BAULKHAM HILLS HIGH SCHOOL

YEAR 12 ASSESSMENT TASK 1 2023

MATHEMATICS ADVANCED

NESA#: ____________________________

Teacher: ____________________________

Mathematics Advanced

General Instructions

- Reading time- 5 minutes
- Working time- 1 hour
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 6-15, show relevant mathematical reasoning and/or calculations

Total marks: 45

Section I - 5 marks (pages 4-6)

- Attempt Questions 1 - 5
- Allow about 8 minutes for this section

Section II - 40 marks (pages 7 - 15)

- Attempt Questions 6 - 15
- Allow about 52 minutes for this section

Section I

5 marks
Attempt Questions 1 - 5
Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1-5.

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1 What type of sequence is 53, 46, 39, 32, … ?

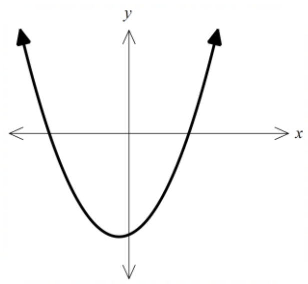
A) Arithmetic with a common difference of 7.

B) Arithmetic with a common difference of (-7).

C) Geometric with a common ratio of 7.

D) Geometric with a common ratio of (-7).

2 The graph of $y = x^2 + x - 12$ is shown below.



For what values of x is $x^2 + x - 12 > 0$?

A) $x < -4, x > 3$

B) $-4 < x < 3$

C) $x < -3, x > 4$

D) $-3 < x < 4$

3 Michael borrows \$25 000 to purchase a car. The amount he owes (A_n) on the car after n months can be given by the recursive equation:

$$A_n = A_{n-1} \times 1.015 - 500$$

Given that $A_0 = 25\ 000$, what amount does Michael owe after 3 months?

A) \$24 641.96

B) \$24 875

C) \$24 748.13

D) \$24 619.35

4 What is the domain of $y = \dfrac{1}{\sqrt{9-x}}$?

A) $x \in (-\infty, 9)$

B) $x \in (-\infty, 9]$

C) $x \in (9, \infty)$

D) $x \in [9, \infty)$

5 A function $f(x)$ is obtained by performing the following operations on $g(x)=\sqrt{x^3+9}$ in the following order:

1. Reflection in the y axis.
2. Horizontal dilation by a factor of 4.
3. Vertical translation of 1 unit up.

Which of the following is the equation of $f(x)$?

A) $-\sqrt{64x^3+9}+1$

B) $\sqrt{9-64x^3}+1$

C) $-\sqrt{\dfrac{x^3}{64}+9}+1$

D) $\sqrt{9-\dfrac{x^3}{64}}+1$

Section II

40 marks
Attempt Questions 6 - 15
Allow about 52 minutes for this section
Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning.

Question 6 (4 marks)

Consider the sequence 14, 19, 24, ….

a) Find the value of the 50th term. **2**

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b) Find the sum of the first 25 terms. **2**

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Question 7 (5 marks)

A frog jumps 180 cm to a lily pad. It then jumps 135 cm to the next lily pad, 101.25 cm to the next lily pad and so on.

a) Show that each successive jump length forms a geometric sequence. **1**

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b) Find the length of the 10^{th} jump. Give your answer correct to 2 decimal places. **2**

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c) The first and final lily pad are 8 metres apart. Following this pattern, can the frog jump from the first lily pad to the last lily pad? Justify your answer with appropriate calculations. **2**

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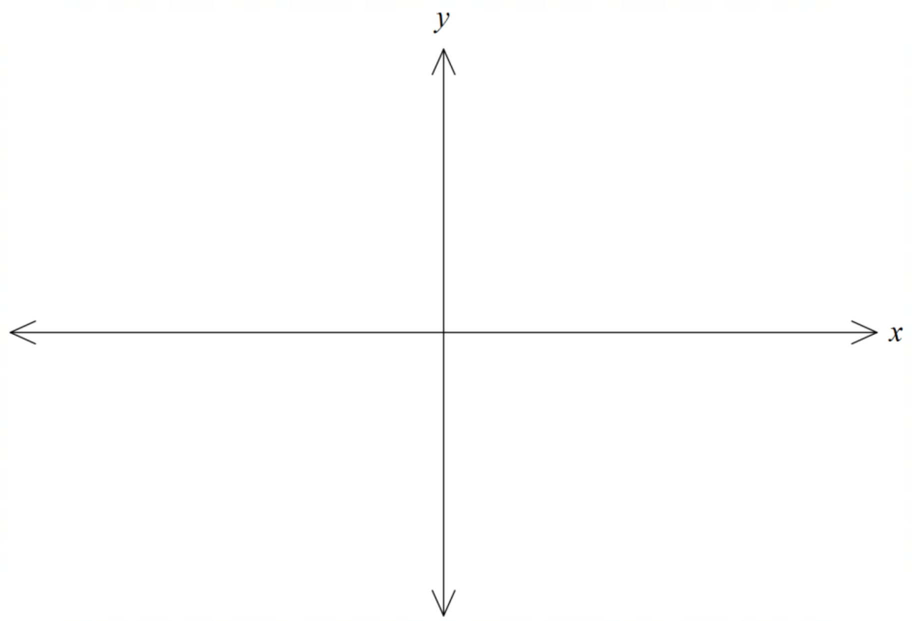
Question 8 (7 marks)

Consider the function $f(x)=\dfrac{1}{x+3}$.

a) Find the x coordinates of the points of intersection of the function $f(x)=\dfrac{1}{x+3}$ with the line $y=x+3$. **2**

b) On the number plane below, sketch the graph of $f(x)=\dfrac{1}{x+3}$ and $y=x+3$. **3**



c) Hence, or otherwise, solve the inequality $\dfrac{1}{x+3}\geq 3+x$. **2**

Question 9 (2 marks)

For what value(s) of x does the sequence $5, 5(2x+1), 5(2x+1)^2, \ldots$ have a limiting sum? **2**

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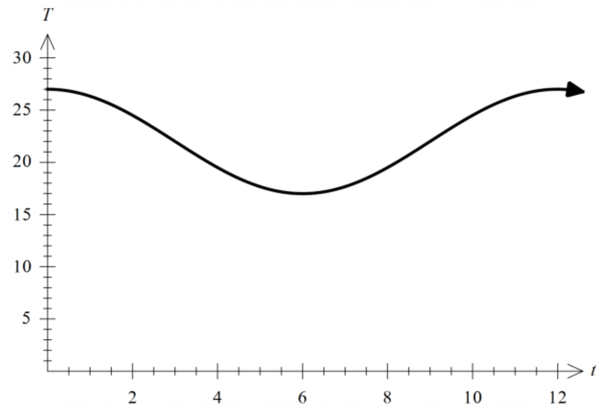
Question 10 (6 marks)

The average monthly maximum temperature in Sydney can be modelled using the cosine function, $T = A\cos\left(\frac{\pi}{6}t\right) + B,$ where T is the average maximum temperature and t is the time elapsed since the start of the year in months (where $0 \le t \le 12$). The lowest average maximum temperature throughout the year is $17°$ and the highest average maximum temperature is $27°$ (in January). The graph showing the average monthly maximum temperature is shown below.



a) Find the value of A and the value of B. **2**

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b) State the period of the function. **1**

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c) An outdoor music festival is being planned. The event is to be held when the average maximum temperature is at least $24°$. For what values of t can the event take place? Give your answer correct to 2 decimal places. **3**

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Question 11 (3 marks)

A farmer is monitoring the salinity of a dam on his property. On the first day of observations, the salinity is measured as 0.15 ppt (parts per thousand). Each subsequent day, the salinity increases by 2% of the previous day's measurement. When the measurement exceeds 10 ppt, the water will no longer be suitable for livestock and the farmer will begin to treat the dam. On which day will the farmer need to start treating his dam? **3**

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Question 12 (2 marks)

The first 3 terms of a geometric series are $2x+3,\ x,\ 2x-3$. Find the value(s) of x. **2**

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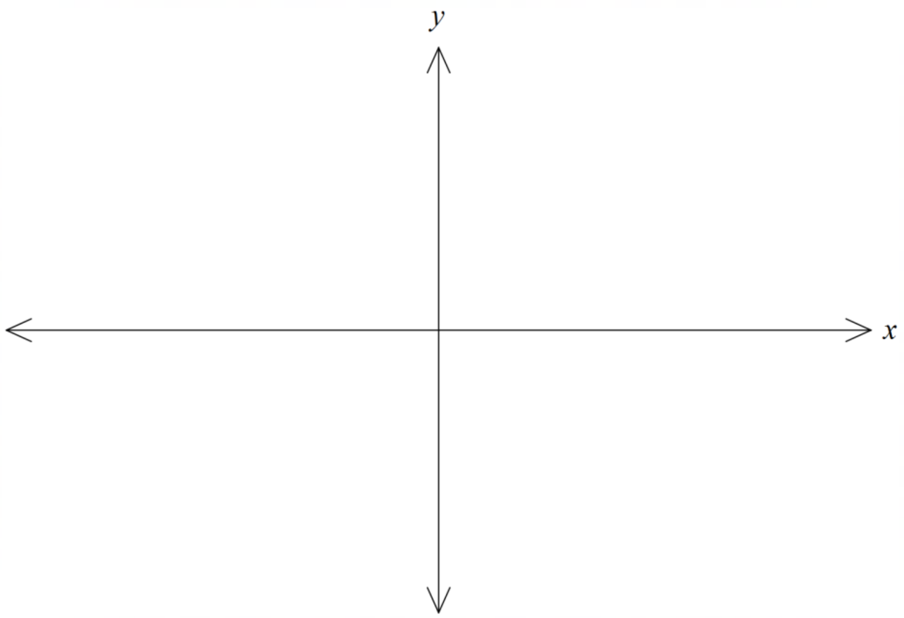
Question 13 (5 marks)

Consider the function $f(x) = \dfrac{8}{x^2 + 4}$.

a) Prove that $f(x)$ is an even function. **1**

b) State the domain and range of $f(x)$. **2**

c) Hence, or otherwise, graph the function $f(x) = \dfrac{8}{x^2 + 4}$. **2**



Question 14 (3 marks)

A company starts a page on social media. Initially, the page has 200 followers ($ie\ T_0 = 200$). Each day the page gains an extra 200 followers but loses 10% of the number of followers on the previous day.

Let T_n be the number of followers on the nth day.

a) Write a recurrence relation for T_n. **1**

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b) Show that $T_n = 2000\left(1-(0.9)^{n+1}\right)$. **2**

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Question 15 (3 marks)

The sum of the first ten terms of an arithmetic sequence is (-50). The next ten terms of this sequence add to (-450). Find the first term and common difference of the sequence. **3**

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End of Exam

Question	Suggested Solution	Answer
1.	As the sequence is decreasing by 7 to find the successive term, the sequence is arithmetic with a common difference of -7.	B
2.	x intercepts occur when $y = 0$ $x^2 + x - 12 = 0$ $(x-3)(x+4) = 0$ $x = 3$ or $x = -4$ $x^2 + x - 12 > 0$ when $x > 3$ or $x < -4$	A
3.	$A_n = A_{n-1} \times 1.015 - 500$ $A_1 = 25000 \times 1.015 - 500$ $= 24875$ $A_2 = 24875 \times 1.015 - 500$ $= 24748.13$ $A_3 = 24748.13 \times 1.015 - 500$ $= \$24619.35$	D
4.	The denominator of a fraction cannot be zero. $9 - x > 0$ $x < 9$ $\therefore$ Domain is $x \in (-\infty, 9)$	A
5.	$g(x) = \sqrt{x^3 + 9}$ becomes $g(-x) = \sqrt{(-x)^3 + 9}$ after the reflection in the y axis $g(\frac{-x}{4}) = \sqrt{\left(\frac{-x}{4}\right)^3 + 9}$ after a horizontal dilation by a factor of 4 $= \sqrt{\frac{-x^3}{64} + 9}$ $= \sqrt{9 - \frac{x^3}{64}}$ $g(\frac{-x}{4}) + 1 = \sqrt{9 - \frac{x^3}{64}} + 1$ after a vertical translation of 1 unit	D

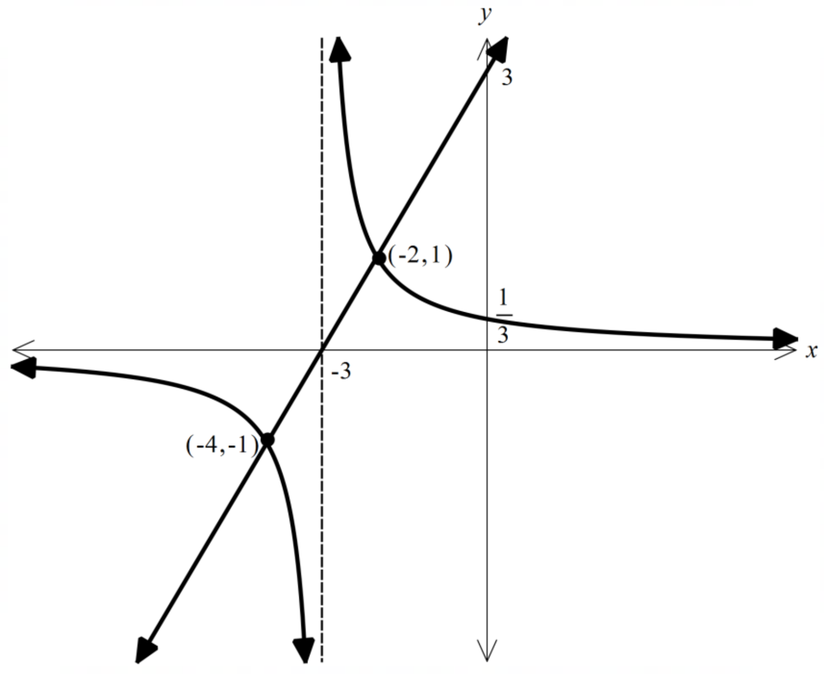
Question 6

	Suggested Solution	Criteria
a	Sequence is arithmetic with $a = 14, n = 50, d = 5$ $T_{50} = 14 + (50-1) \times 5$ $= 259$	2 – correct solution 1 – correctly substitutes a,n and d into $T_n = a + (n-1)d$.
b	$S_{25} = \frac{25}{2}(2 \times 14 + (25-1) \times 5)$ $= 1850$	2 – correct solution 1 – correctly substitutes a,n and d into $S_n = \frac{n}{2}(2a + (n-1)d)$

Question 7

a	Sequence becomes 180,135,101.25,.... $\frac{T_2}{T_1} = \frac{135}{180}$ $= \frac{3}{4}$ $\frac{T_3}{T_2} = \frac{101.25}{135}$ $= \frac{3}{4}$ Since $\frac{T_3}{T_2} = \frac{T_2}{T_1}$, the sequence is geometric	1 – correct colution
b	Geometric Sequence with $a = 180, r = 0.75, n = 10$ $T_{10} = 180(0.75)^{10-1}$ $= 13.51524353$ $= 13.52\,\text{cm}\,(2\text{ dp})$	2 – correct solution 1 - correctly substitutes a, r and n into $T_n = ar^{n-1}$
c	Since frog jumps **to** the first lily pad, the frog will not be able to jump from first to last lily pad if $S_\infty < 980$ $S_\infty = \frac{180}{1-0.75}$ $= 720$ $\therefore$ Frog will not be able to jump from the first to last lily pad	2 – correct solution 1 - correctly substitutes a and r into $S_\infty = \frac{a}{1-r}$

Question 8

a	$\frac{1}{x+3}=x+3$ $(x+3)^2=1$ $x+3=\pm 1$ $x=-2$ or $x=-4$	2 – Correct solution 1 – Obtains $(x+3)^2=1$
b		3 – Correct solution including intercepts, vertical asymptote and points of intersection labelled 2 – Correctly draws one graph with intercepts or vertical asymptote labelled OR Correctly draws both graphs without labelling intercepts. 1 – Correctly draws one graph without labelling intercepts or asymptote OR draws an incorrect graph with correct intercepts or asymptotes
c	$-3<x\le -2$ or $x\le -4$	2 – Correct solution 1 - obtains $-3\le x\le -2$ or $x\le -4$

Question 9

	A geometric sequence has a limiting sum when $-1<r<1$ Since $r=2x+1$, $-1<2x+1$ or $2x+1<1$ $2x>-2$ $\quad$ $2x<0$ $x>-1$ $\quad$ $x<0$ $\therefore -1<x<0$	2 – Correct solution 1- Obtains $x>-1$ or $x<0$ or leaves answer as two separate inequalities

Question 10

a	$A = \frac{1}{2}(27 - 17)$ $= 5$ when $t = 0, T = 27$ $27 = 5\cos\left(\frac{\pi(0)}{6}\right) + B$ $B = 22$	2 – correct solution 1 – finds the correct value of A or B
b	Period $= \frac{2\pi}{\pi/6}$ $= 12$	1 – correct solution
c	Find points of intersection of $5\cos\left(\frac{\pi}{6}t\right) + 22 = 24$ $5\cos\left(\frac{\pi}{6}t\right) = 2$ $\cos\left(\frac{\pi}{6}t\right) = \frac{2}{5}$ $\left(\frac{\pi}{6}t\right) = 1.159279481$ or $\left(\frac{\pi}{6}t\right) = 2\pi - 1.159279481$ $= 5.123905826$ $t = 2.214060718$ or $t = 9.785939282$ $\therefore$ Temperature will be above 24° when $0 < t < 2.21$ or $9.79 < t < 12$ (from graph)	3 – correct solution 2 – obtains 1 correct value for t 1 – obtains $\cos\left(\frac{\pi}{6}t\right) = \frac{2}{5}$

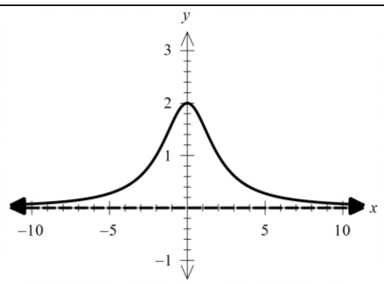
Question 11

	Measurements are 0.15, 0.153, 0.15606 Geometric sequence where a = 0.15, r = 1.02, $T_n = 10$ $10 = 0.15(1.02)^{n-1}$ $\frac{200}{3} = (1.02)^{n-1}$ $\ln_{1.02}\left(\frac{200}{3}\right) = n-1$ $n-1 = \frac{\ln\left(\frac{200}{3}\right)}{\ln(1.02)}$ $n = 213.078176$ $\therefore$ On the 214th day, the salinity will exceed 10 ppt.	3 – correct solution 2 – Obtains $\ln_{1.02}\left(\frac{200}{3}\right) = n-1$ 1 – Obtains $\frac{200}{3} = (1.02)^{n-1}$

Question 12

	Since the series is geometric $\frac{T_3}{T_2} = \frac{T_2}{T_1}$ $\frac{x}{2x+3} = \frac{2x-3}{x}$ $x^2 = (2x-3)(2x+3)$ $4x^2 - 9 = x^2$ $3x^2 = 9$ $x = \pm\sqrt{3}$	2 – correct solution 1 – obtains $\frac{x}{2x+3} = \frac{2x-3}{x}$

Question 13

a	Even function if $f(x) = f(-x)$ $f(-x) = \dfrac{8}{(-x)^2 + 4}$ $= \dfrac{8}{x^2 + 4}$ $= f(x)$ $\therefore f(x) = \dfrac{8}{x^2 + 4}$ is an even function	1 – correct solution
b	Since $x^2 > 0$, domain is all real x Range: Minimum value of $x^2 + 4$ is 4 (when $x = 0$) $\therefore$ Maximum value of $y = \dfrac{8}{x^2 + 4}$ is 2. As $x \to \pm\infty$, $\dfrac{8}{x^2 + 4} \to 0$ Range is $0 < y \leq 2$	2 – correct solution 1 – correct domain or range
c		2 – correct solution 1 – correct asymptotes or correct shape (cusp point ignored)

Question 14

a	$T_n = 200 + 0.9 \times T_{n-1}$ where $T_0 = 200$ and $n \geq 1$	1 – correct solution
b	$T_0 = 200$ $T_1 = 200 + 0.9 \times T_0$ $= 200 + 0.9 \times 200$ $T_2 = 200 + 0.9 \times T_1$ Sequence is $= 200 + 0.9 \times (200 + 0.9 \times 200)$ $= 200 + 200 \times 0.9 + 200 \times 0.9^2$ $T_3 = 200 + 0.9 \times T_2$ $= 200 + 0.9 \times (200 + 200 \times 0.9 + 200 \times 0.9^2)$ $= 200 + 200 \times 0.9 + 200 \times 0.9^2 + 200 \times 0.9^3$ Each term is the sum of a geometric series where $a = 200$, $r = 0.9$, $n = n + 1$ $\therefore T_n = \dfrac{200(1-(0.9)^{n+1})}{1-0.9}$ $= 2000(1-(0.9)^{n+1})$	2 – correct solution 1 – Writes an expression for T_3 in terms of powers of 0.9

Question 15

	$S_{10} = -50$ $\dfrac{10}{2}(2a + (10-1)d) = -50$ $5(2a + 9d) = -50$.....................(1) $2a + 9d = -10$ $S_{20} = (-450) + (-50)$ $= -500$ $\dfrac{20}{2}(2a + (20-1)d) = -500$ $10(2a + 19d) = -500$.....................(2) $2a + 19d = -50$ $(2) - (1)$ $10d = -40$ $d = -4$ $2a + 9d = -10$ $2a + 9(-4) = -10$ $2a = 26$ $a = 13$	3 – correct solution 2 – obtains equation (1) and (2) 1 – obtains equation (1) or (2) or $2a + 29d = -90$