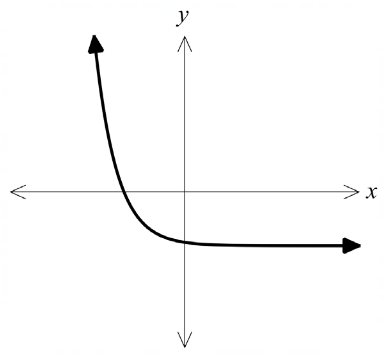
3 Consider the geometric series $1 - 3x + 9x^2 - \cdots$

For what values of x will a limiting sum exist for this infinite geometric series?

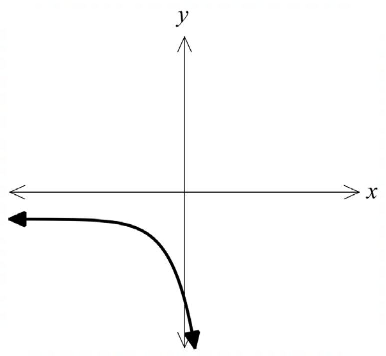
A. $-\frac{1}{3} < x < 1$
B. $-\frac{1}{3} < x < \frac{1}{3}$
C. $-1 < x < 1$
D. $-3 < x < 3$

4 The function $f(x)=2^x$ has been transformed to $g(x)$, where $g(x)=-3f(x+2)-4$. Which of the following graphs best represents $y=g(x)$?

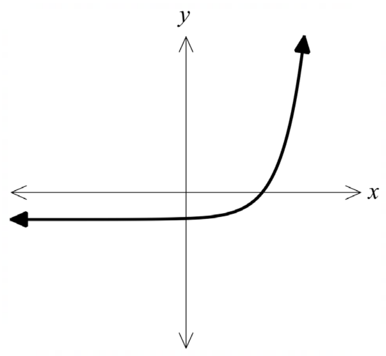
A.



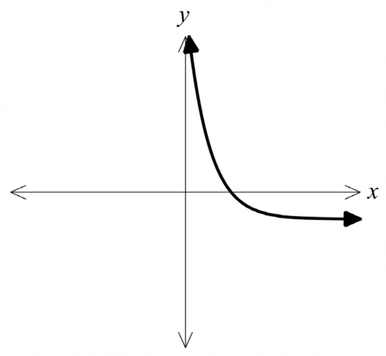
B.



C.



D.



5 The function $y=\dfrac{x-1}{x+1}$ is dilated by a factor of $\dfrac{1}{2}$ horizontally and reflected about the x-axis. What is the new equation for this function?

A. $y=\dfrac{2-x}{2+x}$

B. $y=\dfrac{2+x}{2-x}$

C. $y=\dfrac{1-2x}{1+2x}$

D. $y=\dfrac{2x-1}{2x+1}$

End of Section I

Section II

36 marks
Attempt Questions 6-15
Allow about 52 minutes for this section
Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning.

Question 6 (4 marks)

The sum of the first n terms of an arithmetic series is given by $S_n = 25n - 2n^2$.

a) Find the first term. **1**

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b) Find the common difference. **2**

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c) Show that the general term is given by $T_n = 27 - 4n$. **1**

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End of Question 6

Question 7 (2 marks)

The circle $x^2 + 4x + y^2 - 6y + 12 = 0$ is transformed by a vertical translation 3 units down and a horizontal translation 5 units right. What are the coordinates of the new centre of the transformed circle? **2**

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Question 8 (2 marks)

A factory gets permission to dispose, at the start of every day, 600kg of waste into a stream of water. The running stream removes 40% of any waste present, by the end of the day. Show that the amount of waste present in the stream at the end of the n^{th} day is $900(1-0.6^n)$. **2**

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End of Question 8

Question 9 (6 marks)

Consider the functions $f(x) = x^2 - x - 2$ and $g(x) = x + 1$.

a) Find the points of intersection of $y = f(x)$ and $y = g(x)$. **2**

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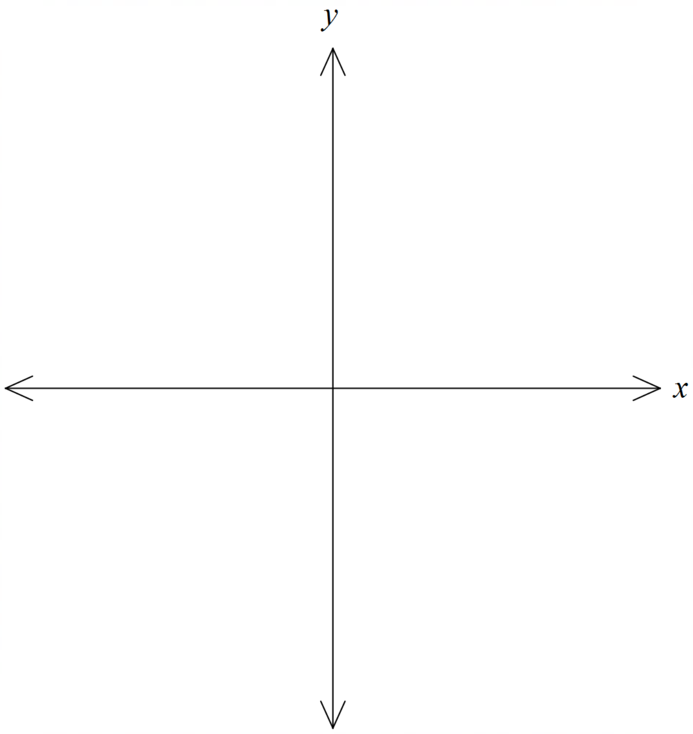
b) Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same set of axes, showing all important features, including the points of intersection. **3**



c) Hence or otherwise, solve $g(x) < f(x)$. **1**

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End of Question 9

Question 10 (3 marks)

A square chess board contains alternating black and white squares with eight squares along each side. Andrew places one grain of rice on the first square, two grains of rice on the second square, four grains on the third, eight on the fourth, and so on, until every square is covered.

a) Show that the number of grains of rice in each subsequence square form a geometric sequence. **1**

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b) Find the number of grains of rice he places on the last square. Leave your answer in index form. **1**

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c) How many grains of rice does he need altogether? Leave your answer in simplified index form. **1**

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End of Question 10

Question 11 (7 marks)

Consider the function $f(x) = -\dfrac{1}{x^2 - 1}$.

a) State the domain and range of this function. **2**

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b) Show that $f(x)$ is an even function. **1**

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c) Find the equations of the vertical and horizontal asymptotes. **2**

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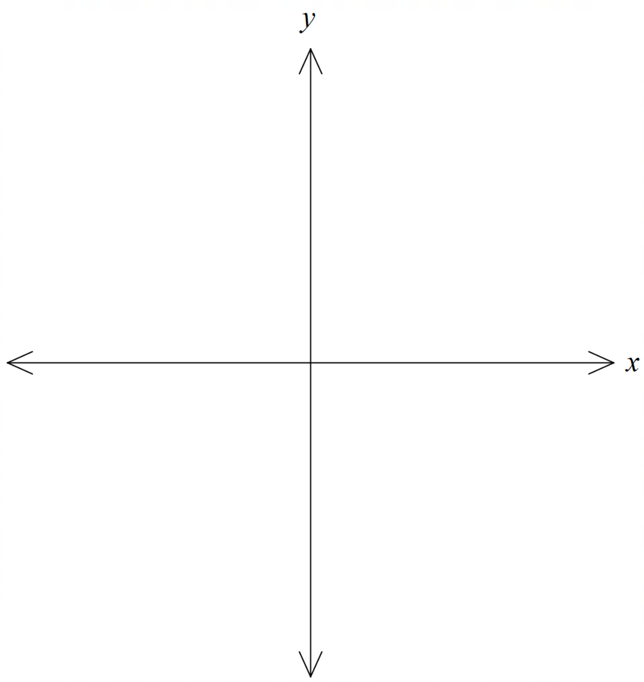
d) Using the above information, sketch the function $f(x) = -\dfrac{1}{x^2 - 1}$. Show all of the above features as well as any intercepts with the axes. **2**



End of Question 11

Question 12 (3 marks) **3**

The third and fifth terms of a geometric progression are 6 and $\frac{3}{2}$ respectively. Find the common ratio and first term for this progression.

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End of Question 12

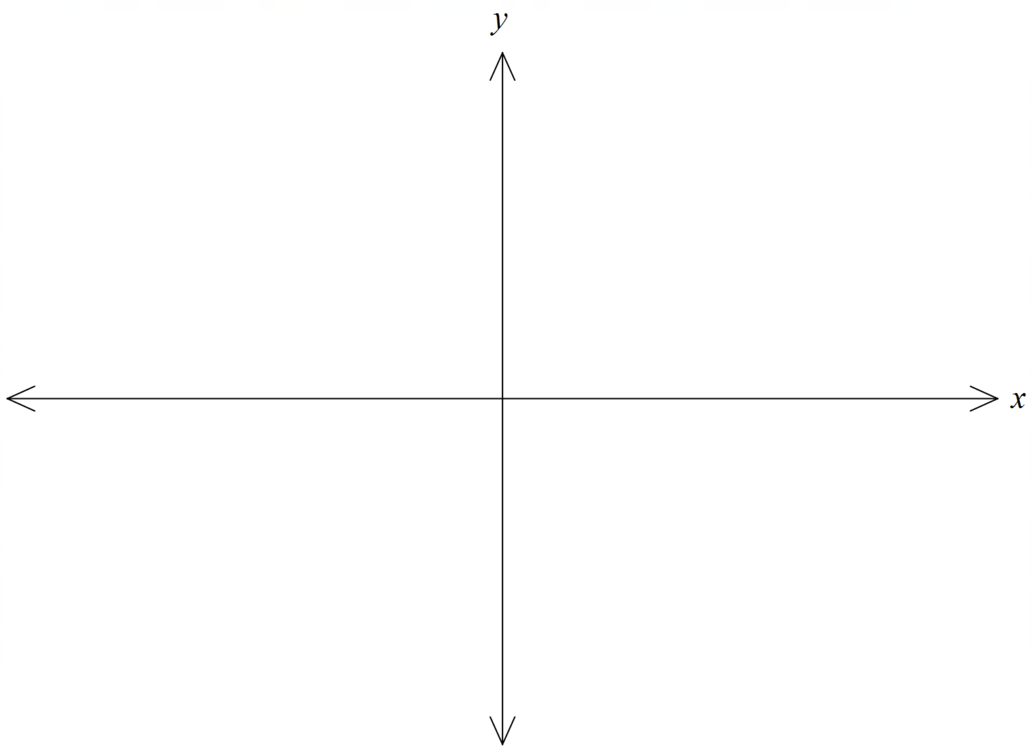
Question 13 (3 marks)

If $4,\ x,\ y$ form a geometric sequence and $x,\ y, 12$ form an arithmetic sequence, find the values of x and y. **3**

End of Question 13

Question 14 (3 marks)

On the axes provided sketch the function $f(x) = \sin(-2x) - 1$ in the domain $0 \le x \le \pi$, showing any important features. **3**



End of Question 14

Question 15 (3 marks)

A and B are two arithmetic series, where all the terms are positive.

In series A, the first term is a and common difference is d. In series B, the first term is M and the common difference is D.

Let $P =$ the sum of the first n terms of A and let $Q =$ the sum of the first n terms of B.

a) Show $\dfrac{P}{Q} = \dfrac{a + \left(\dfrac{n-1}{2}\right)d}{M + \left(\dfrac{n-1}{2}\right)D}$. **1**

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b) Express $\dfrac{10^{th} \text{ term of A}}{10^{th} \text{ term of B}}$ in terms of a, d, M and D. **1**

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c) Given that $\dfrac{P}{Q} = \dfrac{5n+4}{9n+6}$, evaluate $\dfrac{10^{th} \text{ term of A}}{10^{th} \text{ term of B}}$. **1**

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End of Exam

Baulkham Hills High School
Task 1 December Examination 2022

Marking Guideline- Yr 12 Mathematics Advanced

Section I (5 marks)

Award 1 marks to each correct answer.

Answers: 1.B 2.A 3.B 4. B 5. C

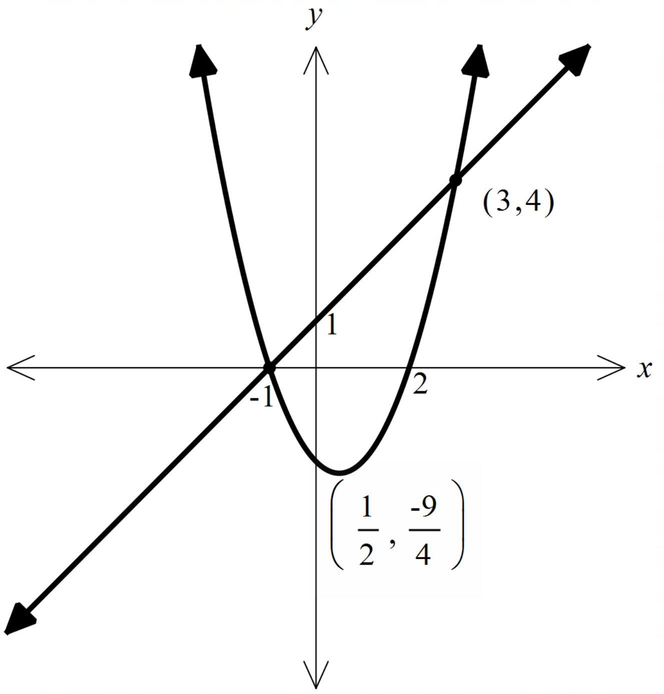
Question	Suggested solutions	Answer
1	There are 2 points of intersections within the restricted domain.	B
2	$\tan nx$ has period $\frac{\pi}{n}$. $\therefore$ period is $\frac{\pi}{4}$	A
3	$r = -3x$ $-1 < r < 1$ $-1 < -3x < 1$ $\frac{1}{3} > x > -\frac{1}{3}$ $\therefore -\frac{1}{3} < x < \frac{1}{3}$	B
4	The function has been reflected along the x-axis. $\therefore$option B	B
5	$y = \frac{x-1}{x+1} \Rightarrow y = \frac{2x-1}{2x+1} \Rightarrow -y = \frac{2x-1}{2x+1}$ $\therefore y = \frac{1-2x}{1+2x}$	C

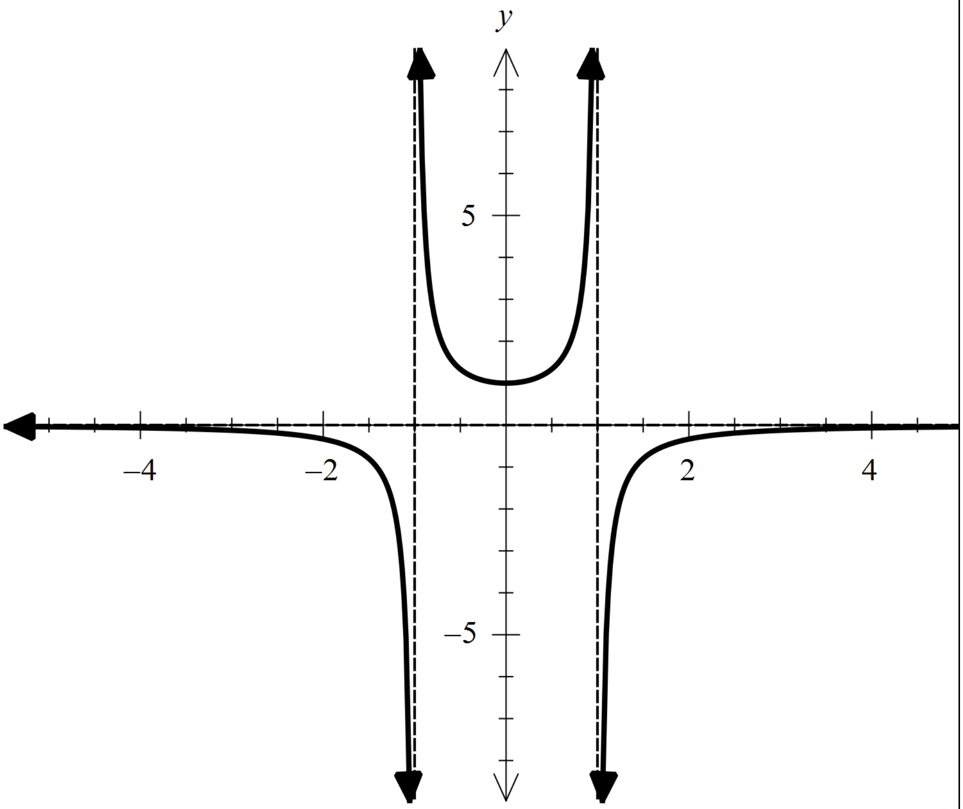
Section II (36 marks)

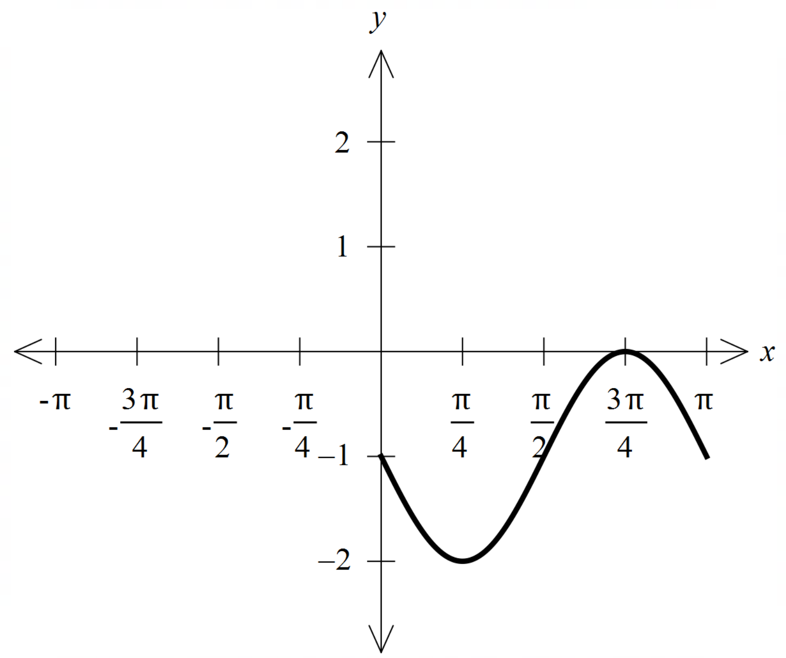
In all questions, award full marks for correct answers with necessary working.

Use the suggested solutions in conjunction with the marking criteria.

Question	Suggested Solutions	Marking criteria
6a	$n=1,$ $25(1)-2(1)^2=23$ $\therefore 1^{st}$ term is 23	1- Correct solution
6b	$T_1=23$ $n=2, S_2=42$ $\therefore T_2=19$ $n=3, S_3=57$ $\therefore T_3=15$ $23,19,15,\ldots$ $d=-4$	2- Correct solution 1- finding correctly the 2nd term
6c	$T_n=a+(n-1)d$ $T_n=23+(n-1)\times -4$ $T_n=23-4n+4$ $T_n=27-4n$	1- Correctly substitutes and simplifies
7	$x^2+4x+\left(\frac{4}{2}\right)^2-\left(\frac{4}{2}\right)^2+y^2-6y+\left(\frac{-6}{2}\right)^2-\left(\frac{-6}{2}\right)^2+12=0$ $(x+2)^2+(y-3)^2=1$ $(x+2-5)^2+(y-3+3)^2=1$ $(x-3)^2+y^2=1$ New centre $=(3,0)$	2- correct solution 1- correctly completes the square to obtain $(x+2)^2+(y-3)^2=1$ or states the original centre (-2,3)
8	Day 1: 360 Day 2: $360+0.6\times$ Day $1=360+360\times 0.6$ Day 3: $360+0.6\times$ Day $2=360+360\times 0.6+360\times 0.6^2$ Day n: $360+360\times 0.6+360\times 0.6^2+\ldots+360\times 0.6^{n-1}$ $=360(1+0.6+0.6^2+0.6^3+\ldots+0.6^{n-1})$ $=360\left(\frac{1\times(1-.06^n)}{1-0.6}\right)$ $=900(1-0.6^n)$	2- correctly shows a GP and uses the sum of a gp formula 1- identifies the gp
9a	$x^2-x-2=x+1$ $x^2-2x-3=0$ $(x-3)(x+1)=0$ $x=3, x=-1$ $\therefore$ pts of intersection are $(3,4)$ and $(-1,0)$	2- correct solution 1- attempts to use simultaneous equations

9b		3- correct shapes, intercepts, vertex and point of intersection 2- correct line and parabola with 2 out of the 4 points needed on the diagram (Shape, intercepts, vertex and points of intersection) 1- correct line or parabola
9c	When the line is below the parabola. $x<-1$ or $x>3$	1- correct solution
10a	$1, 2, 2^2, 2^3, \ldots$ $a=1,$ ratio: $\frac{2}{1}=2$ $\frac{2^2}{2}=2$ $\therefore$ the series is a geometric sequence	1- correct shows a gp
10b	$T_n = ar^{n-1}$ $T_{64} = 1\times 2^{63}$ $T_{64} = 2^{63}$	1- correct solution
10c	$S_n = \frac{a(r^n-1)}{r-1}$ $S_{64} = \frac{1(2^{64}-1)}{2-1}$ $S_{64} = 2^{64}-1$	1- correct solution
11a	Domain: all real x, $x \neq \pm 1$ Range: $y<0$ or $y \geq 1$	2- correct solution 1- correctly finds domain or range correctly
11b	$f(-x) = -\frac{1}{(-x)^2-1}$ $= \frac{-1}{x^2-1}$ $= f(x)$ $\therefore f(x)$ is even	1- correct solution

11c	Vertical asymptotes: $x = \pm 1$ Horizontal asymptotes : $y = 0$	2- correct solution 1- correctly finds either both vertical asymptotes or horizontal asymptotes
11d		2- correct shape with all information provided 1- correct shape or applies all the information from part a-c
12	$T_n = ar^{n-1}$ $\quad T_n = ar^{n-1}$ $6 = ar^2 \quad \frac{3}{2} = ar^4$ $a = \frac{6}{r^2} \quad a = \frac{3}{2r^4}$ $\frac{6}{r^2} = \frac{3}{2r^4}$ $12r^4 - 3r^2 = 0$ $3r^2(4r^2 - 1) = 0$ $r = 0, \pm\frac{1}{2}$ $\therefore r = \pm\frac{1}{2}$ $\therefore$ if $r = \pm\frac{1}{2}, a = 24$	3- Correct solution 2- correctly finds $r = \pm\frac{1}{2}$ 1- correctly forms a pair of simultaneous equations

13	$4,\ x,\ y \Rightarrow a = 4,\ r: \frac{x}{4} = \frac{y}{x}$ $x^2 = 4y$ $y = \frac{x^2}{4}$ (1) $x,\ y,\ 12 \Rightarrow a = x,\ d: y - x = 12 - y$ $2y - x = 12$ (2) Sub (1) into (2): $2\left(\frac{x^2}{4}\right) - x = 12$ $\frac{x^2}{2} - x = 12$ $x^2 - 2x - 24 = 0$ $(x-6)(x+4) = 0$ $\therefore x = 6$ or $x = -4$ When $x = 6,\ y = 9$ When $x = -4,\ y = 4$	3- correct solution 2- correctly forms a quadratic to solve for x 1- correctly forms a pair of equations to solve simultaneously
14		3- correct graph with vertex and endpoints labelled 2- attempts to translate down 1 and compress horizontally 1- attempts to translate down 1 or compress horizontally Note: must label x intercept.

15a	For the first AP **A** : Sum of n terms P $= \frac{n}{2}\left[2a+(n-1)d\right]$ and nth term a$_n$ $= a+(n-1)d$ For the second AP **B**: Sum of n terms Q $= \frac{n}{2}\left[2M+(n-1)D\right]$ and nth term M$_n$ $= M+(n-1)D$ It is given that $\frac{\text{Sum of n terms of first AP}}{\text{Sum of n terms of second AP}}$ $= \dfrac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2M+(n-1)D\right]}$ $= \dfrac{[2a+(n-1)d]}{[2M+(n-1)D]}$ $= \dfrac{a+\left(\frac{n-1}{2}\right)d}{M+\left(\frac{n-1}{2}\right)D}$	1- correctly shows the sum of both AP's and simplifies to obtain the ratio needed
15b	The ratio of the 10th term of A and B: $\frac{10^{th}\text{ term of A}}{10^{th}\text{ term of B}} = \frac{a_{10}\text{ of first AP}}{M_{10}\text{ of second AP}}$ $= \frac{a+(10-1)d}{M+(10-1)D}$ $= \frac{a+9d}{M+9D}$	1- correct solution
15c	Hence, $\frac{n-1}{2} = 9 \Rightarrow n = 19$ Substituting $n = 19$ in (1) $\frac{a+9d}{M+9D} = \frac{5(19)+4}{9(19)+6}$ $= \frac{33}{59}$	1-correct solution