Solutions

BAULKHAM HILLS HIGH SCHOOL

2022 Year 12 Assessment Task 2

Mathematics Advanced

General Instructions	• Reading time – 10 minutes • Working time – 2 hours • Write using black pen • NESA approved calculators may be used • A reference sheet has been provided at the end of this paper • In Questions 9 – 26, show relevant mathematical reasoning and/or calculations
Total marks: 84	**Section I – 8 marks** (pages 4 – 6) • Attempt Questions 1 – 8 • Allow about 12 minutes for this section **Section II – 76 marks** (pages 7 – 25) • Attempt Questions 9 – 26 • Allow about 1 hours and 48 minutes for this section

Section I

8 marks
Attempt Questions 1 – 8
Allow about 12 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 8.

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1 Given $e^{3x} = 15$, which of the following is the correct expression for x?

A. $x = \dfrac{\log_{10} 15}{3}$

B. $x = \dfrac{\log_e 3}{15}$

C. $x = \log_e 5$

D. $x = \dfrac{\log_e 15}{3}$

$3x = \log_e 15$
$x = \dfrac{\log_e 15}{3}$

2 What is the value of $\int_1^4 \frac{dx}{x}$?

A. $\dfrac{15}{16}$

B. $\log_{10} 4$

C. $e^4 - e$

D. $2\log_e 2$

$= \left[\log_e |x|\right]_1^4$
$= \log_e 4 - \log_e 1$
$= 2\log_e 2$

3 Given $f(x) = \cos(e^x)$, which of the following is the correct expression for $f'(x)$?

$f'(x) = -\sin(e^x) \times e^x$

A. $\sin(e^x)$

B. $-\sin(e^x)$

C. $e^x \sin(e^x)$

D. $-e^x \sin(e^x)$

4 Which of the following is the derivative of $\dfrac{\sin x}{\cos x}$ with respect to x?

A. 1

B. $\tan x$

C. $\dfrac{1}{\cos^2 x}$

D. $\cos^2 x$

$$y' = \frac{\cos x \times \cos x - (-\sin x)\sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

5 Given $y = \log_{10}(x^2 z^3)$, which of the following expressions is correct?

A. $y = 6\log_{10}(xz)$

B. $y = 5\log_{10}(xz)$

C. $y = 2\log_{10} x + 3\log_{10} z$

D. $y = 6(\log_{10} x + \log_{10} z)$

$$\log_{10}(x^2 z^3) = \log_{10} x^2 + \log_{10} z^3$$

$$= 2\log_{10} x + 3\log_{10} z$$

6 Which of the following is the primitive of $\sqrt{x}\left(x^{\frac{7}{2}} + \sqrt{2x}\right)$?

A. $\dfrac{x^5}{5} + \dfrac{\sqrt{2}x^2}{2} + c$

B. $\dfrac{x^4}{4} + \sqrt{2}x + c$

C. $\dfrac{x^5}{5} + \dfrac{2\sqrt{2}x^{\frac{5}{2}}}{5} + c$

D. $x^5 + \sqrt{2}x^2 + c$

$$\int \sqrt{x}\left(x^{\frac{7}{2}} + \sqrt{2}\, x^{\frac{1}{2}}\right) dx$$

$$= \int x^{\frac{1}{2}}\left(x^{\frac{7}{2}} + \sqrt{2}\, x^{\frac{1}{2}}\right) dx$$

$$= \int (x^4 + \sqrt{2}\, x)\, dx$$

$$= \frac{x^5}{5} + \frac{\sqrt{2}}{2} x^2 + C$$

7 Given $y=5\times2^{x-1}$, which of the following expression is correct?

A. $x=\log_2\left(\frac{y}{5}\right)+1$

$2^{x-1}=\frac{y}{5}$

B. $y=10^{x-1}$

$x-1=\log_2\left(\frac{y}{5}\right)$

C. $\log_{10}y=x-1$

$x=\log_2\left(\frac{y}{5}\right)+1$

D. $x=5\times2^{y-1}$

8 Which of the following definite integrals is used to evaluate the area of the region bounded by the parabolas $y=x^2+1$ and $y=3-x^2$?

$x^2+1=3-x^2$

$2x^2=2$

$x=\pm1$

A. $\int_{-1}^{1}\left(x^2-1\right)dx$

B. $\int_{-1}^{1}\left(2-2x^2\right)dx$

$\int_{-1}^{1}(3-x^2)-(x^2+1)\,dx$

C. $\int_{-2}^{2}\left(2x^2-2\right)dx$

$=\int_{-1}^{1}\left(2-2x^2\right)dx$

D. $\int_{0}^{2}\left(2-2x^2\right)dx$

End of Multiple Choice Section

2022 Year 12 Assessment Task 2

Mathematics Advanced

Section II Answer Booklet

76 marks
Attempt Questions 9 – 26
Allow about 1 hour and 48 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 9 (6 marks)

Differentiate each of the following with respect to x:

a) $x\log_e x$ 	$y = x\log_e x$ **2**

$y' = x'\log_e x + x(\log_e x)'$

$= 1\times\log_e x + x\times\frac{1}{x}$

$= \log_e x + 1$

(2) correct solution

(1) Attempt to use correct product rule.

b) $\dfrac{\cos x}{x-2}$ 	$y = \dfrac{\cos x}{x-2}$ **2**

$y' = \dfrac{(\cos x)'(x-2) - (x-2)'\cos x}{(x-2)^2}$

$= \dfrac{-\sin x(x-2) - 1\times\cos x}{(x-2)^2}$

$= \dfrac{-x\sin x + 2\sin x - \cos x}{(x-2)^2}$

(2) correct soln

(1) Attempt to use correct quotient rule

c) $\left(e^{2x}+1\right)^{-4}$ 	$y = (e^{2x}+1)^{-4}$ **2**

$y' = -4(e^{2x}+1)^{-4-1}(e^{2x}+1)'$

$= -4(e^{2x}+1)^{-5}\times e^{2x}\times 2$

$= \dfrac{-8e^{2x}}{(e^{2x}+1)^5}$ or $8e^{2x}(e^{2x}+1)^{-5}$

(2) Correct solution

(1) Attempt to use correct chain rule

Question 10 (3 marks)

Find the equation of the tangent to the curve $y = \sin 2x$ at the point where $x = \frac{\pi}{12}$.

$y = \sin 2x$ $\quad f\left(\frac{\pi}{12}\right) = \sin\left(2\times\frac{\pi}{12}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

$y' = \cos 2x \times 2$

$f'\left(\frac{\pi}{12}\right) = 2\cos\left(2\times\frac{\pi}{12}\right)$

Equation of the tangent to the curve $\quad = 2\cos\frac{\pi}{6} = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$

$y - y_1 = m(x - x_1)$ $\quad$ (3) correct solution

$y - \frac{1}{2} = \sqrt{3}\left(x - \frac{\pi}{12}\right)$ $\quad$ (2) Evaluate either $f\left(\frac{\pi}{12}\right)$ or $f'\left(\frac{\pi}{12}\right)$ correctly and the equation of the tangent

$y = \sqrt{3}x - \frac{\pi\sqrt{3}}{12} + \frac{1}{2}$ $\quad$ (1) obtain correct y'

Question 11 (2 marks)

For what values of x is the function $y = \frac{1}{x^2+1}$ decreasing? $\quad$ (2) correct solution

$y' = \frac{-2x}{(x^2+1)^2}$ $\quad$ (1) obtain correct y'

$y' < 0$ when $x > 0$

y is decreasing function when $x > 0$

Note : students do not need to use y'.

Accept : if $x > 0$, as x increases, y decreases.

Question 12 (3 marks)

a) Differentiate $y = \sin^4 x$. **1**

$\frac{dy}{dx} = 4\sin^3 x \cos x$

b) Hence, find $\int_0^{\frac{\pi}{2}} 4\cos x \sin^3 x \, dx$. **2**

Hence, $\int_0^{\frac{\pi}{2}} 4\cos x \sin^3 x \, dx = \left[\sin^4 x\right]_0^{\frac{\pi}{2}}$ (from (a))

$= \sin^4 \frac{\pi}{2} - \sin^4 0$

$= 1 - 0$

$= 1$

(2) correct solution

(1) correct primitive with correct ~~upper & lower~~ bounds

(1) correct answer but not using hence

Question 13 (3 marks)

Solve $\log_e(4x-5) - \log_e 3 = 2$, giving your answer in exact form.

$\log_e(4x-5) - \log_e 3 = 2$

$\log_e \frac{(4x-5)}{3} = 2$

$\frac{4x-5}{3} = e^2$

$x = \frac{3e^2+5}{4}$

(3) Correct solution

(2) significant progress

(1) obtain correct expression $\log_e \frac{(4x-5)}{3}$

Question 14 (3 marks)

Use the trapezoidal rule with 3 function values to evaluate $\int_2^4 x\log_e x\,dx$, express your answer correct to 3 significant figures.

x	2	3	4
y	1.38629	3.29584	5.54518

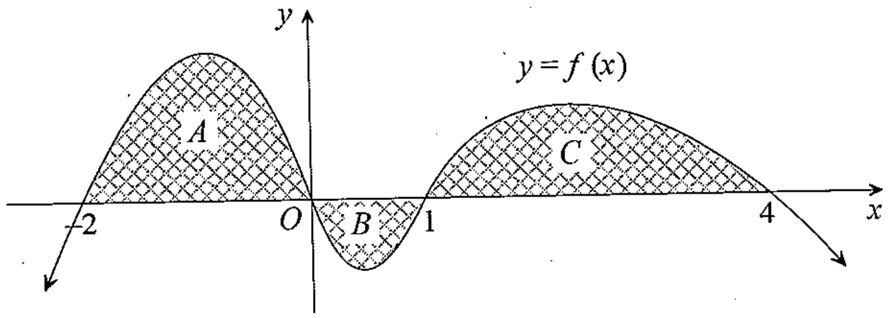
$\int_2^4 x\log_e x\,dx \approx \frac{1}{2}\times 1\times[1.38629+3.29584]+\frac{1}{2}\times 1\times[3.29584+5.54518]$

$= 6.76$

(3) correct solution
(2) Attempt to use two trapezoidal applications or Trapezoidal formula correctly.
(1) one trapezoidal application correctly
(−1) for rounding in the middle of the working out.

Question 15 (1 mark)

The graph of the function $f(x)$ is shown in the diagram below. The shaded areas are bounded by $y=f(x)$ and the x-axis.



Given the shaded area A is 10 square units, the shaded area B is 4 square units and the shaded area C is 8 square units, evaluate $\int_{-2}^{4} f(x)\,dx$.

$\int_{-2}^{4} f(x)\,dx = 10-4+8$

$= 14$ square units

Question 16 (4 marks)

a) Differentiate $\log_e(\sin x)$ with respect to x. **1**

$y = \log_e(\sin x)$

$y' = \frac{1}{\sin x} \times \cos x$

$= \cot x$

b) Hence, or otherwise, evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x \, dx$, giving your answer in exact form. **3**

Hence $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x \, dx = \left[\log_e|\sin x|\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \log_e\left|\sin \frac{\pi}{2}\right| - \log_e\left|\sin \frac{\pi}{4}\right|$

$= \log_e 1 - \log_e \frac{1}{\sqrt{2}}$

$= 0 - \log_e 2^{-\frac{1}{2}}$

$= \frac{1}{2}\log_e 2$

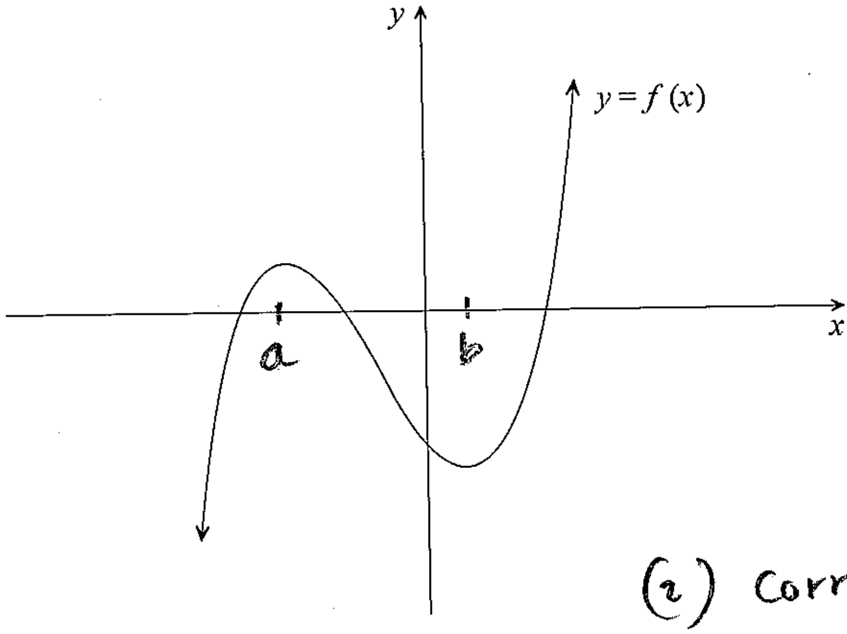
(3) Correct solution

(2) correct primitive and correct substitution.

(1) Correct primitive with lower & upper bounds

Question 17 (2 marks)

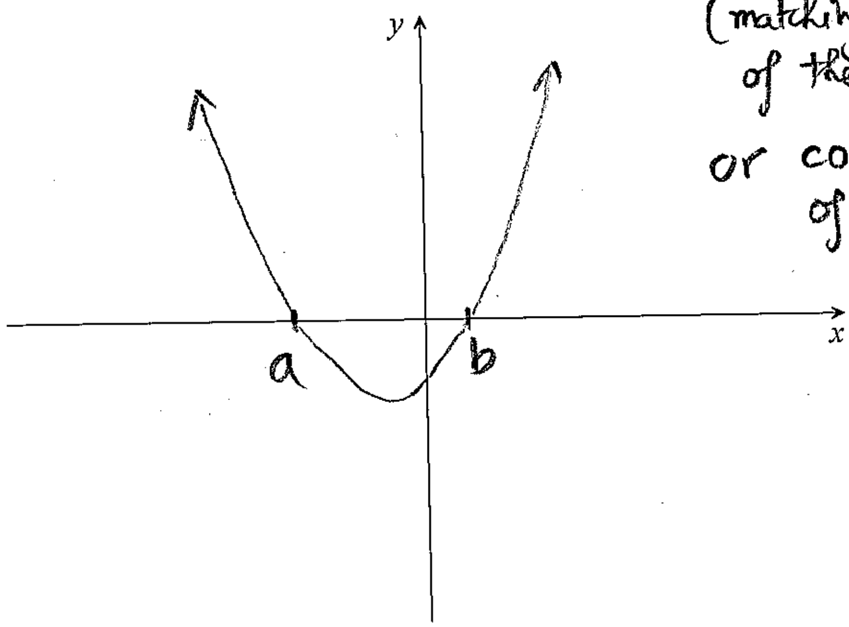
The graph of a function $y = f(x)$ is shown below.



On the set of axes given below, draw the graph of $y = f'(x)$.

(2) correct solution

(1) correct x-intercepts (matching turning points of the curve $y = f(x)$)
or correct shape of y'



Question 18 (3 marks)

Find the equation of the curve $y = f(x)$ given $\dfrac{d^2y}{dx^2} = 6x - 4$ and $y = f(x)$ has a stationary point at $(1, 2)$.

$y'' = 6x - 4$

$y' = \int (6x - 4)\,dx$

$= 3x^2 - 4x + C_1$

$f'(1) = 0$

$f'(1) = 3\times1^2 - 4\times1 + C_1 = 0$

$= 3 - 4 + C_1 = 0$

$\therefore C_1 = 1$

$\therefore f'(x) = 3x^2 - 4x + 1$

$f(x) = \int (3x^2 - 4x + 1)\,dx$

$= x^3 - 2x^2 + x + C_2$

$f(1) = 2$

$f(1) = 1^3 - 2\times1^2 + 1 + C_2 = 2$

$1 - 2 + 1 + C_2 = 2$

$\therefore C_2 = 2$

$\therefore f(x) = x^3 - 2x^2 + x + 2$

(3) Correct solution

(2) correct expression for y' and y'' (but either incorrect C_1 or incorrect C_2)

(1) Correct expression of y' and correct C_1

Question 19 (7 marks)

a) Find $\int \frac{1+x}{x^2}\,dx$. **3**

$$\int \frac{1+x}{x^2}\,dx = \int\left(\frac{1}{x^2}+\frac{x}{x^2}\right)dx = \int\left(x^{-2}+\frac{1}{x}\right)dx$$

$$= \frac{x^{-2+1}}{-2+1} + \log_e|x| + C$$

$$= -\frac{1}{x} + \log_e|x| + C$$

(3) Correct solution

(2) correct primitive (C not included)

(1) either primitive has $-\frac{1}{x}$ or $\log_e|x|$

b) Find $\int \cos\left(\frac{x^\circ}{2}\right)dx$. **2**

$x^\circ = \frac{\pi x}{180}$ radians

$$\int \cos\left(\frac{x^\circ}{2}\right)dx = \int \cos\left(\frac{\pi x}{360}\right)dx$$

$$= \frac{360}{\pi} \times \sin\left(\frac{\pi x}{360}\right) + C$$

(2) correct solution

(1) either change x° to radians or correct primitive

c) Find $\int \frac{1}{3}e^{2x-5}\,dx$. **2**

$$\int \frac{1}{3}e^{2x-5}\,dx = \frac{1}{6}e^{2x-5} + C$$

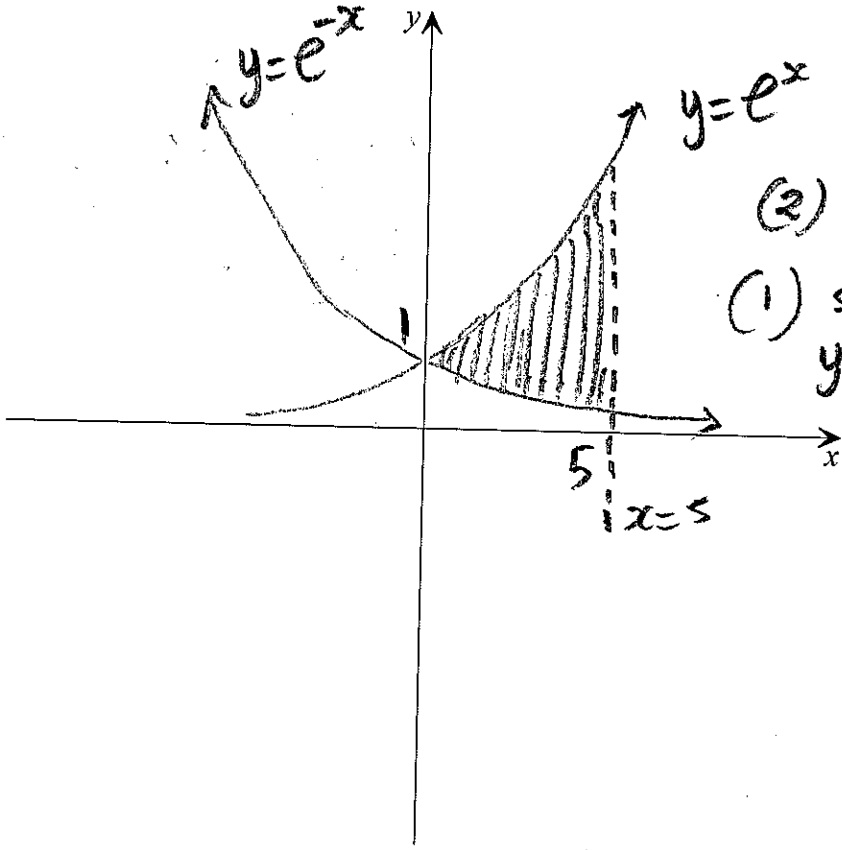
(2) Correct solution

(1) Incorrect coefficient (number) in front of e^{2x-5}.

Question 20 (4 marks)

a) On the axes given below, graph the region bounded by the following curves: **2**

$$y = e^x \ , \ y = e^{-x} \ , \ x = 5$$



(2) correct solution

(1) sketch either $y=e^x$ or $y=e^{-x}$ correctly.

b) Hence calculate the area of the region bounded by the curves $y = e^x$, $y = e^{-x}$ and $x = 5$. **2**

$$\text{Area} = \int_0^5 (e^x - e^{-x})\,dx$$

(2) Correct solution

(1) obtain correct primitive.

$$= \left[e^x + e^{-x}\right]_0^5$$

$$= e^5 + e^{-5} - (e^0 + e^{0})$$

$$= e^5 + \frac{1}{e^5} - 2$$

Question 21 (4 marks)

a) Given $f(x)=\log_e\left(x+\sqrt{x^2+1}\right)$, show that $f'(x)=\frac{1}{\sqrt{x^2+1}}$. **2**

$$f'(x) = \frac{1}{x+\sqrt{x^2+1}} \times \left(x+(x^2+1)^{\frac{1}{2}}\right)'$$

$$= \frac{1}{x+\sqrt{x^2+1}} \times \left(1+\frac{1}{2}(x^2+1)^{\frac{1}{2}-1} \times (x^2+1)'\right)$$

$$= \frac{1}{x+\sqrt{x^2+1}} \left[1+\frac{1}{2(x^2+1)^{\frac{1}{2}}} \times 2x\right]$$

$$= \frac{1}{x+\sqrt{x^2+1}} \times \left[1+\frac{x}{\sqrt{x^2+1}}\right]$$

$$= \frac{1}{x+\sqrt{x^2+1}} \times \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}$$

$$= \frac{1}{\sqrt{x^2+1}}$$

(2) correct solution

(1) correctly use product rule

b) Hence, or otherwise, evaluate $\int_0^1 \frac{dx}{\sqrt{x^2+1}}$, leaving your answer in exact form. **2**

Hence $\int_0^1 \frac{dx}{\sqrt{x^2+1}} = \left[\log_e\left|x+\sqrt{x^2+1}\right|\right]_0^1$

$$= \log_e\left(1+\sqrt{1^2+1}\right) - \log_e\left(0+\sqrt{0^2+1}\right)$$

$$= \log_e\left(1+\sqrt{2}\right) - 0$$

$$= \log_e\left(1+\sqrt{2}\right)$$

(2) correct solution

(1) obtain correct primitive

Question 22 (2 marks)

Differentiate the following with respect to x:

a) 5^{x+2} $y = 5^{x+2}$ **1**

$y' = 5^{x+2} \times \log_e 5$

b) $\log_4(3x)$ **1**

$y = \log_4(3x)$

$= \log_4 3 + \log_4 x$

$y' = 0 + \dfrac{1}{x \log_e 4}$

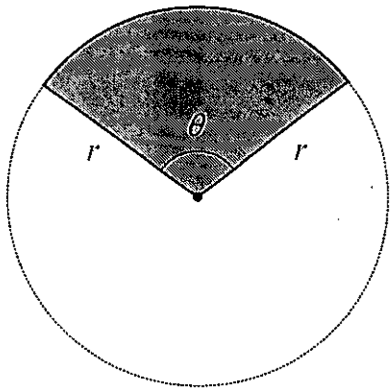
$= \dfrac{1}{x \log_e 4}$

End of Question 22

Exam continues on the next page

Question 23 (6 marks)

Arjun is fencing a paddock using M metres of fencing. The paddock is in the shape of a sector of a circle with radius r metres and an angle of θ (in radians) at the centre, as shown in the diagram below.



a) Show that the length of fencing required is given by the equation $M = r(\theta + 2)$. **1**

$M = 2r +$ length of arc

$= 2r + r\theta$

$= r(2+\theta)$

b) Show that the area of the paddock, in square metres, is given by $A = \frac{1}{2}Mr - r^2$. **2**

Area $A = \frac{1}{2}r^2\theta$

from a) $M = 2r + r\theta$

$\therefore \theta = \frac{M-2r}{r}$

Sub into (1)

$A = \frac{1}{2}r^2 \times \left(\frac{M-2r}{r}\right)$

$= \frac{1}{2}Mr - r^2$

(2) Correct solution

(1) either obtain $\theta = \frac{M-2r}{r}$ correctly or incorrect expression of θ and do substitution into $A = \frac{1}{2}r^2\theta$.

Question 23 continues on the following page

Question 23 (continued)

c) Calculate the size of the angle θ such that the area of the paddock is a maximum. **3**

$A = \frac{1}{2}Mr - r^2$

$A' = \frac{dA}{dr} = \frac{1}{2}M - 2r$

$A' = 0 \quad \frac{1}{2}M - 2r = 0$

$\therefore M = 4r$

$A'' = \frac{d^2A}{dr^2} = 0 - 2 < 0 \quad \therefore$ Maximum.

$\therefore \theta = \frac{M - 2r}{r} = \frac{4r - 2r}{r} = 2$ radians

ie $\theta = 2$ radians will give maximum area of the paddock.

(3) correct solution

(2) * obtain A' and value of $M = 4r$ but failed to use table or A'' to ascertain $M = 4r$ will give maximum area. And obtain θ.

* obtain y' and $M = 4r$ and use table or y' to ascertain correctly at $M = 4r$ will give max area but not obtain θ.

(1) obtain $\frac{dA}{dr} = 0$ correctly.

(3) Correct solution

(2) obtain two stationary points correctly but failed use the table or y'' to determine their nature.

OR obtain one correct stationary pt and check its nature

(1) obtains $x = 1, -1$

Question 24 (7 marks)

Consider the function $y = \frac{1}{3}x^3 - x + 1$.

a) Find the stationary points on this curve and determine their nature. 3

$y' = x^2 - 1$

$y' = 0 \quad x^2 - 1 = 0$

$x = \pm 1$

$y'' = 2x$

$f''(1) = 2 \times 1 = 2 > 0 \qquad f(1) = \frac{1}{3} \times 1^3 - 1 + 1 = \frac{1}{3}$

$\therefore \text{Min}\left(1, \frac{1}{3}\right)$

$f''(-1) = 2 \times -1 = -2 < 0 \qquad f(-1) = \frac{1}{3}(-1)^3 - (-1) + 1 = -\frac{1}{3} + 2 = \frac{5}{3}$

$\therefore \text{Max}\left(-1, \frac{5}{3}\right)$

b) Show that there is a point of inflection at a point where $x = 0$. 2

$y'' = 2x$

x	-1	0	1
y''	-2	0	2

$f''(x) = 0 \Leftrightarrow x = 0$

$\therefore$ Concavity changes

$\therefore$ there is a point of inflection at $x = 0$

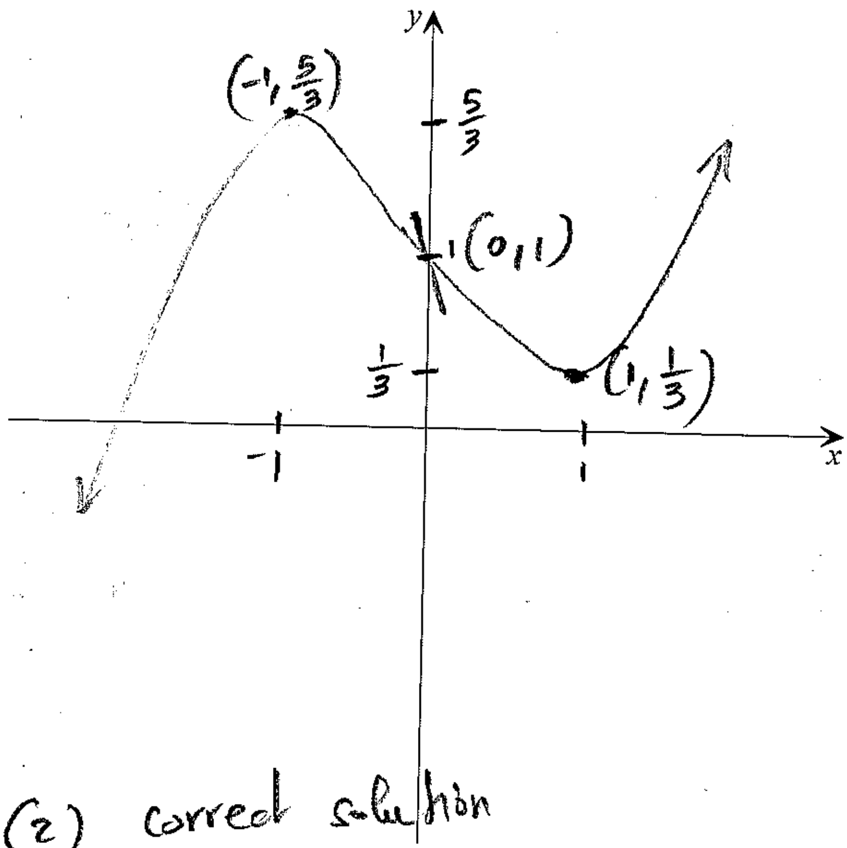
(2) obtain correct solution

(1) solve for $y'' = 0$ OR check concavity changes.

Question 24 continues on the following page

Question 24 (continued)

c) Sketch the curve $y = \frac{1}{3}x^3 - x + 1$, labelling the stationary points, the point of inflection and the y-intercept. **2**



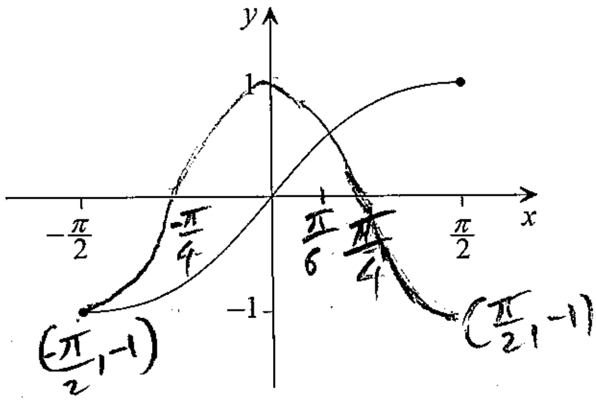
(2) correct solution

(1) either label the turning points correctly
or label point of inflection correctly
or correct shape but fail to
label the above points.

a) (2) Correct sketch.
(1) obtain correct shape
OR obtain x and y intercepts

Question 25 (7 marks)

The diagram below shows the graph of $y = \sin x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.



a) On the set of axes above, sketch the graph of $y = \cos(2x)$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. **2**

b) Show by substitution that the two graphs intersect where $x = \frac{\pi}{6}$ and $x = -\frac{\pi}{2}$. **2**

When $x = \frac{\pi}{6}$, $\sin\frac{\pi}{6} = \frac{1}{2}$, $\cos 2\times\frac{\pi}{6} = \cos\frac{\pi}{3} = \frac{1}{2}$

when $x = -\frac{\pi}{2}$, $\sin(-\frac{\pi}{2}) = -1$, $\cos(2\times(-\frac{\pi}{2})) = \cos(-\pi) = -1$

$\therefore$ Two graphs intersect at $x = \frac{\pi}{6}$ and at $x = -\frac{\pi}{2}$

(2) obtain correct solution
(1) Correctly substitutes one point and obtain $y = \frac{1}{2}$ for $x = \frac{\pi}{6}$ OR $y = -1$ for $x = \frac{\pi}{2}$

c) Calculate the exact area enclosed by $y = \sin x$ and $y = \cos(2x)$.

$$\text{Area} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} (\cos 2x - \sin x)\, dx$$

$$= \left[\frac{1}{2}\sin 2x + \cos x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{6}}$$

$$= \frac{1}{2}\sin\left(2\times\frac{\pi}{6}\right) + \cos\frac{\pi}{6} - \left[\frac{1}{2}\sin\left(2\left(-\frac{\pi}{2}\right)\right) + \cos\left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{2}\sin\frac{\pi}{3} + \cos\frac{\pi}{6} - \frac{1}{2}\sin(-\pi) - \cos\left(-\frac{\pi}{2}\right)$$

$$= \frac{1}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} - 0 - 0$$

$$= \frac{3\sqrt{3}}{4}$$

(3) obtain Correct solutn
(2) correct primitive and correct substitution but incorrect answer.
(1) obtain correct integral.

Question 26 (9 marks)

Consider the function $y = e^{\sin x}$ for $0 \le x \le 2\pi$.

a) Find all the stationary points on the curve in the given domain. **2**

$y' = e^{\sin x} \times \cos x$

$y' = 0 \quad e^{\sin x} \times \cos x = 0 \quad$ Note $e^{\sin x} > 0$

$\therefore \cos x = 0 \quad x = \frac{\pi}{2}, \frac{3\pi}{2}$

$f(\frac{\pi}{2}) = e^{\sin\frac{\pi}{2}} = e^{1} = e$

$f(\frac{3\pi}{2}) = e^{\sin\frac{3\pi}{2}} = e^{-1} = \frac{1}{e}$

$\therefore$ stationary points are $(\frac{\pi}{2}, e), (\frac{3\pi}{2}, \frac{1}{e})$

(2) obtain correct solution

(1) obtain one correct stationary pt or $x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$.

b) By finding the second derivative or otherwise, determine the nature of stationary points. **3**

$y'' = (e^{\sin x})' \times \cos x + (e^{\sin x}) \times (\cos x)'$

$= e^{\sin x} \times \cos x \times \cos x + e^{\sin x} \times (-\sin x)$

$= e^{\sin x}(\cos^2 x - \sin x) \quad$ Note $e^{\sin x} > 0$

$f''(\frac{\pi}{2}) = e^{\sin\frac{\pi}{2}}(\cos^2\frac{\pi}{2} - \sin\frac{\pi}{2}) = e^{1}(0-1) = -e < 0$

$\therefore$ Max $(\frac{\pi}{2}, e)$

$f''(\frac{3\pi}{2}) = e^{\sin\frac{3\pi}{2}}(\cos^2\frac{3\pi}{2} - \sin\frac{3\pi}{2}) = e^{-1}(0-(-1)) = e^{-1} > 0$

$\therefore$ Min $(\frac{3\pi}{2}, \frac{1}{e})$

(3) correct solution

(2) correctly checks the nature for 2 stationary pts

(1) Correct second derivative

Question 26 continues on the following page

OR correctly check the nature of one stationary point

(2) obtains two correct x-values
(1) obtain $\sin x = \frac{-1 \pm \sqrt{5}}{2}$

Question 26 (continued)

c) Find the x-values of the points of inflexion correct to 1 decimal place. **2**

$y'' = e^{\sin x}(\cos^2 x - \sin x) = 0$ Note $e^{\sin x} > 0$

$\therefore \cos^2 x - \sin x = 0$

$1 - \sin^2 x - \sin x = 0$

$\sin^2 x + \sin x - 1 = 0$

$\sin x = \frac{-1 \pm \sqrt{5}}{2}$ $\therefore x \doteqdot 0.7$ rad or $x \doteqdot 2.5$ rad

x	0	0.7	$\frac{\pi}{2}$	2.5	$\frac{3\pi}{2}$
y''	1	0	$-e$	0	$\frac{1}{e}$
	$+$		$-$		$+$

$f(0.7) = e^{\sin 0.7} = 1.01$

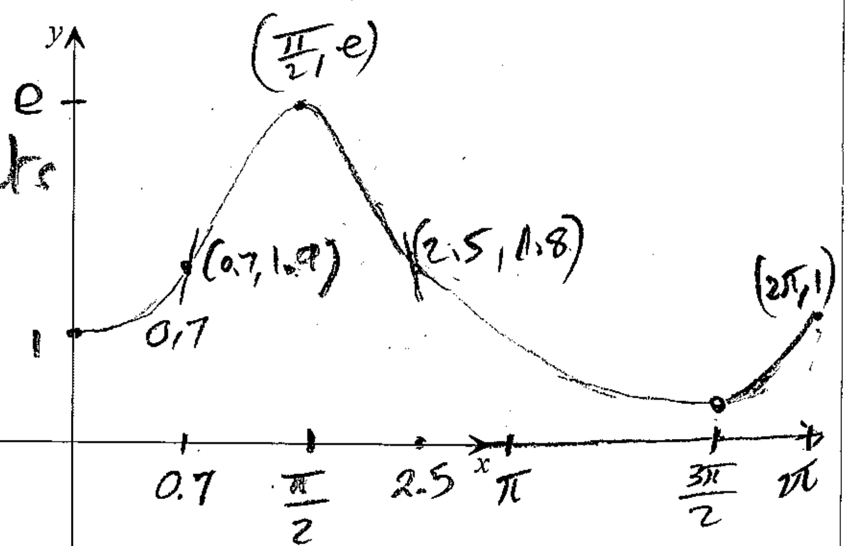
$f(2.5) = 1.04$

Concavity changes $(0.7, 1.9)$ Concavity changes $(2.5, 1.8)$

d) On the given axes, draw the graph of $y = e^{\sin x}$ showing all of the information obtained in parts a) to c). **2**

(2) Correct shape + label two inflection points + min + max points correctly + end points

(1) correct shape but fail to label all the points



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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

BAULKHAM HILLS HIGH SCHOOL

2022

YEAR 12 Assessment Task 2 EXAMINATION

Mathematics Advanced

General Instructions	• Reading time – 10 minutes • Working time – 2 hours • Write using black pen • NESA approved calculators may be used • A reference sheet is provided • In Questions 9 – 26, show relevant mathematical reasoning and/or calculations
Total marks: 90	**Section I – 8 marks** (pages 4-6) • Attempt Questions 1 – 8 • Allow about 12 minutes for this section **Section II – 76 marks** (pages 7–25) • Attempt Questions 9 – 26 • Allow about 1 hours and 48 minutes for this section

Section I

8 marks
Attempt Questions 1 – 8
Allow about 12 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 8.

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1 If $e^{3x} = 15$ then what is the correct expression of x ?

A. $x = \dfrac{\log_{10} 15}{3}$

B. $x = \dfrac{\log_e 3}{15}$

C. $x = \log_e 5$

D. $x = \dfrac{\log_e 15}{3}$

2 What is the value of $\int_1^4 \frac{dx}{x} =$

A. $\dfrac{15}{16}$

B. $\log_{10} 4$

C. $e^4 - e$

D. $2\log_e 2$

3 Given $f(x) = \cos(e^x)$, what of the following is the correct expression for $f'(x)$?

A. $\sin(e^x)$

B. $-\sin(e^x)$

C. $e^x \sin(e^x)$

D. $-e^x \sin(e^x)$

4 Which of the following is the derivative of $\frac{\sin x}{\cos x}$ with respect to x?

A. 1

B. $\tan x$

C. $\frac{1}{\cos^2 x}$

D. $\cos^2 x$

5 Given $y = \log_{10}(x^2 z^3)$, which of the following expression is correct?

A. $y = 6\log_{10}(xz)$

B. $y = 5\log_{10}(xz)$

C. $y = 2\log_{10} x + 3\log_{10} z$

D. $y = 6(\log_{10} x + \log_{10} z)$

6 Which of the following is the primitive of $\sqrt{x}\left(x^{\frac{7}{2}} + \sqrt{2x}\right)$?

A. $\frac{x^5}{5} + \frac{\sqrt{2}x^2}{2} + C$

B. $\frac{x^4}{4} + \sqrt{2}x + C$

C. $\frac{x^5}{5} + \frac{2\sqrt{2}x^{\frac{5}{2}}}{5} + C$

D. $x^5 + \sqrt{2}x^2 + C$

7 Given $y = 5 \times 2^{x-1}$, which of the following expression is correct?

A. $x = \log_2\left(\frac{y}{5}\right) + 1$

B. $y = 10^{x-1}$

C. $\log_{10} y = x - 1$

D. $x = 5 \times 2^{y-1}$

8 Which of the following definite integrals is used to evaluate the area of the region bounded by the parabolas $y = x^2 + 1$ and $y = 3 - x^2$?

A. $\int_{-1}^{1} (x^2 - 1)dx$

B. $\int_{-1}^{1} (2 - 2x^2)dx$

C. $\int_{-2}^{2} (2x^2 - 2)dx$

D. $\int_{0}^{2} (2 - 2x^2)dx$

End of Multiple Choice Section

2022 Year 12 Assessment Task 2 Examination

Mathematics Advanced

Section II Answer Booklet

80 marks
Attempt Questions 9 – 26
Allow about 1 hour and 48 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 9 (6 marks)

Differentiate each of the following with respect to x:

a) $x \log_e x$ 2

b) $\dfrac{\cos x}{x-2}$ 2

c) $(e^{2x}+1)^{-4}$ 2

Question 10 (3 marks)

Find the equation of the tangent to the curve $y = \sin 2x$ at the point $x = \frac{\pi}{12}$.

Question 11 (2 marks)

For what values of x the function $y = \frac{1}{x^2 + 1}$ decreasing?

Question 12 (3 marks)

a) Differentiate $y = \sin^4 x$ 1

b) Hence, find $\int_0^{\frac{\pi}{2}} 4\cos x \sin^3 x dx$ 2

Question 13 (3 marks)

Solve $\log_e(4x-5) - \log_e 3 = 2$, giving your answer in exact form.

Question 14 (3 marks)

Use the trapezoidal rule with 3 function values to evaluate $\int_2^4 x \log_e x \, dx$, express you answer correct to 3 significant figures.

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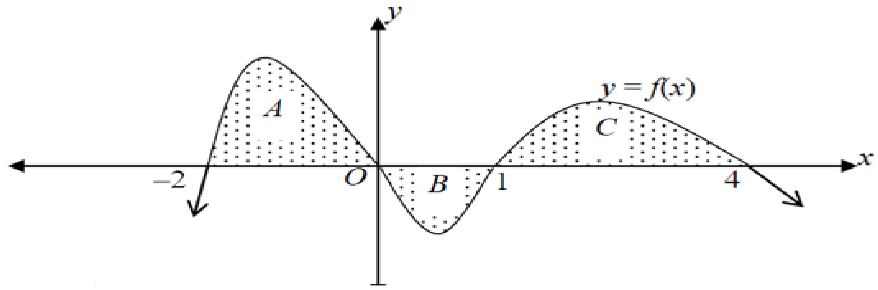
Question 15 (1 mark)

The graph of the function $f(x)$ is shown in the diagram. The shaded areas are bounded by $y = f(x)$ and the x-axis.



GiventThe shaded area A is 10 square units, the shaded area B is 4 square units and the shaded area C is 8 square units, evaluate: $\int_{-2}^{4} f(x)dx$.

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Question 16 (4 marks)

a) Differentiate $\log_e(\sin x)$ with respect to x 1

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b) Hence, or otherwise , evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx,$ giving answer in exact form. 3

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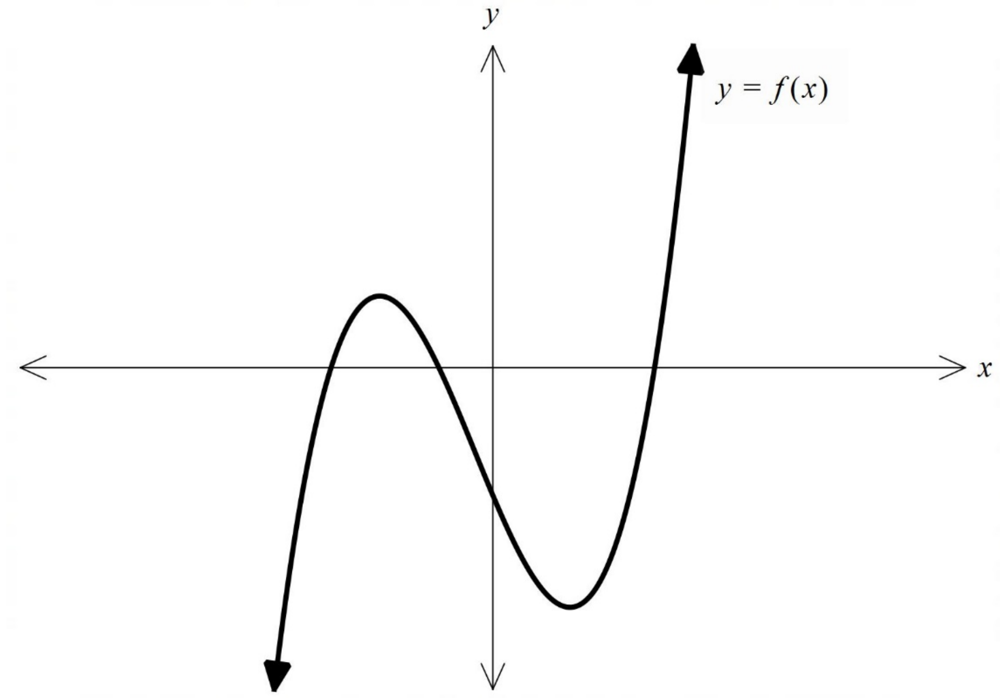
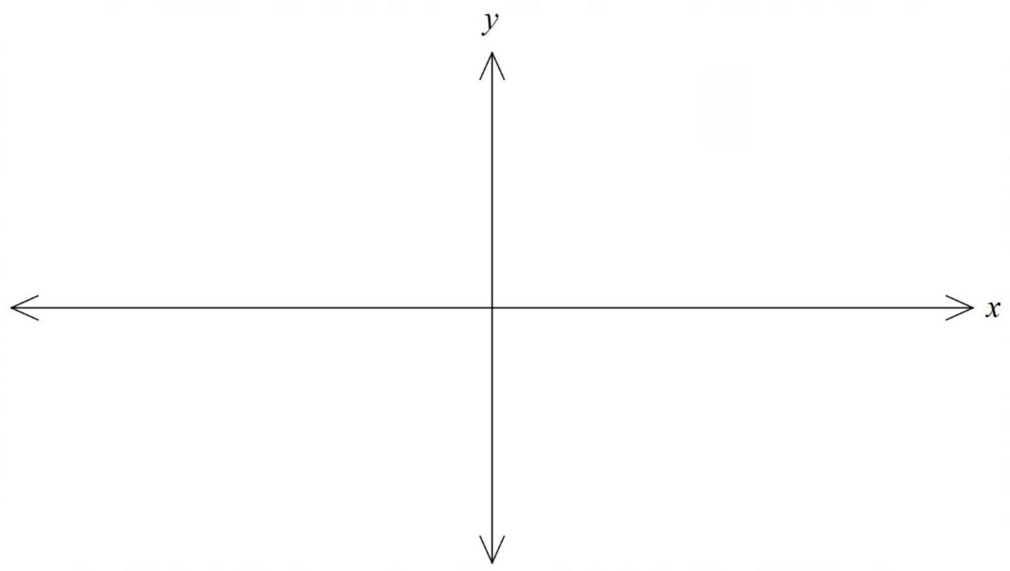
Question 17 (2 marks)

The graph of a function $y = f(x)$ is shown below.



On the given set of axes below, draw a sketch of the derivative $y = f'(x)$ of the function $y = f(x)$.



Question 18 (3 marks)

Find the equation of the curve $y = f(x)$ given that $\frac{d^2y}{dx^2} = 6x - 4$, and $y = f(x)$ has a stationary point at (1, 2)

Question 19 (7 marks)

a) Evaluate $\int \frac{1+x}{x^2} dx$. 3

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b) Find $\int \cos(\frac{x^{\circ}}{2})$ 2

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c) $\int \frac{1}{3} e^{2x-5} dx$. 2

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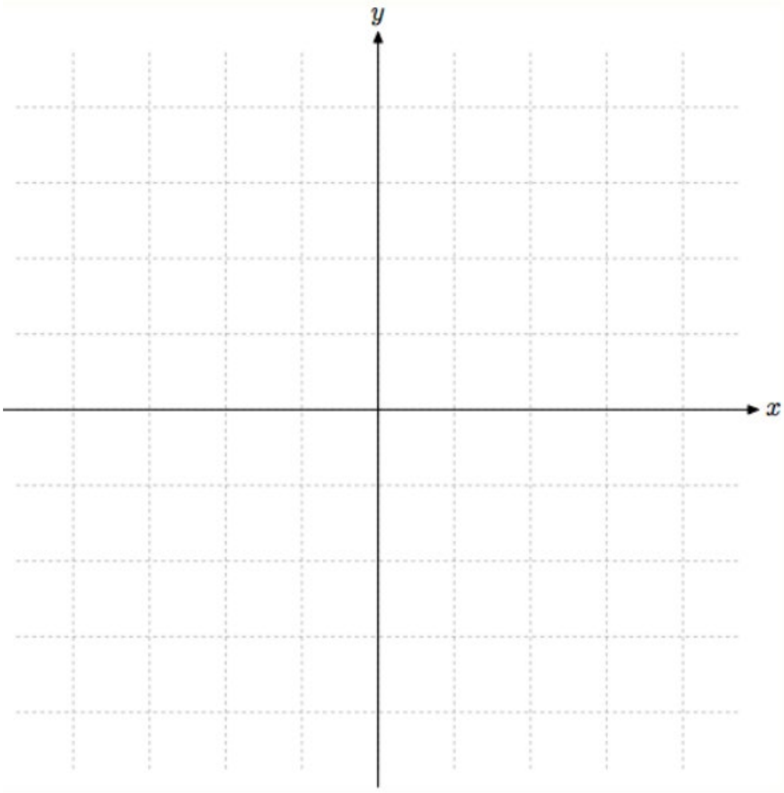
Question 20 (4 marks)

a) On the axes givenbelow, graph the region bounded by the curves: 2

$y = e^{x}, y = e^{-x}$ and $x = 5$.



b) Hence calculate the area of the region bounded by curve 2

$y = e^{x}, y = e^{-x}$ and $x = 5$

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Question 21 (4 marks)
Evaluate:

a) Given $f(x) = \log_e(x + \sqrt{x^2 + 1})$, show that $f'(x) = \frac{1}{\sqrt{x^2+1}}$. 2

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b) Hence, or otherwise, find $\int_0^1 \frac{dx}{\sqrt{x^2+1}}$, leaving your answer in exact form. 2

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Question 22 (2 marks)

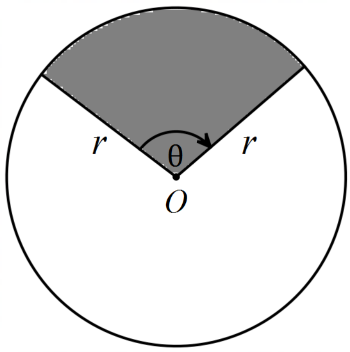
Differentiate the following with respect to x:

a) 5^{x+2} 1

b) $\log_4(3x)$ 1

Question 23 (5 marks)

Arjun is fencing a paddock using M metres of fencing. The paddock is to be in the shape of a sector of a circle with radius r and sector angle θ (in radians), as shown below.



a) Show that the length M of fencing required is given by the equation $M = r(\theta + 2)$. 1

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b) Show that the area of the paddock , in square metres, is given by $A = \frac{1}{2}Mr - r^2$ 2

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c) Calculate the value of angle θ such that gives the area of the paddock is a maximum. 3

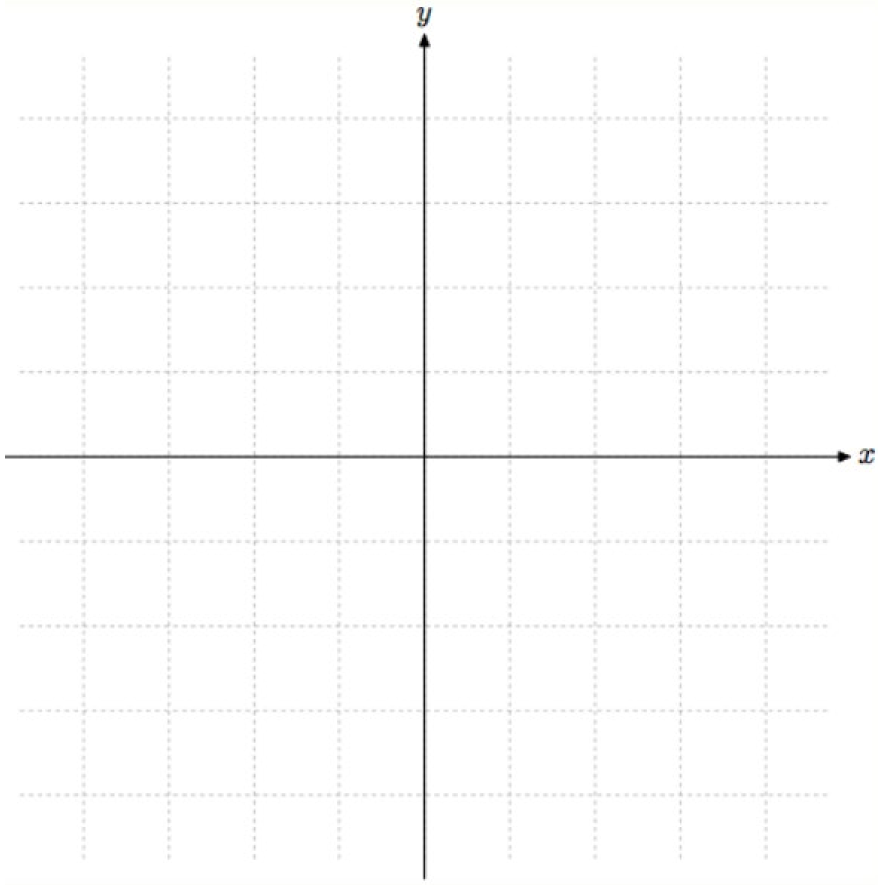
Question 24 (7 marks)

Consider the function $y = \frac{1}{3}x^3 - x + 1$.

a) Find the stationary points on this curve and determine their nature. 3

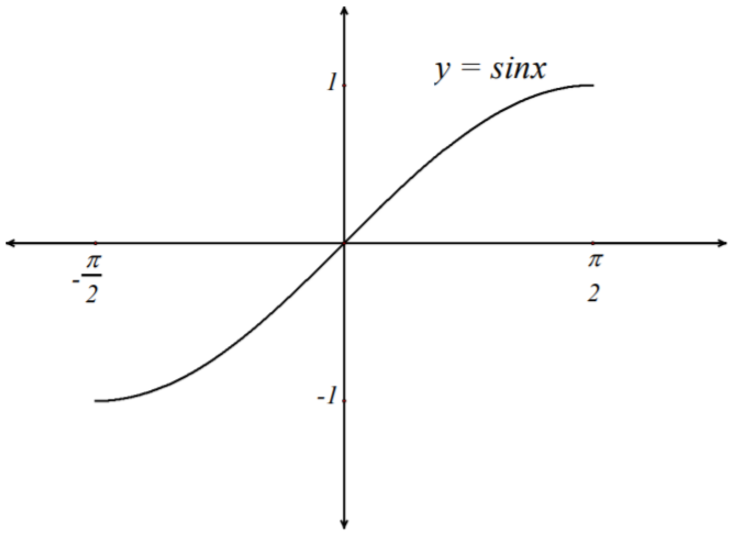
b) Show that there is a point of inflexion at point where $x = 0$. 2

f) Sketch the curve $y=\frac{1}{3}x^3-x+1$, labelling the stationary points, thevpoint of inflexion and $y-$ intercept. 2



Question 25 (6 marks)

The diagram above shows the graph of $\mathrm{y} = \sin x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.



a) On the set of axes above, sketch the graph of $y = \cos(2x)$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. **2**

b) Show by substitution that the two graphs intersect where $x = \frac{\pi}{6}$ and $x = -\frac{\pi}{2}$. **1**

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c) Calculate the exact area enclosed by $y = \sin x$ and $y = cox(2x)$. **3**

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Question 26 (8 marks)

Consider the function $y = e^{\sin x}$ for $0 \leq x \leq 2\pi$.

a) Find all the stationary points on the curve in the given domain. 2

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b) By finding the second derivative or otherwise, determine the nature of stationary points. 2

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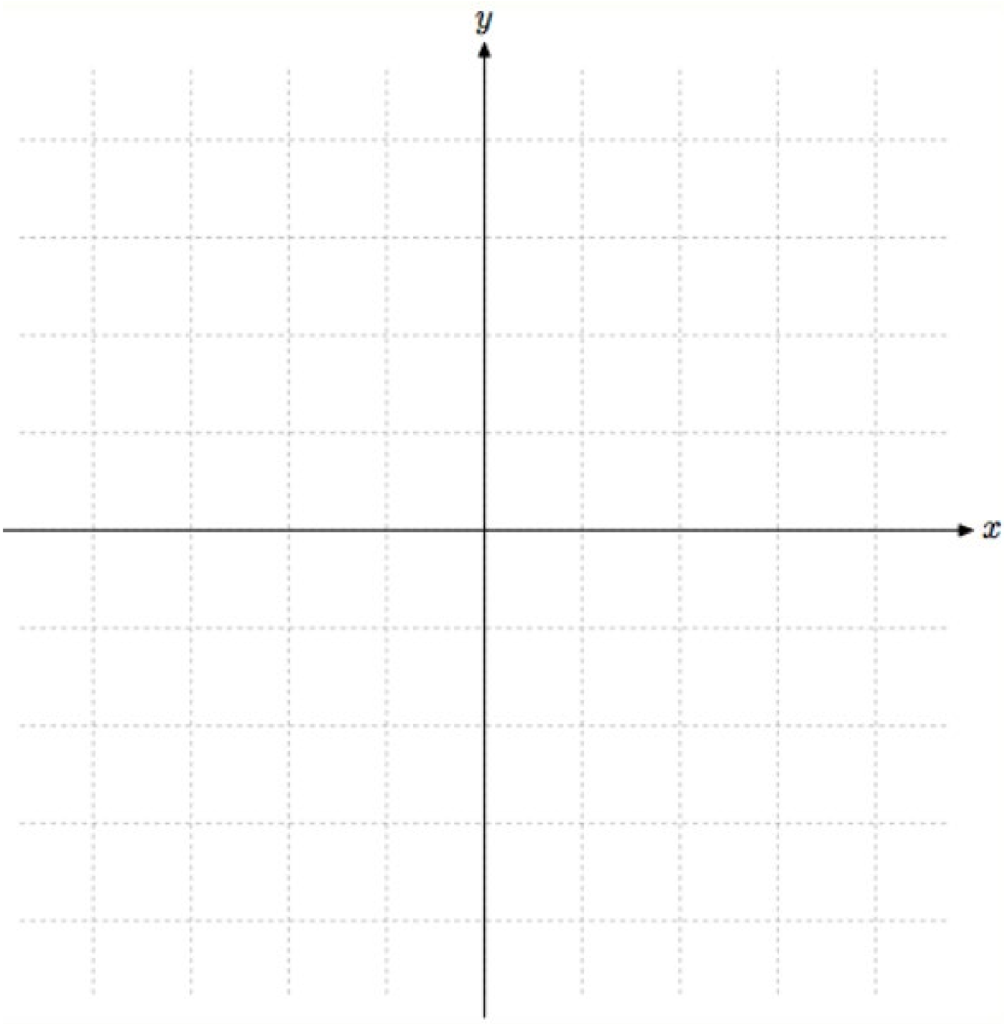
c) Find the $x-$values of points of inflexion, correct to 1 decimal place. 2

d) Sketch thegiven axex, draw the graph of $y = e^{\sin x}$, showing all the information obtained .in parts a) to c). 2



Section II extra writing space

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End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.