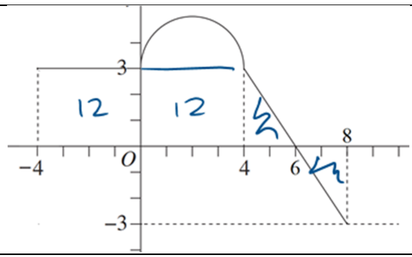
Baulkham Hills High School
Task 2 Examination 2023

Marking Guideline- Yr 12 Advanced

Section I (5 marks)

Award 1 marks to each correct answer.

Answers	1. B	2. B	3. D	4. B	5. D

Question	Suggested solutions	Answer
1	$\frac{dy}{dx} = -4 \times 3e^{-4x}$ $\frac{dy}{dx} = -4y$	B
2	Note: e^2 is a constant so its derivative is e^2x	B
3	$\int \tan x \; dx = \int \frac{\sin x}{\cos x} dx$	D
4		B
5	$f''(x) = 0$ means no concavity (does not bend)	D

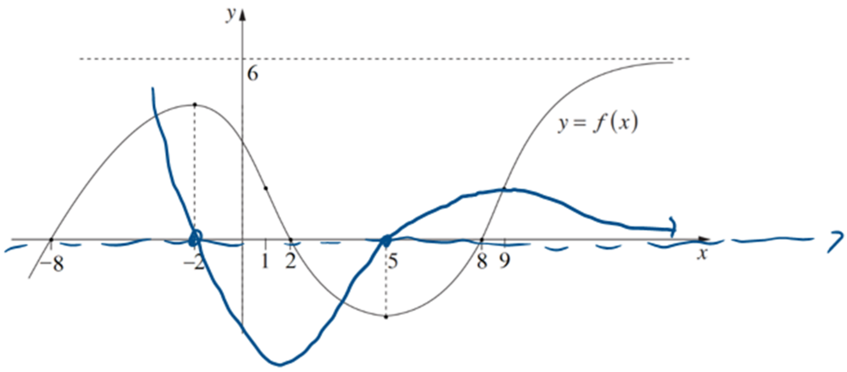
Section II (33 marks)

In all questions, award full marks for correct answers with necessary working.

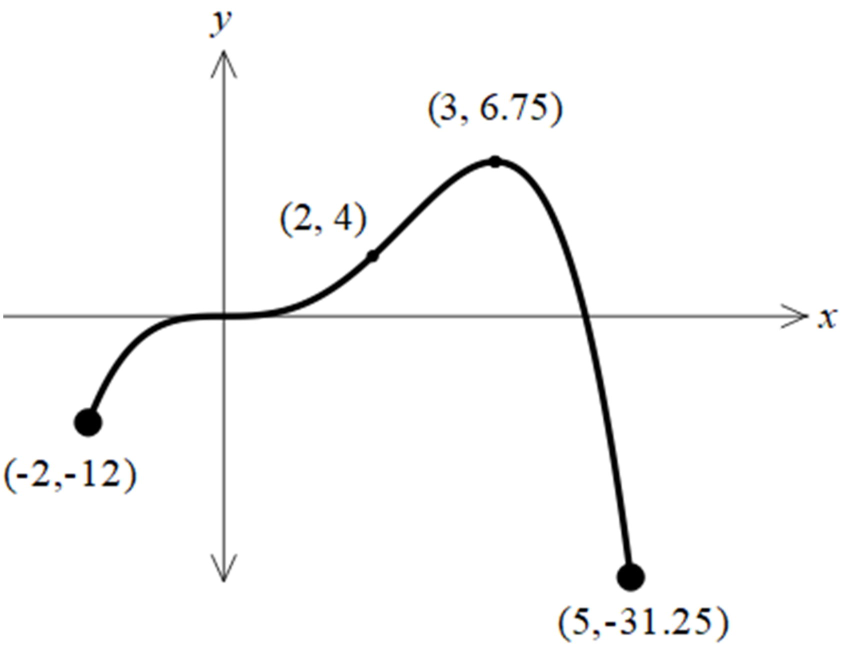
Use the suggested solutions in conjunction with the marking criteria.

Q	Suggested Solutions	Marking criteria
6a	$3\sec^2(3x-4)$	1- correct solution
6b	$\frac{\cos x}{\sin x} = \cot x$	1- correct solution
6c	$u = \cos 3x$, $u' = -3\sin 3x$; $v = x$, $v' = 1$ $y' = \frac{-3x\sin x - \cos 3x}{x^2}$	2- correct solution 1- derives $\cos 3x$ or correct use of product/quotient rule
6d	$u = 2x-1$, $u' = 2$; $v = e^{2x}$, $v' = 2e^{2x}$ $y' = 2e^{2x} + 2(2x-1)e^{2x}$ $y' = e^{2x} + 4xe^{2x} - 2e^{2x}$ $y' = 4xe^{2x}$	2- correct solution 1- derives e^{2x}or correct use of product rule
7	$\frac{4}{5}$	1- correct solution
8	$f'(x) = 4x^3 + 3x^2 + 2kx - 3$ $f''(x) = 12x^2 + 6x + 2k$ Concave up with $f''(x) > 0$ $2(6x^2 + 3x + k) > 0$ $a > 0\ or\ \Delta < 0$ $3^2 - 4 \times 6 \times k < 0$ $-24k < -9$ $k > \frac{3}{8}$	3- correct solution 2- finds $k = \frac{3}{8}$ 1- finds $f''(x)$
9	$A = \frac{2.7}{2}[0 + 2(3.5 + 1.3 + 2.3) + 0]$ $A = 19.17\ \text{m}^2$	2- correct solution 1- finds one iteration of the trapezoidal rule
10	$f(x) = x^3 - 2x^2 + x + c$ $f(2) = 5 = 2^3 - 2 \times 2^2 + 2 + c$ $c = 3$ $\therefore f(x) = x^3 - 2x^2 + x + 3$	2- correct solution 1- integrates correctly
11	$A = \int_2^4 \frac{2}{3x}dx = \frac{2}{3}[lnx]_2^4$ $= \frac{2}{3}(ln4 - ln2)$ $= \frac{2}{3}\ln 2$ OR $A = \int_2^4 \frac{2}{3x}dx = \frac{2}{3}[ln3x]_2^4$ $= \frac{2}{3}(ln12 - ln6)$ $= \frac{2}{3}\ln 2$	2- correct solution 1- correctly integrates
12	$\frac{dy}{dx} = e^x \sin x + \cos x\, e^x$ At $x = \pi$ $m_T = e^\pi \sin\pi + \cos\pi\, e^\pi$ $m_T = -e^\pi$ $m_N = \frac{1}{e^\pi}$ Equation of the normal $y - 0 = \frac{1}{e^\pi}(x - \pi)$ $e^\pi y - x + \frac{\pi}{e^\pi} = 0$	3- correct solution 2- finds the equation of a tangent OR finds the gradient of the normal. 1- correct derivative OR finds the equation of the normal with an incorrect derivative.

13a)	$$2^{2x}\times 2^3-4\times 2^x=0$$ $$2^2\times 2^x(2^x\times 2-1)=0$$ $2^x=0$ Not possible; $2\times 2^x=1$, $2^x=\frac{1}{2}$, $x=-1$ $$\therefore x=-1$$	3- correct solution 2- solves a quadratic equation 1- has an equation in terms of 2^x
13b)	$$\log_5 3^2=\log_5\left(\frac{x}{6}\right)$$ $$3^2=\frac{x}{6}$$ $$x=54$$	2- correct solution 1- uses at least one log law correctly.
14a)	$\int(2x+1)^{-2}dx=\frac{(2x+1)^{-1}}{-1\times 2}+c$ $=\frac{-1}{2(2x+1)}+c$	2- correct solution must have +c 1- integrates $(2x+1)^{-2}$
14b)	$\frac{1}{10}\log_e\lvert 5x^2+3\rvert+c$	2- correct solution 1- integrates without chain (missing $\frac{1}{10}$)
14c)	$\int 3x+\frac{4}{x}dx$ $\frac{3}{2}x^2+4\log_e\lvert x\rvert+c$	1- correct solution
14d)	$\frac{1}{2}e^{2x}+c$	1- correct solution
14e)	$-\frac{1}{4}\cos(4x-3)+c$	2- correct solution 1- integrates without chain (missing $-\frac{1}{4}$)
14f)	$$\int 5\times 5^x dx=\frac{5}{\log_e 5}5^x+c$$ $$=\frac{5^{x+1}}{\log_e 5}+c$$	1- correct solution
14g)	$\int_0^{\frac{\pi}{4}}\frac{2}{\cos^2 x}dx=\int_0^{\frac{\pi}{4}}2\sec^2 x\,dx$ $=2[\tan x]_0^{\frac{\pi}{4}}$ $=2(\tan\frac{\pi}{4}-\tan 0)$ $=2$	3- correct solution 2- integrates $\sec^2 x$ 1- changes $1-\sin^2 x$ to $\cos^2 x$
15a)	$2x=-4x^2+16x$ $0=-4x^2+14x$ $0=2x(-2x+7)$ $x=0$ or $x=\frac{7}{2}$	1- correct solution
15b)	$\int_0^{\frac{7}{2}}-4x^2+16x-2x\,dx=\left[-\frac{4}{3}x^3+7x^2\right]_0^{\frac{7}{2}}$ $$=\left(-\frac{4}{3}\times\left(\frac{7}{2}\right)^3+7\times\left(\frac{7}{2}\right)^2\right)-0$$ $$\frac{343}{12}=28.583$$	2- correct solution using part (a) 1- correct integral

16a)	$x < 1 \; or \; x > 9$	1- correct solution
16b)		3- correct solution 2- satisfies 2 criteria points 1- satisfies one criteria point Criteria point: - stationary points - max/min points align with inflection points - asymptote at $y = 0$ as $x \to \infty$
17a(i)	$y = (3x^2+6)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}(3x^2+6)^{-\frac{1}{2}} \times 6x$ $= \frac{3x}{\sqrt{3x^2+6}}$	2- correct solution 1- derives without chain rule
17a(ii)	$\frac{1}{3}\left[\sqrt{3x^2+6}\right]_0^1 = \frac{1}{3}\left[\sqrt{9}-\sqrt{6}\right]$ $= 1 - \frac{\sqrt{6}}{3}$	2- correct solution 1- correct integral
17b)	$y = [\sin(2x)]^5$ $\frac{dy}{dx} = 5[\sin(2x)]^4 \times \cos(2x) \times 2$ $\frac{dy}{dx} = 10\cos(2x)\sin^4(2x)$	2- correct solution 1- correct integral able to apply chain rule once
17c)	$y = \sin\left(\frac{\pi}{180}x\right)$ $\frac{dy}{dx} = \frac{\pi}{180}\cos\left(\frac{\pi}{180}x\right)$ $or \; \frac{dy}{dx} = \frac{\pi}{180}\cos x^{\circ}$	2- correct solution 1- converts degrees to radians and dervies without chain rule.
17d)	$y = \frac{1}{2}\log_e[e^x(2x+1)]$ $y = \frac{1}{2}[\log_e e^x + \log_e(2x+1)\,]$ $y = \frac{1}{2}[x + \log_e(2x+1)\,]$ $y' = \frac{1}{2} + \frac{1}{2} \times \frac{2}{2x+1}$ $y' = \frac{1}{2} + \frac{1}{2x+1} = \frac{2x+3}{4x+2}$	2- correct solution 1- uses at least one log law to simplify the question.
18	$x = \frac{3}{y} + 1$ $A = \frac{1}{2} \times 2 \times 3 + \int_3^5 \frac{3}{y} + 1 \, dx$ $A = 3 + \left[3\log_e(y) + y\right]_3^5$ $A = 3 + [(3\log 5 + 5) - (3\log_e 3 + 3)]$ $A = 5 + 3\log_e\left(\frac{5}{3}\right)$	3- correct solution 2- Correct expression to find the area 1-finds the area of the triangle, or finds $\int_3^5 \frac{3}{y} + 1 \, dx$ or finds $\int \frac{3}{x-1} dx$

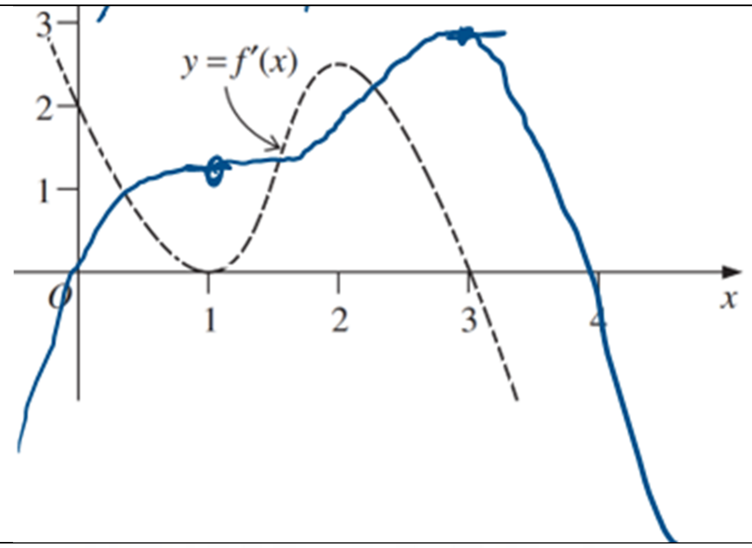
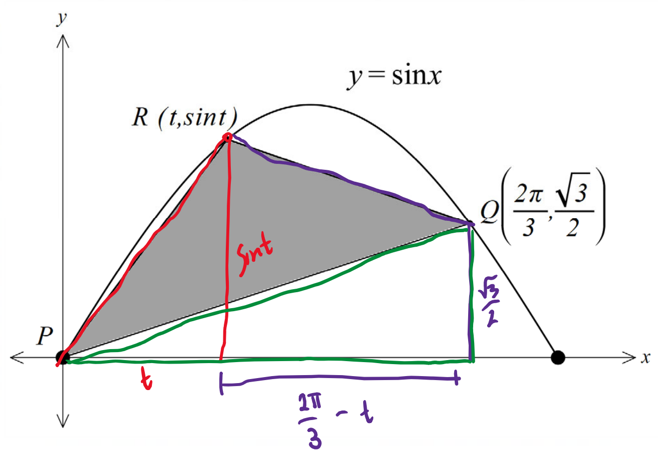
<table>
<tr>
<td>19a)</td>
<td>
$$f'(x) = 3x^2 - x^3$$
Stat point when $f'(x) = 0$
$$0 = 3x^2 - x^3$$
$$0 = x^2(3 - x)$$
$$x = 0 \text{ or } x = 3$$
$$f(0) = 0, f(3) = 6.75$$
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>$f'(x)$</td><td>4</td><td>0</td><td>4</td><td>0</td><td>-16</td></tr>
<tr><td>Gradient</td><td>/</td><td>-</td><td>/</td><td>-</td><td>\</td></tr>
</table>
∴ horizontal inflection point at $(0,0)$ and maximum point at $(3, 6.75)$.
</td>
<td>3- correct solution
2- test nature of the points by a table or second derivative.
1- Finds $x = 0,3$

Note: Must conclude a horizontal inflection point to obtain full marks.</td>
</tr>
<tr>
<td>19b)</td>
<td>
$$f''(x) = 6x - 3x^2$$
Inflection when $f''(x) = 0$
$$3x(2 - x) = 0$$
$$x = 0 \text{ or } x = 2$$
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>$f''(x)$</td><td>-9</td><td>0</td><td>3</td><td>0</td><td>-9</td></tr>
<tr><td>Concavity</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
$$f(2) = 4$$
∴inflection at $(0,0)$ and $(2,4)$
</td>
<td>3- correct solution
2- tests for the change in concavity
1- Finds $x = 0,3$</td>
</tr>
<tr>
<td>19c)</td>
<td>
$$f(-2) = -12$$
$$f(5) = -31.25$$


</td>
<td>3- correct solution
2- satisfies 2 criteria points
1- satisfies one criteria point

Criteria point:
- Finds and correctly plots endpoints
- Shows max/min and inflection points
- Correct shape

Note: important that the horizontal inflection point shows a flat gradient at (0,0) and the inflection point at (2,4) does NOT have a bump or is flat.</td>
</tr>
<tr>
<td>19d)</td>
<td>$$-13.25 \le y \le 6.75$$</td>
<td>1- correct solution</td>
</tr>
</table>

20		2- correct solution 1- correct shape or a graph that passes through (0,0)
21a)	 A =Area below PR + Area below RQ – Area below PQ $A = \left(\frac{1}{2}\times t\times \sin t\right) + \left(\frac{1}{2}\times\left(\frac{2\pi}{3}-t\right)\times\left(\sin t+\frac{\sqrt{3}}{2}\right)\right)$ $-\left(\frac{1}{2}\times\frac{2\pi}{3}\times\frac{\sqrt{3}}{2}\right)$ $A = \cancel{\frac{1}{2}t\sin t} + \frac{1}{2}\left(\frac{2\pi}{3}\sin t + \cancel{\frac{\sqrt{3}\pi}{3}} - \cancel{t\sin t} - \frac{\sqrt{3}}{2}t\right) - \cancel{\frac{\pi\sqrt{3}}{6}}$ $A = \frac{4\times\pi}{4\times 3}\sin t - \frac{\sqrt{3}\times 3}{4\times 3}t$ $A = \frac{1}{12}(4\pi\sin t - 3\sqrt{3}t)$	2- correct solution 1- Finds an area that will lead to significant progress to a solution.
21b)	$A = \frac{1}{12}(4\pi\sin t - 3\sqrt{3}t)$ $\frac{dA}{dt} = \frac{1}{12}[4\pi\cos t - 3\sqrt{3}]$ Stat point when $\frac{dA}{dt} = 0$ $\therefore \cos t = \frac{3\sqrt{3}}{4\pi}$ $t = 1.145\ldots$ $\frac{d^2A}{dt^2} = \frac{1}{12}(-4\pi\sin t)$ $when\ t = 1.145$ $\frac{d^2A}{dt^2} = -0.9534$ ∴concave down $A = \frac{1}{12}(4\pi\sin 1.145 - 3\sqrt{3}\times 1.145)$ $A = 0.46$	3- correct solution 2- satisfies 2 criteria points 1- satisfies one criteria point Criteria point: - Finds t - test for concavity - Finds maximum A

BAULKHAM HILLS HIGH SCHOOL 2023
YEAR 12 ASSESSMENT TASK 2
MATHEMATICS ADVANCED

MARKING COVER SHEET

FINAL MARK

NESA#:______________________________ **Teacher:**______________________________

Question	Curve Sketching	Integration	Exponentials & Logarithmic	Trigonometric Functions	Total
	Outcomes				
Multiple Choice	Q5 /1	Q4 /1	Q1,2 /2	Q3 /1	**/5**
6			d /2	abc /4	**/6**
7,8,9,10	8 /3	9,10 /4		7 /1	**/8**
11,12		11 /2		12 /3	**/5**
13			/5		**/5**
14		a /2	bcdf /5	eg /5	**/12**
15, 16	16 /4	15 /3			**/7**
17		a /4	d /2	bc /4	**/10**
18, 19	19 /10	18 /3			**/13**
20, 21	20 ,21 /7				**/7**
TOTAL	**/25**	**/19**	**/16**	**/18**	**/78**

Baulkham Hills High School

2023 Year 12 - Assessment Task 2

Mathematics
Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ☒ B ☒ C ○ D ○

Start Here →

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○

BAULKHAM HILLS HIGH SCHOOL
YEAR 12 ASSESSMENT TASK 2 2023
MATHEMATICS ADVANCED

Mathematics Advanced

General Instructions	• Reading time- 10 minutes • Working time- 2 hours • Write using permanent black or blue pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • In Questions 6-21, show relevant mathematical reasoning and/or calculations
Total marks: 78	**Section I - 5 marks** (pages 4-5) • Attempt Questions 1-5. • Allow about 10 minutes for this section. **Section II - 73 marks** (pages 6-22) • Attempt Questions 6-21. • Allow about 1 hour 50 minutes for this section.

Section I

5 marks
Attempt Questions 1-5
Allow about 10 minutes for this section
Use the multiple-choice answer sheet for Questions 1-5.

1 Which of the following is the derivative of the function, $y = 3e^{-4x}$?

A. $\dfrac{dy}{dx} = 4y$

B. $\dfrac{dy}{dx} = -4y$

C. $\dfrac{dy}{dx} = 3y$

D. $\dfrac{dy}{dx} = -3y$

2 Which of the following is $\int (e^2 + e^{3x})dx$?

A. $2ex + 3e^{3x} + c$

B. $e^2 x + \dfrac{1}{3}e^{3x} + c$

C. $2e^2 + 3e^{3x} + c$

D. $\dfrac{1}{2}e + \dfrac{1}{3}e^{3x} + c$

3 Which of the following is equivalent to $\int \tan x \, dx$?

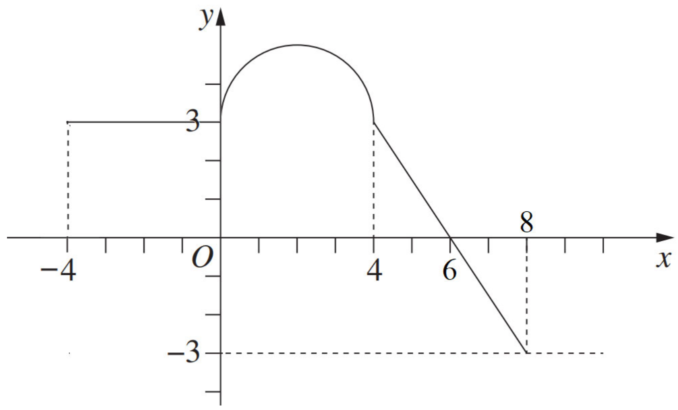
A. $\sec^2 x + c$

B. $-\text{cosec}^2 x + c$

C. $\log_e(\sin x) + c$

D. $-\log_e(\cos x)+c$

4 The diagram shows the graph $y = f(x)$, which is made up of line segments and a semi-circle.



Which of the following represents the exact value of $\int_{-4}^{8} f(x)\,dx$?

A. $30 + 2\pi$

B. $24 + 2\pi$

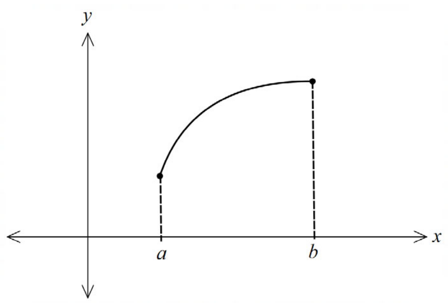
C. $6 + 2\pi$

D. 2π

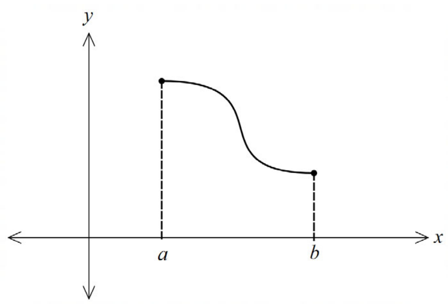
5 The function $f(x)$ is defined for $a \le x \le b$. On this interval, $f'(x) > 0$ and $f''(x) = 0$.

Which graph best represents $y = f(x)$?

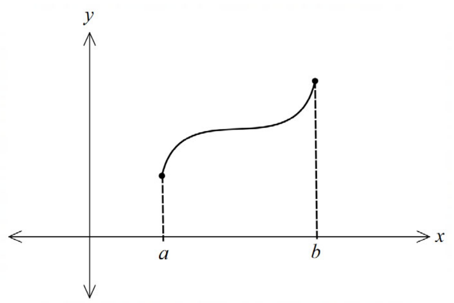
A.



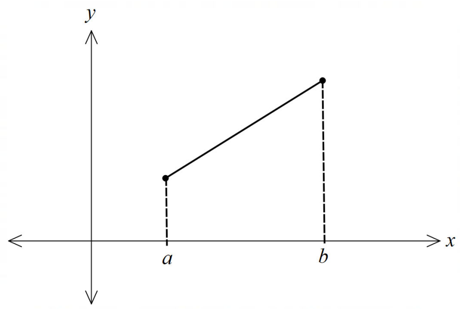
B.



C.



D.



End of Section I

Section II

73 marks
Attempt Questions 6-21
Allow about 1 hour 50 minutes for this section
Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning.

Question 6 (6 marks)

Differentiate the following with respect to x

a) $y = \tan(3x-4)$ **1**

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b) $y = \log_e(\sin x)$ **1**

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c) $y = \dfrac{\cos 3x}{x}$ **2**

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d) $y = (2x-1)e^{2x}$ **2**

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End of Question 6

Question 7 (1 mark)

Find $\lim_{x \to 0} \frac{\sin 4x}{5x}$ **1**

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Question 8 (3 marks)

Let $f(x) = x^4 + x^3 + kx^2 - 3x - 5$, where k is a constant.

Find the values of k for which $f(x)$ is always concave up. **3**

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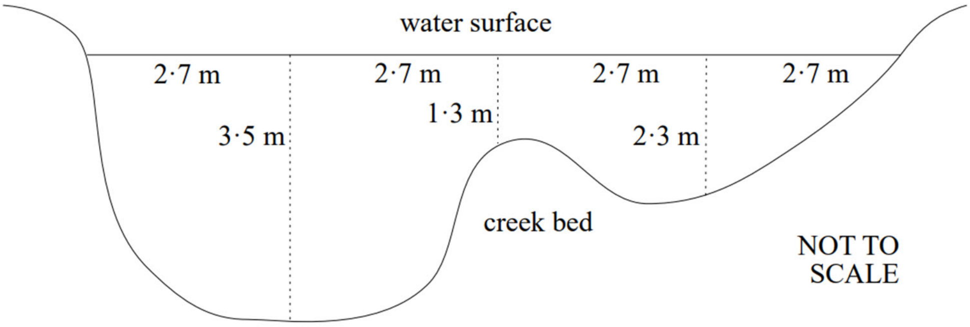
End of Question 8

Question 9 (2 marks)

The diagram below shows a vertical cross-section of a creek which is 10.8 metres wide.
At this location the depth of the river, in metres, has been measured at 2.7 metre intervals.



Use the trapezoidal rule with the five depth measurements to calculate the approximate area of the cross-section. **2**

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Question 10 (2 marks)

Find the equation of the curve passing through the point (2,5) with gradient function $f'(x) = 3x^2 - 4x + 1$. **2**

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End of Question 10

Question 11 (2 marks)

Find the exact area of the region bounded by the curve $y = \frac{2}{3x}$, the x-axis and the lines $x = 2$ and $x = 4$. **2**

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Question 12 (3 marks)

Find the equation of the normal to the curve $y = e^x \sin x$ at the point $(\pi, 0)$. **3**

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End of Question 12

Question 13 (5 marks)

For the following equations, solve for x

a) $2^{2x+3} - 4 \times 2^x = 0$ **3**

b) $2\log_5 3 = \log_5 x - \log_5 6$ **2**

End of Question 13

Question 14 (12 marks)

a) Find $\int \frac{dx}{(2x+1)^2}$. **2**

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b) Find $\int \frac{x}{5x^2+3}\,dx$. **2**

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c) Find $\int \frac{3x^2+4}{x}\,dx$. **1**

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d) Find $\int \frac{e^{3x}}{e^x}\,dx$. **1**

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e) Find $\int \sin(4x-3)\,dx$. **2**

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f) Find $\int 5^{x+1}\,dx$. **1**

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g) Find the value of $\int_0^{\frac{\pi}{4}} \frac{2}{1-\sin^2 x}\,dx$. **3**

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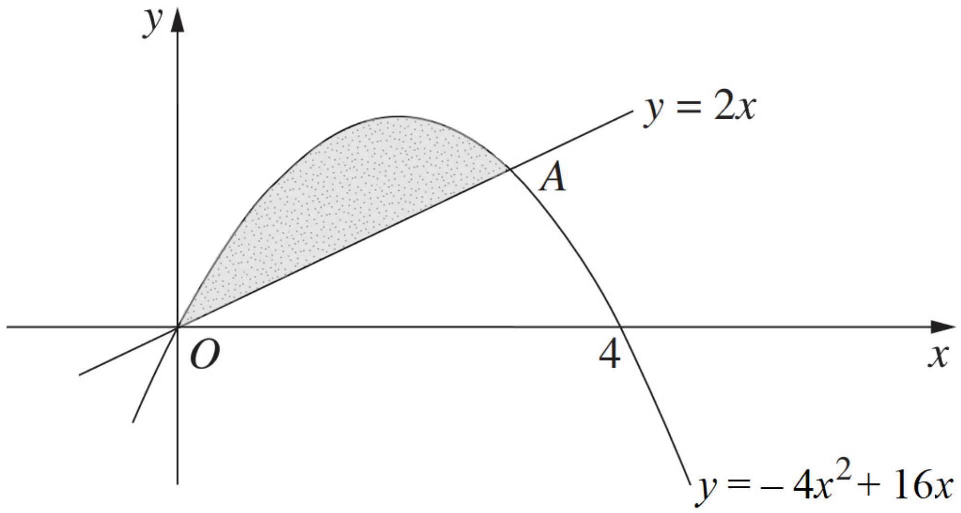
End of Question 14

Question 15 (3 marks)

The parabola $y = -4x^2 + 16x$ and the line $y = 2x$ intersect at the origin and point A.



a) Find the x-coordinate of the point A. **1**

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b) Calculate the area enclosed by the parabola and the line. **2**

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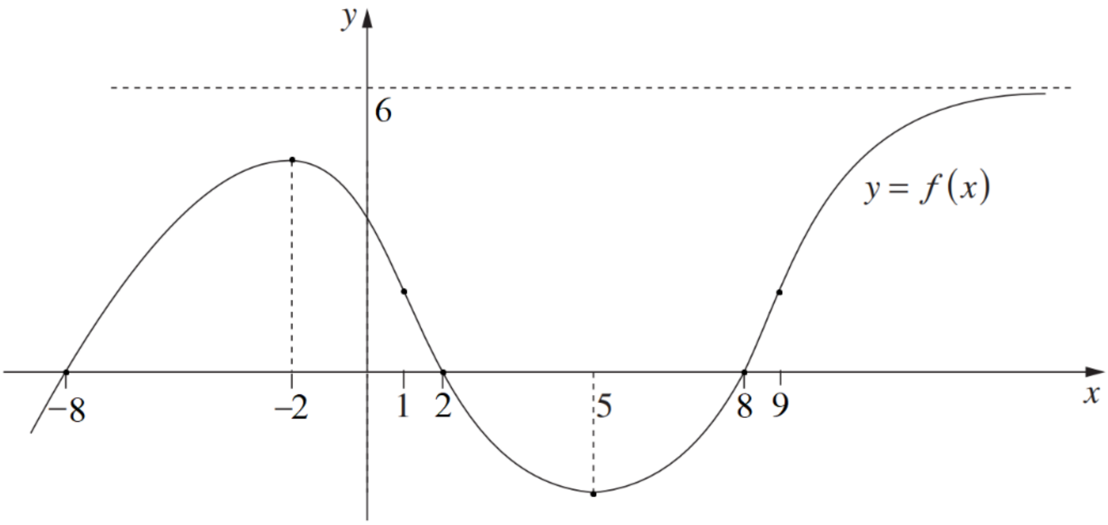
End of Question 15

Question 16 (4 marks)



The diagram shows the graph of a function $y = f(x)$

a) For which values of x is the curve concave down? **1**

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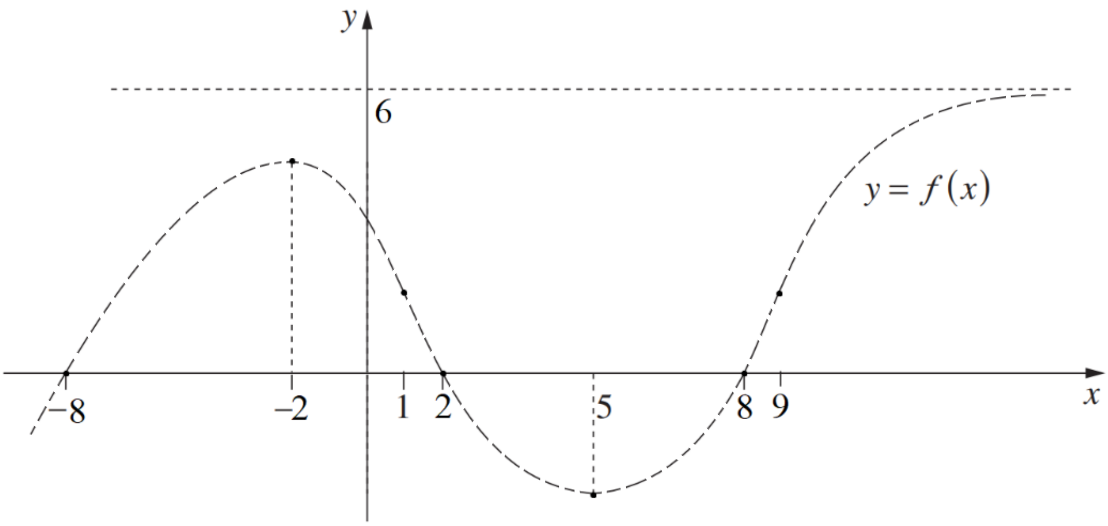
b) Sketch $y = f'(x)$ on the graph below. **3**



End of Question 16

Question 17 (10 marks)

a) (i) Differentiate $y=\sqrt{3x^2+6}$. **2**

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(ii) Hence find $\int_0^1 \frac{x}{\sqrt{3x^2+6}}\,dx$. **2**

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b) Differentiate $y=\sin^5(2x)$ **2**

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c) Differentiate $y = \sin x°$. **2**

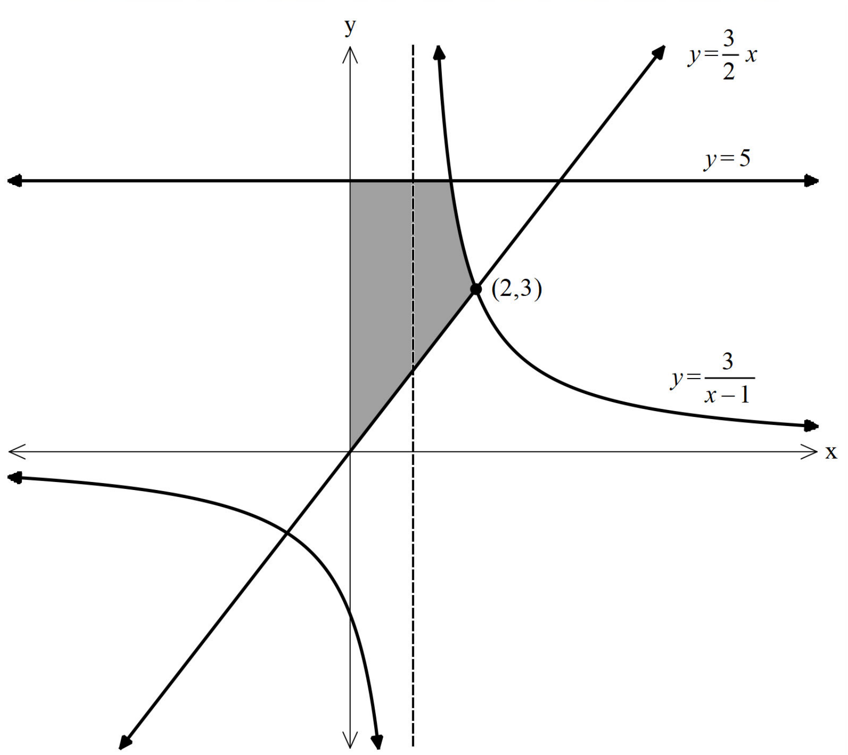
d) Differentiate $y = \log_e \sqrt{e^x(2x + 1)}$ **2**

End of Question 17

Question 18 (3 marks)

The curve $y = \frac{3}{x-1}$ intersects the line $y = \frac{3}{2}x$ at the point $(2,3)$.

The region bound by the curve $y = \frac{3}{x-1}$, the line $y = \frac{3}{2}x$, the y-axis and the line $y = 5$ is shaded in the diagram.



Find the exact area of the shaded region. **3**

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End of Question 18

Question 19 (10 marks)

Consider the function $f(x) = x^3 - \frac{1}{4}x^4$.

a) Find any stationary points of the curve $y = f(x)$, and determine their nature. **3**

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b) Find any points of inflection of the curve $y = f(x)$. **3**

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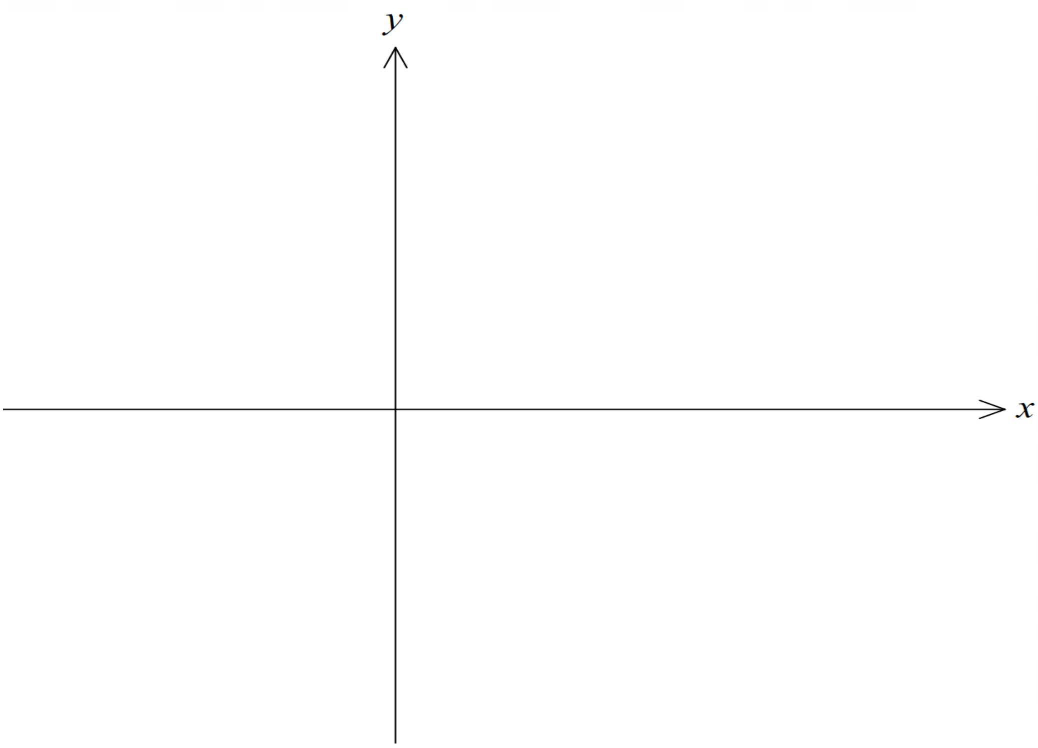
c) Using the information above, sketch $y = f(x)$ for $-2 \leq x \leq 5$ showing all important features. **3**

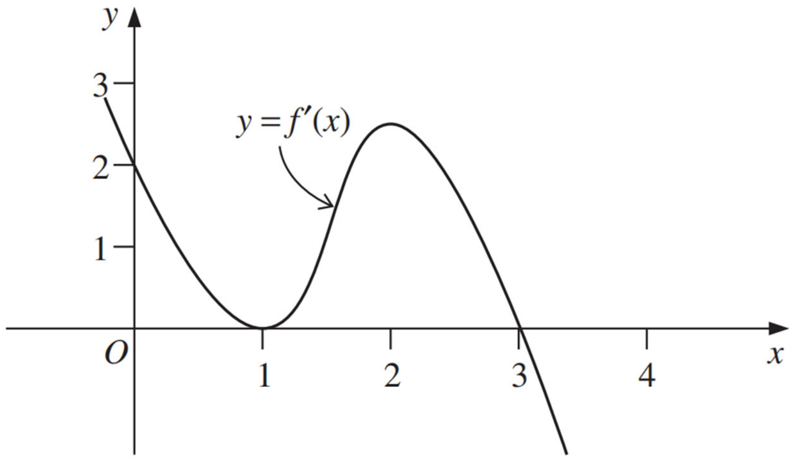


d) State the range of the function for the given domain. **1**

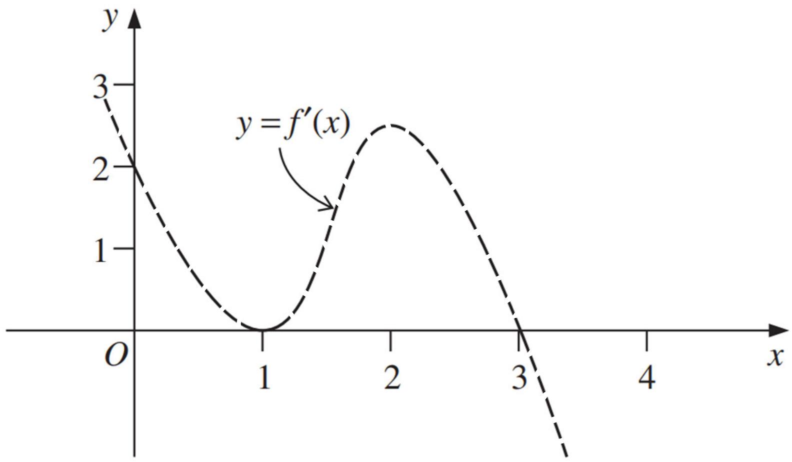
End of Question 19

Question 20 (2 marks)

The graph below shows a sketch of the gradient function of the curve $y = f(x)$.
It is given that $f(0) = 0$. **2**



Draw a sketch of $y = f(x)$.



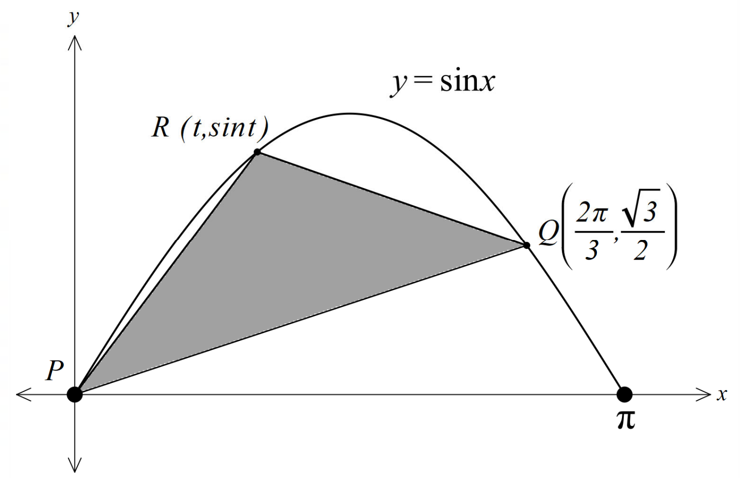
End of Question 20

Question 21 (5 marks)

The diagram below shows the graph of the function $f(x) = \sin x$ for $0 \le x \le \pi$.

The points $P(0,0)$ and $Q\left(\dfrac{2\pi}{3}, \dfrac{\sqrt{3}}{2}\right)$ both lie on the curve.

The point $R(t, \sin t)$ is also a point on the curve between P and Q, such that $0 \le t \le \frac{2\pi}{3}$.



a) Show that A, the area of ΔPQR, is given by **2**

$$A = \frac{1}{12}\left(4\pi \sin t - 3\sqrt{3}\,t\right) \text{ when } t \le \frac{2\pi}{3}.$$

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b) Find the maximum value of A, the area of ΔPQR.
Justify your answer, leaving your answer correct to two decimal places. **3**

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End of Exam