



BAULKHAM HILLS HIGH SCHOOL
YEAR 12 ASSESSMENT TASK 1 2024
MATHEMATICS ADVANCED

NESA#: _____

Teacher: _____

Mathematics Advanced

**General
Instructions**

- Reading time- 5 minutes
- Working time- 1 hour
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 6-12, show relevant mathematical reasoning and/or calculations

**Total marks:
37**

Section I - 5 marks (pages 4-5)

- Attempt Questions 1-5
- Allow about 8 minutes for this section

Section II - 32 marks (pages 6-14)

- Attempt Questions 6-12
- Allow about 52 minutes for this section

Section I

5 marks

Attempt Questions 1-5

Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1-5.

- 1 Which of the following formulas could be used to calculate the sum of the first n **positive even** numbers?

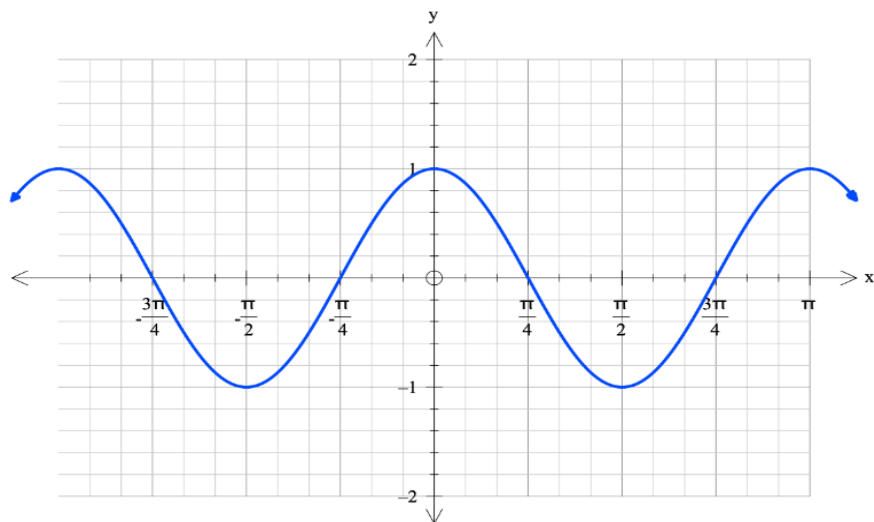
A) $S_n = 2n(n+1)$

B) $S_n = n(n+1)$

C) $S_n = n(2^n - 1)$

D) $S_n = \frac{n}{2}(6 - 2n)$

- 2 What is the equation of the graph below ?



A) $y = \sin(2x + \pi)$

B) $y = \sin(2x + \frac{\pi}{3})$

C) $y = \sin(x + \pi)$

D) $y = \sin(2x + \frac{\pi}{2})$

- 3 For what values of m will the geometric series

$1 + 2m + 4m^2 + 8m^3 + \dots$ have a limiting sum?

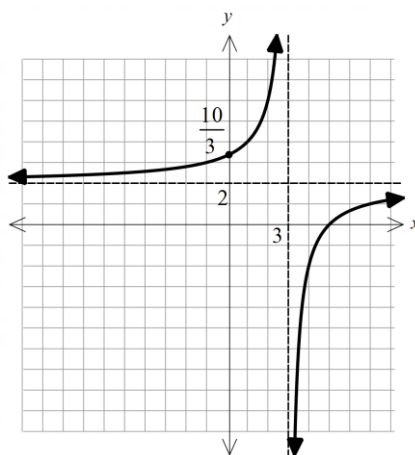
- A) $-1 \leq m \leq 1$
- B) $-\frac{1}{2} \leq m \leq \frac{1}{2}$
- C) $-\frac{1}{2} < m < \frac{1}{2}$
- D) $m < \frac{1}{2}$

- 4 The graph of $y = \frac{5}{x+2}$ is translated 4 units right and dilated vertically by a factor of $\frac{1}{2}$.

Which of the following gives the equation of the new function?

- A) $\frac{y}{2} = \frac{5}{x-2}$
- B) $2y = \frac{5}{x-4}$
- C) $2y = \frac{5}{x-2}$
- D) $\frac{y}{2} = \frac{5}{x+4}$

- 5 Consider the graphs of $y = \frac{k}{x+b} + c$ below, with y -intercept $\left(0, \frac{10}{3}\right)$.



Which of the following is true?

- A) $k < b < c$
- B) $b < k < c$
- C) $k < c < b$
- D) $c < k < b$

End of Section I

Section II

32 Marks

Attempt Questions 6-12

Allow about 52 minutes for this section

Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning.

Question 6 (3 marks)

a) Show that the series $x + \frac{x}{4} + \frac{x}{16} + \dots$ is a geometric series. 1

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b) This series $x + \frac{x}{4} + \frac{x}{16} + \dots$ has a limiting sum of $\frac{7}{8}$. Calculate the value of x . 2

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Question 7(4 marks)

Jack is a weightlifter training for competition in 4 weeks. On the first day, he lifts 120 kg, and each subsequent day he increases the weight by 1.5 kg until he reaches his goal of lifting 150 kg. Then he continues to lift 150 each day.

- a) How much does Jack lift on the 11th day? **1**

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- b) On which day does he first lift 150 kg? **1**

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- c) Jack states that the total weight he lifted over these 28 days is more than the weight of an elephant. If an elephant has a weight of 3500 kg, is Jack's statement true? Justify your answer with calculations. **2**

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Question 8 (5 marks)

The graph of $f(x) = \cos x$ is subjected to a sequence of transformations in the following order

- horizontal dilation by scale factor $\frac{1}{2}$,
- reflection in the x -axis,
- translation upward by 3 units.

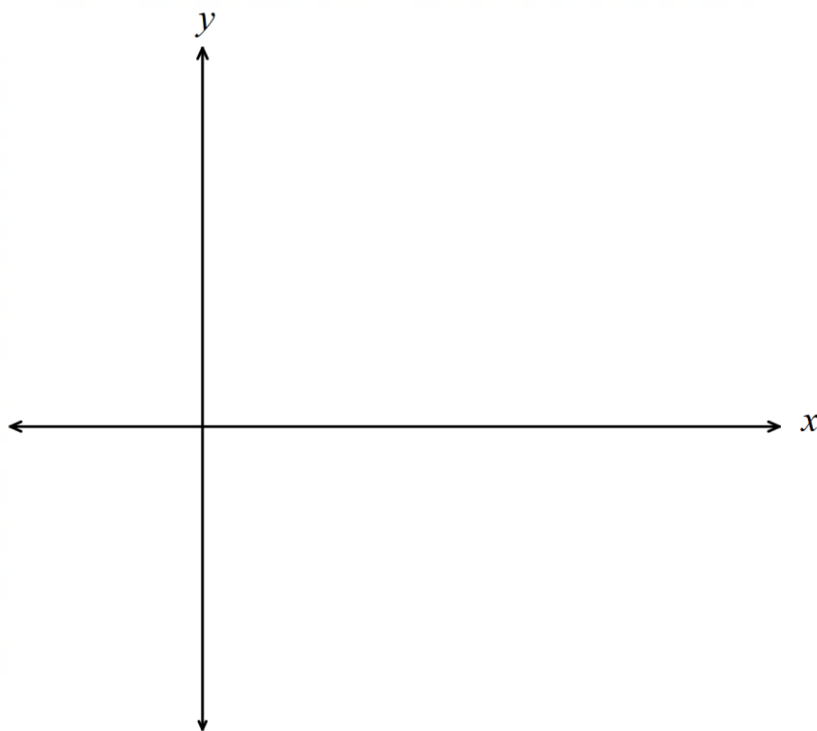
a) Write the equation of the transformed graph, in the form of $y = g(x)$. 2

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b) Sketch $y = g(x)$, for $0 \leq x \leq 2\pi$. 2



c) Hence write the range of $y = g(x)$. 1

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Question 9 (4 marks)

A man has planted a tree that was 1.2 m tall. It grew 90 cm in the first year. Each subsequent year it grew 20% less than it did in the previous year.

a) How much did the tree grow in the 5th year after it was planted? **1**

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b) Calculate the growth of the tree over the first 5 years after it was planted. **1**

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Question 9 continues on the following page

Question 9 (continues)

- c) Find the maximum height that the tree can grow. 2

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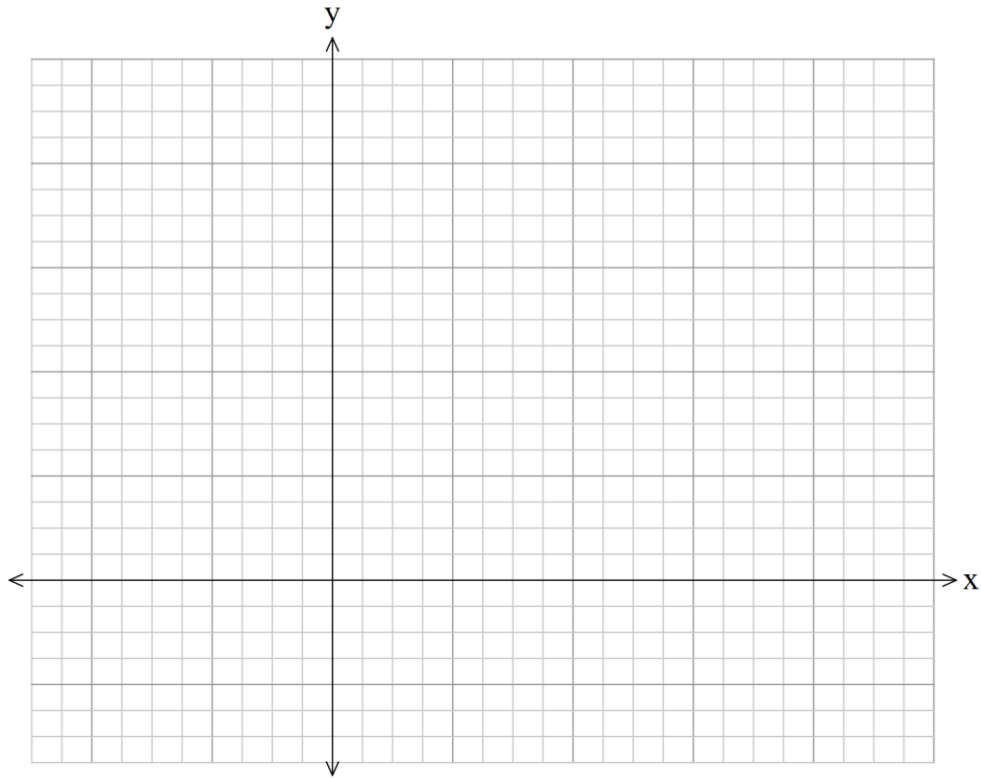
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Question 10 (5 marks)

a) Sketch the function $f(x) = (x-4)^2 + 2$ and $g(x) = 13 - 2x$, on the number plane given below.

Clearly label x and y intercepts.

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b) Find the point of intersection of $f(x) = (x-4)^2 + 2$ and $g(x) = 13 - 2x$, Hence or otherwise using the graph above, find the values of x that satisfy the inequality

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$$(x-4)^2 + 2 < 13 - 2x.$$

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Question 11 (6 marks)

Consider the function $y = \frac{5-3x}{(x-1)(x-3)}$.

- a) State the zeroes of the above function.

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- b) Find the equations of the vertical and horizontal asymptotes.

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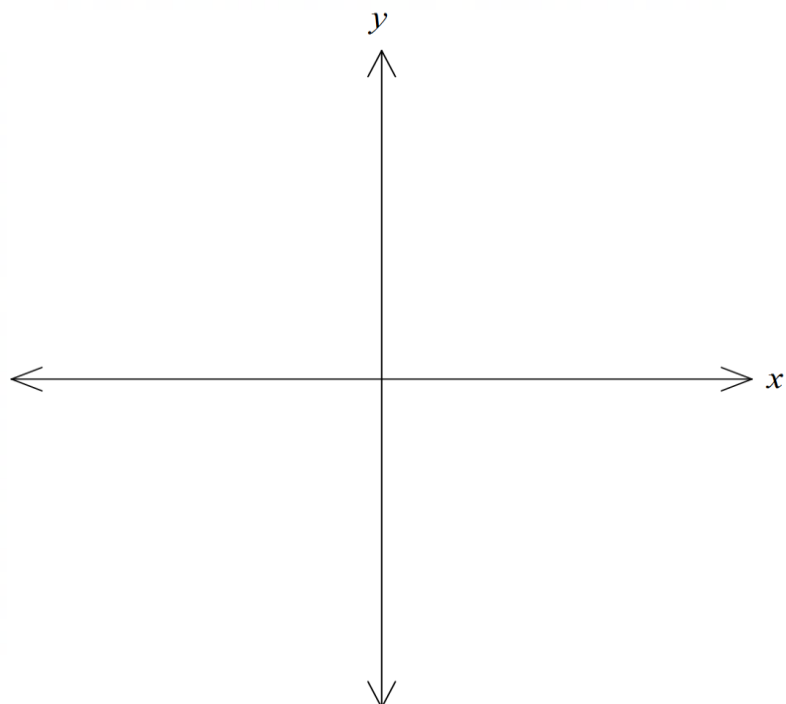
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- c) Using the above information, sketch the function $y = \frac{5-3x}{(x-1)(x-3)}$. Show all the above features including intercepts. (Without using calculus)

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Question 12 (5 marks)

The sum of the first n terms of a series is given by $S_n = \frac{1}{n(n+1)}$.

- a) Show that $S_{n-1} = \frac{1}{n(n-1)}$ for $n \geq 2$. **1**

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- b) Show that the n^{th} term, T_n , can be written as $T_n = \frac{2}{n(1-n^2)}$ for $n \geq 2$. **2**

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Question 12 continues on the following page

Question 12 (continues)

c) Hence, or otherwise, evaluate $\sum_{n=10}^{30} \frac{1}{n(1-n^2)}$.

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End of Question 12

End of Exam

Baulkham Hills High School
Task 1 December Examination 2024

Marking Guideline- Yr 12 Mathematics Advanced

Section I (5 marks)

Award 1 marks to each correct answer.

Answers: 1.B 2.D 3.C 4. C 5.A

Question	Suggested solutions	Answer
1	$2 + 4 + 6 + \dots$ $S_n = \frac{n}{2}(2a + (n-1)d)$ $= \frac{n}{2}(2 \times 2 + (n-1) \times 2)$ $= \frac{n}{2}(4 + 2n)$ $= n(n+1)$	B
2	Amplitude = 1, Period = π and phase shift is $\frac{\pi}{4}$ to the left	D
3	$r = 2m$ $\therefore -1 < 2m < 1$ $-\frac{1}{2} < m < \frac{1}{2}$	C
4	Replace x with $x - 4$ as translated 4 units to the right. $y = \frac{5}{x - 4 + 2}$ $= \frac{5}{x - 2}$ Then vertical dilation by $\frac{1}{2}$ $2y = \frac{5}{x - 2}$	C
5	$c = 2$ $b = -3$ $y = \frac{k}{x - 3} + 2$ y intercept: $\frac{10}{3} = \frac{k}{-3} + 2$ $-10 = k - 6$ $k = -4$	A

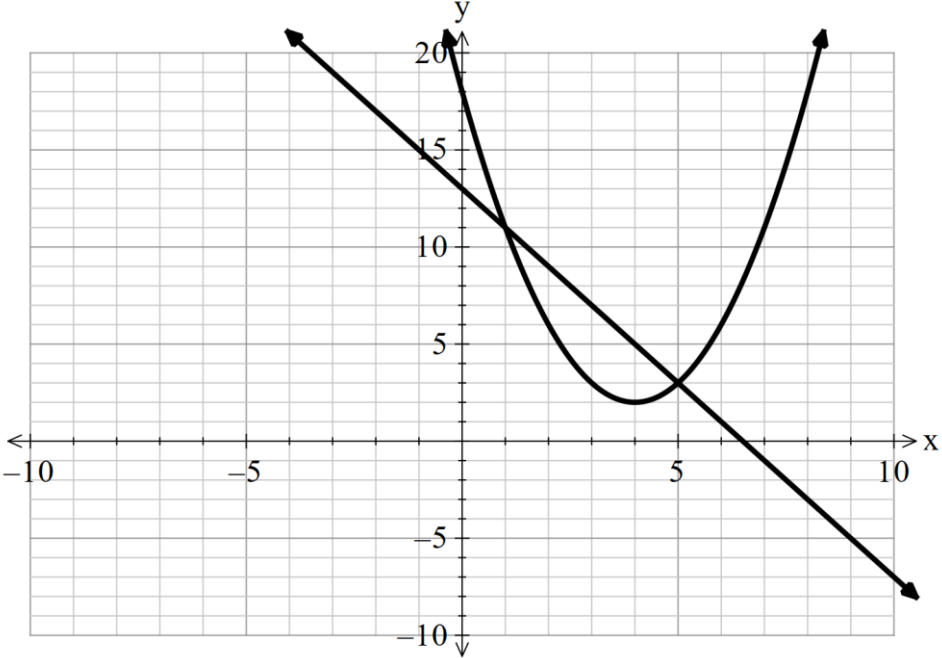
Section II (33 marks)

In all questions, award full marks for correct answers with necessary working.

Use the suggested solutions in conjunction with the marking criteria.

Question	Suggested Solutions	Marking criteria
6a	$\frac{T_2}{T_1} = \frac{\frac{x}{4}}{x} = \frac{1}{4}$ $\frac{T_3}{T_2} = \frac{\frac{x}{16}}{\frac{x}{4}} = \frac{1}{4}$ $\therefore \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{1}{4}$ Hence geometric progression	1- Correct test for GP
6b	$S_{\infty} = \frac{a}{1-r}$ $\frac{7}{8} = \frac{x}{1 - \frac{1}{4}}$ $\frac{7}{8} = \frac{4x}{3}$ $x = \frac{21}{32}$	2- correct solution 1- correctly applying S_{∞}
7a	The weight that Jack lifted each day form an arithmetic sequence with $a = 120$ kg and $d = 1.5$ kg. On 11th day he lifted $T_{11} = a + 10d$ $= 120 + 10 \times 1.5 = 135 \text{ Kg}$	1- correct solution
7b	$150 = 120 + (n-1) \times 1.5$ $30 = (n-1) \times 1.5$ $20 = n-1$ $n = 21$ $\therefore$ Jack first lifts 150 kg on 21st day.	1-correct solution

7c	The weight that Jack lifted each day form an arithmetic sequence with $a = 120\text{kg}$ and $d = 1.5\text{kg}$. On 11th day he lifted $T_{11} = a + 10d$ $= 120 + 10 \times 1.5 = 135\text{Kg}$ $= \frac{21}{2}(120 + 150)$ $= 2835\text{kg.}$ In the first 21 days he lifted $120 + 121.5 + \dots + 150$ On the remaining 7 days he lifted $150 \times 7 = 1050\text{kg}$ So total weight lifted is $2835 + 1050 = 3885\text{kg}$ Hence Jack is correct as he has lifted 3885 kg which is more than elephants weight	2- correct solution 1- correctly finding the sum of 21 days or correctly finds S_{28}.
8a	$y = 3 - \cos 2x.$	2- correct answer 1- If two of the transformations are correct.
8b		2- correct graph 1- If two of the following three are correct • Period • Reflection in the x-axis • Vertical translation of 3 units up
8c	Range $2 \leq y \leq 4.$	1- Correct solution
9a	$T_n = ar^{n-1}$ $T_5 = 90 \times (0.8)^{5-1}$ $= 36.864\text{cm}$	1- Correct solution
9b	$h = S_5$ $= \frac{90 \times (1 - 0.8^5)}{1 - 0.8}$ $= 302.544\text{cm}$	1- Correct solution
9c	Maximum height $h = 120 + S_{\infty}$ $= 120 + \frac{90}{1 - 0.8}$ $= 570\text{cm or } 5.7\text{m.}$	2- Correct solution 1- correctly finding value of S_{∞}.

10a		3- graphs with correct x and y intercepts. 2- correct graphs without the intercepts 1- 1 correct graph with intercepts
10c	$(x-4)^2 + 2 = 13 - 2x$ $x^2 - 8x + 18 = 13 - 2x$ $x^2 - 6x + 5 = 0$ $x = 1 \qquad x = 5$ $y = 11 \qquad y = 1$ $(1, 11) \quad (5, 1)$ $1 < x < 5$	2- Correct solution 1- correctly recognising point of intersection.
11a	$y = \frac{5-3x}{(x-1)(x-3)}$ Zeros: $x = \frac{5}{3}$	1- Correct solution
11b	Vertical: $x = 1$ and 3 Horizontal: $\lim_{x \rightarrow \infty} \frac{5-3x}{x^2-4x+3}$ $= \lim_{x \rightarrow \infty} \frac{\frac{5}{x^2} - \frac{3}{x}}{\frac{x^2}{x^2} - \frac{4x}{x^2} + \frac{3}{x^2}}$ $= \lim_{x \rightarrow \infty} \frac{\frac{5}{x^2} - \frac{3}{x}}{1 - \frac{4}{x} + \frac{3}{x^2}} = 0$ $\therefore y = 0$ is the horizontal asymptote	2- correct solution 1- correctly find both vertical asymptotes or 1 vertical asymptote and 1 horizontal asymptote

11c		3- correct graph with asymptotes shown and intercepts labelled 2- correct graph without the features 1- correct asymptotes shown or equivalent
12a	$S_n = \frac{1}{n(n+1)} \quad \text{for } n \geq 2$ $S_{n-1} = \frac{1}{(n-1)(n-1+1)}$ $= \frac{1}{(n-1)n}$	1- correct solution
12b	$t_n = S_n - S_{n-1}$ $= \frac{1}{n(n+1)} - \frac{1}{(n-1)n}$ $= \frac{(n-1) - (n+1)}{n(n-1)(n+1)}$ $= \frac{-2}{n(n^2-1)}$ $= \frac{2}{n(1-n^2)}$	2- correct solution 1- some progress towards solution
12c	$\sum_{n=10}^{30} \frac{1}{n(1-n^2)} = \frac{1}{2} \sum_{n=10}^{30} \frac{2}{n(1-n^2)}$ $= \frac{1}{2} (S_{30} - S_9)$ $= \frac{1}{2} \left(\frac{1}{30 \times 31} - \frac{1}{9 \times 10} \right)$ $= \frac{1}{2} \left(\frac{1}{930} - \frac{1}{90} \right)$ $= \frac{-7}{1395}$	2- correct solution 1- Correctly recognising $S_{30} - S_9$ or correctly using part b