



**BAULKHAM HILLS HIGH SCHOOL**

**2024**

**YEAR 12 ASSESSMENT TASK 3**

# Mathematics Advanced

## **General Instructions**

- Reading time – 5 minutes.
- Working time – 1 hour.
- Write using black or blue pen.
- NESA-approved calculators may be used.
- Answer all questions in the answer booklet provided.
- Show all necessary working in Questions 6-13.
- Marks may be deducted for careless or badly arranged work.
- A reference sheet is provided.

**Total marks – 46**

**The exam consists of 18 pages.**

This paper consists of TWO sections.

## **Section I– Page 5 - 7 (5 marks)**

### **Questions 1 - 5**

- Attempt Questions 1 - 5  
Allow about 8 minutes for this section.

## **Section II – Pages 8 – 19 (41 marks)**

- Attempt questions 6 - 13  
Allow about 52 minutes for this section.



# BAULKHAM HILLS HIGH SCHOOL

## YEAR 12 ASSESSMENT TASK 3 2024

NESA#: \_\_\_\_\_

Teacher: \_\_\_\_\_

### Mathematics Advanced Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:  $2 + 4 =$  (A) 2 (B) 6 (C) 8 (D) 9  
A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☒ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A ☒ B ☒ C ☐ D ☐  
*correct* (arrow pointing to B)

- Start Here →
1. A ☐ B ☐ C ☐ D ☐
  2. A ☐ B ☐ C ☐ D ☐
  3. A ☐ B ☐ C ☐ D ☐
  4. A ☐ B ☐ C ☐ D ☐
  5. A ☐ B ☐ C ☐ D ☐

## **Section I - 5 marks**

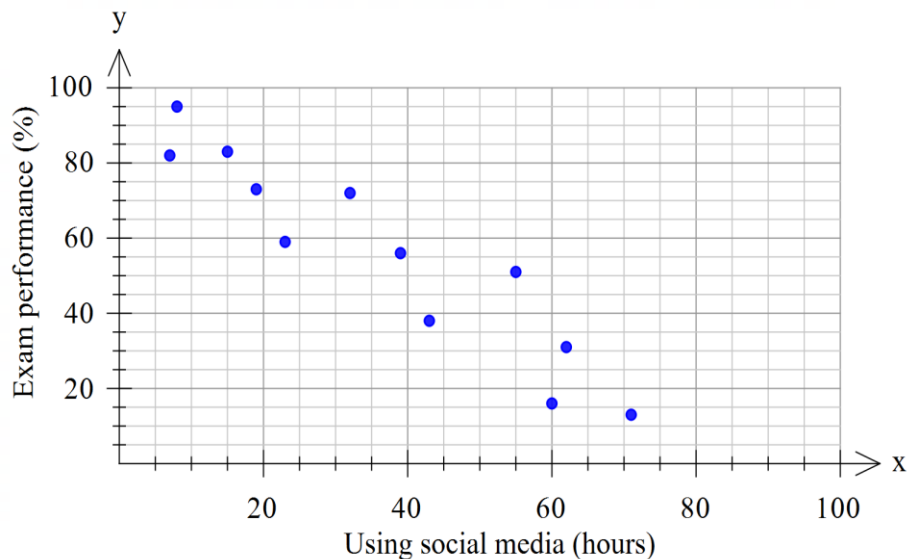
Use the multiple-choice answer sheet for questions 1-5.

1. A set of data is shown below.

| <i>Score</i> | <i>Frequency</i> | <i>Cumulative Frequency</i> |
|--------------|------------------|-----------------------------|
| 10           | 6                | 6                           |
| 11           | 18               | 24                          |
| 12           | 6                | 30                          |
| 13           | 5                | 35                          |

Which pair of measures have the same value?

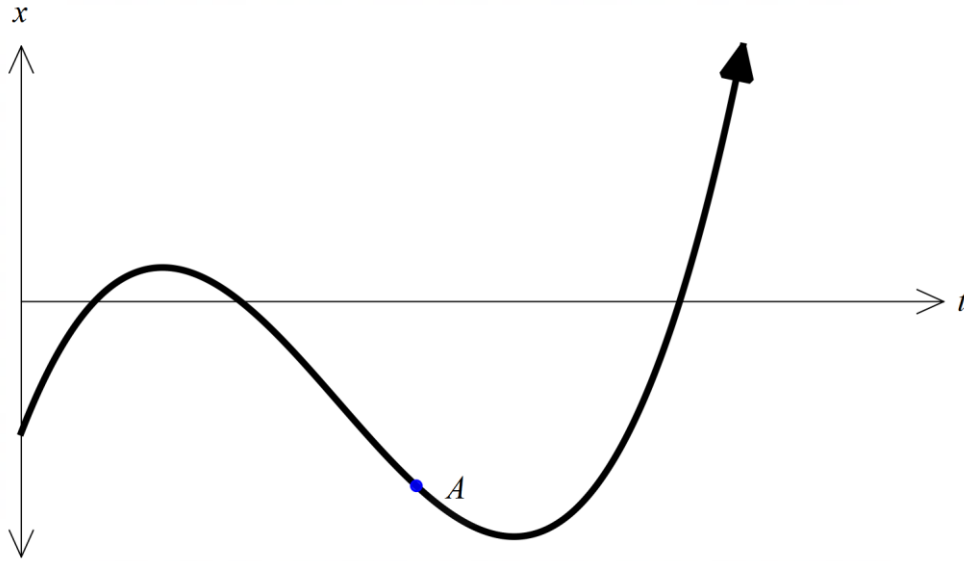
- (A) The median and upper quartile.
  - (B) The mean and upper quartile.
  - (C) The median and lower quartile.
  - (D) The mean and lower quartile.
2. Students were surveyed about their use of social media and their performance in a recent examination. The results are shown in the scatterplot below.



Which of the following best describes the correlation between the examination performance and the use of social media?

- (A) A strong negative correlation
- (B) A weak negative correlation
- (C) A strong positive correlation.
- (D) No correlation.

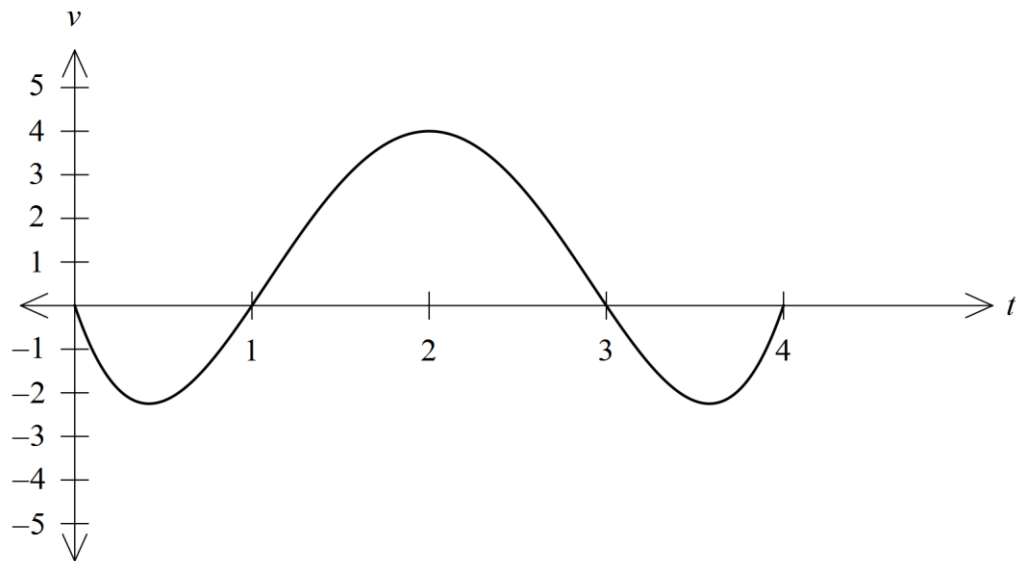
3. The diagram below shows the displacement,  $x$  metres, of a moving object at time  $t$  seconds.



Which of the following statements describes the motion of the object at point A.

- (A) Velocity is negative and acceleration is positive.
  - (B) Velocity is negative and acceleration is negative.
  - (C) Velocity is positive and acceleration is negative.
  - (D) Velocity is positive and acceleration is positive.
4. What amount to the nearest dollar must be invested now at 6% per annum, compounded monthly, so that in four years time it will have grown to \$50,000?
- (A) \$39,605
  - (B) \$39,355
  - (C) \$47,095
  - (D) \$49,012

5. A particle moves along a straight line. Its velocity  $v$  at time  $t$  is shown in the graph below.



For what value of  $t$  is the displacement of the particle a maximum?

- (A)  $t = 0$ .
- (B)  $t = 1$ .
- (C)  $t = 3$ .
- (D)  $t = 4$ .

**End of Section I**

## **Section II**

**41 marks**

**Attempt Questions 6 - 13**

**Allow about 52 minutes for this section**

### **Instructions:**

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing paper is available. If you use extra paper, write your NESA number, your teacher's name on the paper and indicate clearly which question you are answering.

### **Question 6 (6 marks)**

A particle is moving in a straight line with velocity  $v = 2e^t + 4e^{-t}$ ,  $t$  is in seconds and  $v$  m/s. Initially, the particle is at the origin  $O$ .

(a) What is its initial velocity?

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(b) Is the particle ever at rest? Give reasons.

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(c) Find the displacement function,  $x$ , of the particle, at time  $t$  seconds. 2

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(d) Find the time when the particle is at  $x = 4$ . (Give your answer in exact form) 2

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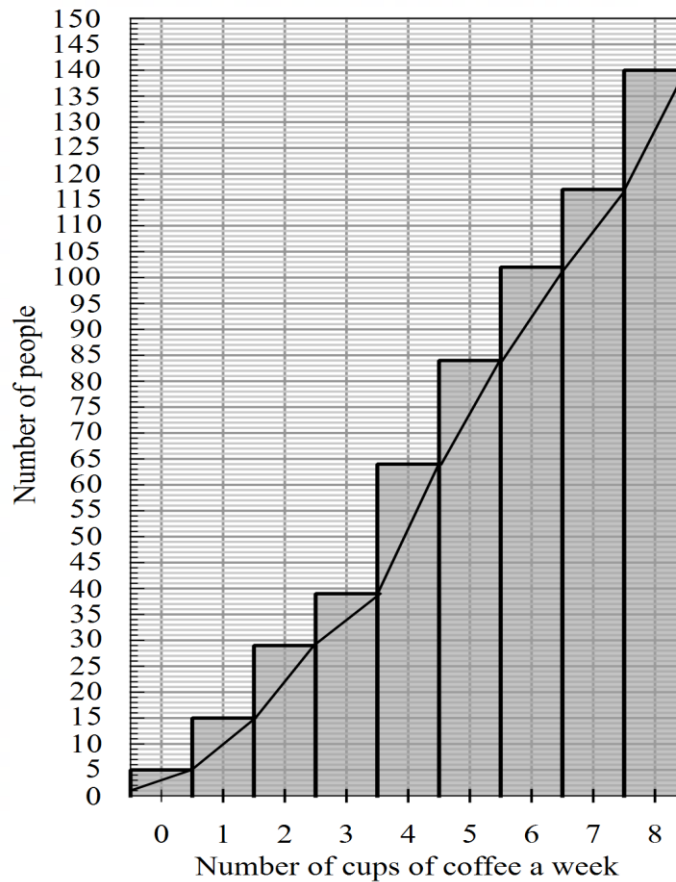
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**Question 7** (4 marks)

Minh is doing research on coffee consumption by university students. He asks his course mates to indicate how many cups of coffee they typically consume each week. The data collected is displayed below in a cumulative frequency histogram and ogive.



- (a) Find the lower quartile, median and upper quartile.

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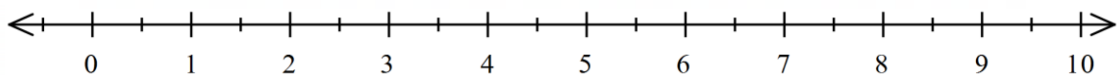
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- (b) Construct a box-plot to represent the data.

2



**Question 8** (4 marks)

When Fred was born, his parents opened a new savings account and made a deposit of \$300. At the start of each subsequent six-month period, his parents make a deposit of \$300. The account earns an interest rate of 6% per annum, compounded half-yearly.

Let  $A_n$  be the amount (in dollars) in the account at the end of  $n^{\text{th}}$  half-year.

- (a) Show that  $A_2 = 300 \times (1.03)^2 + 300 \times (1.03)$ . 1

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- (b) Show that  $A_n = 10300(1.03^n - 1)$ . 2

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- (c) Show that the amount available for withdrawal at the end of the 18<sup>th</sup> year will be \$19552 to the nearest dollar. (Assume that no withdrawals are made from the fund during this time.) 1

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**Question 9** (4 marks)

Caitlin is a scientist studying the growth of a particular type of giraffe over several years. The data she recorded is shown in the table below:

Note: She did not record the height of the giraffe in the 5<sup>th</sup> year.

|                               |     |     |     |     |   |     |     |     |     |     |
|-------------------------------|-----|-----|-----|-----|---|-----|-----|-----|-----|-----|
| Years since born, $t$         | 1   | 2   | 3   | 4   | 5 | 6   | 7   | 8   | 9   | 10  |
| Height of giraffe, $h$ metres | 0.6 | 1.5 | 2.1 | 2.7 | * | 3.1 | 3.6 | 4.2 | 4.7 | 5.0 |

- (a) Calculate the Pearson's correlation coefficient. Give your answer correct to 3 decimal places. **1**

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- (b) Find the equation of the least-squares line of best fit, expressing the height ( $h$ ) in terms of years ( $t$ ). Give your answer correct to 2 decimal places. **1**

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- (c) Use the equation of the least-squares line of best fit to predict the giraffe's height in the fifth year. Give your answer correct to 1 decimal place. **1**

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- (d) Estimate how many years it will take for the giraffe to reach a height of 6.1 metres. Give your answer correct to 1 decimal place. **1**

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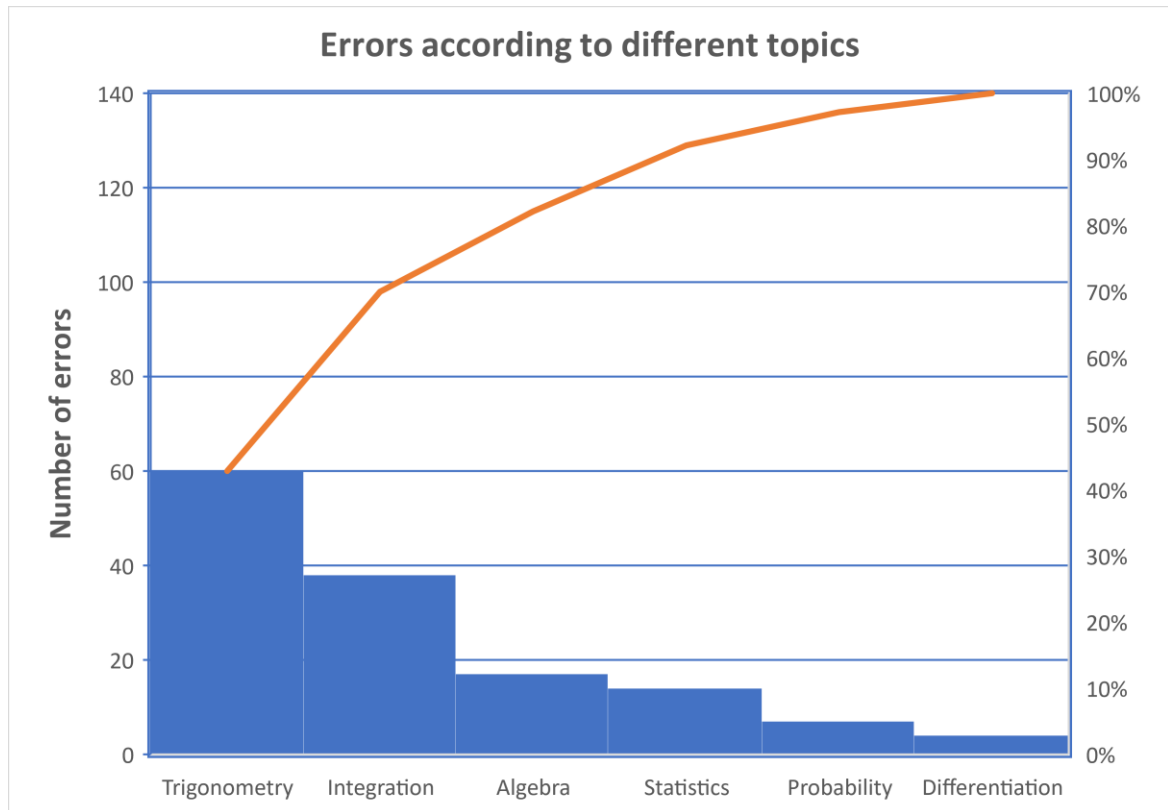
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**Question 10** (2 marks)

A student analyses errors according to different topics on his recent mathematics examinations.

The data is shown in the following Pareto chart:



- (a) Approximately what percentage of errors were from the Integration topic?

**1**

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- (b) The student wishes to focus on as few topics as possible that address 80% of errors in preparation for their next examination. What topics should be the focus of his revision?

**1**

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**Question 11** (8 marks)

An object that was initially at the origin moves with velocity ( $v$  m/s) given by  $v = 24 + 10t - t^2$ .

- (a) Find the maximum speed of the object. 2

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- (b) Find the acceleration when the object is at rest. 3

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- (c) Find the total distance travelled by the object during the first 15 seconds.

[illegible]

**Question 12** (7 marks)

Arjun borrows \$1,000,000 for a period of 10 years at an interest rate of 0.5% per month, compounded monthly. At the end of each month, a repayment of \$  $M$  is made.

Let \$  $A_n$  be the amount owing after the  $n^{\text{th}}$  repayment has been made.

- (a) Show that  $A_2 = 1000000(1.005)^2 - M(1.005)^1 - M$ . 1

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- (b) Show that  $A_n = (1000000 - 200M)(1.005)^n + 200M$ . 2

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- (c) Hence show that the monthly instalment  $M$  is given by:  $M = \frac{5000(1.005)^{120}}{1.005^{120} - 1}$ . 2

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- (d) Find the amount still owing to the bank after 5 years, correct to two decimal places.

[illegible]

**Question 13** (6 marks)

The acceleration ( $a \text{ m/s}^2$ ) of an object at time  $t$  seconds is given by  $a = \frac{-1}{(5t+1)^2}$ . The object is initially 3 metres to the left of the origin travelling with velocity 1 m/s.

- (a) Find an expression for the object's velocity ( $v$ ) at time  $t$ .

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- (b) Find the limiting speed of the object.

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(c) Find the position of the object at the end of the 10<sup>th</sup> second. (Answer to the nearest metre) 3

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**End of Exam**



Solutions

**BAULKHAM HILLS HIGH SCHOOL**

**2024**

**YEAR 12 ASSESSMENT TASK 3**

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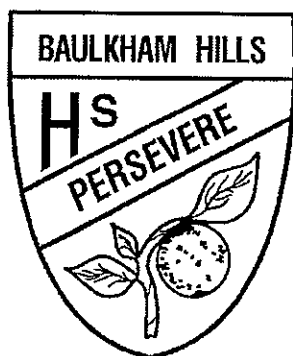
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**Questions 1 - 5**

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Allow about 8 minutes for this section.

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# BAULKHAM HILLS HIGH SCHOOL

## YEAR 12 ASSESSMENT TASK 3 2024

NESA#: \_\_\_\_\_

Teacher: \_\_\_\_\_

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A ☒ B ☒ C ☐ D ☐  
correct

- Start Here** → 1. A ☐ B ☐ C ☒ D ☐  
2. A ☒ B ☐ C ☐ D ☐  
3. A ☒ B ☐ C ☐ D ☐  
4. A ☐ B ☒ C ☐ D ☐  
5. A ☐ B ☐ C ☒ D ☐

## Section I - 5 marks

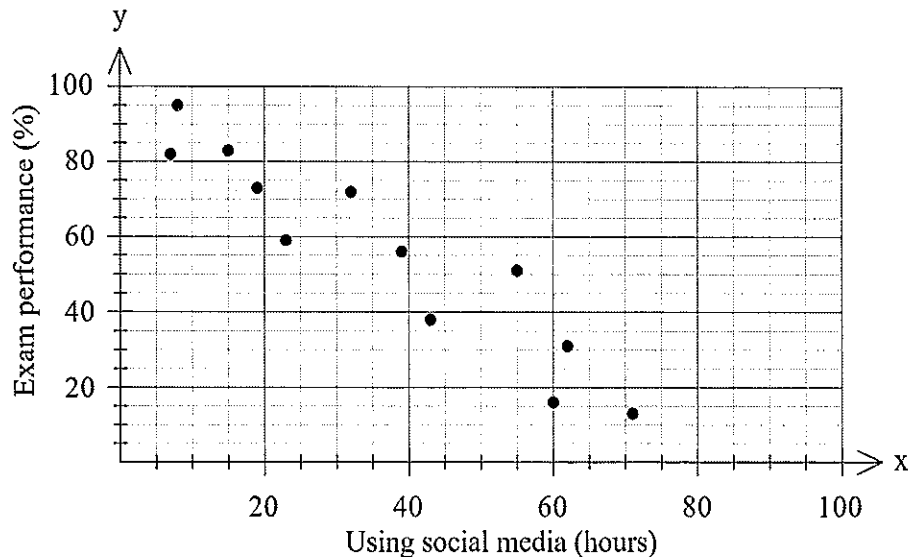
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1. A set of data is shown below.

| Score | Frequency | Cumulative Frequency |
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| 12    | 6         | 30                   |
| 13    | 5         | 35                   |

Which pair of measures have the same value?

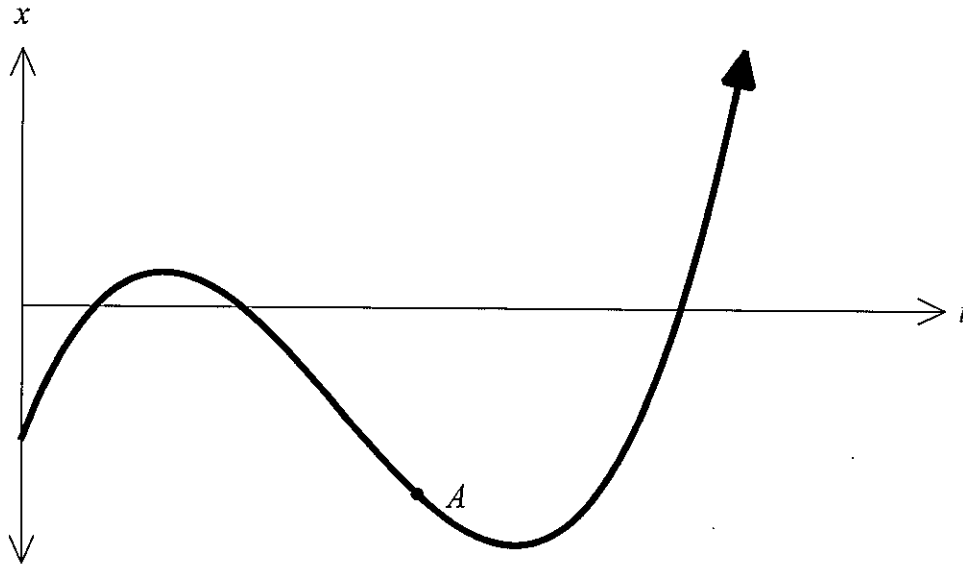
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2. Students were surveyed about their use of social media and their performance in a recent examination. The results are shown in the scatterplot below.



Which of the following best describes the correlation between the examination performance and the use of social media?

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(C) A strong positive correlation.  
(D) No correlation.

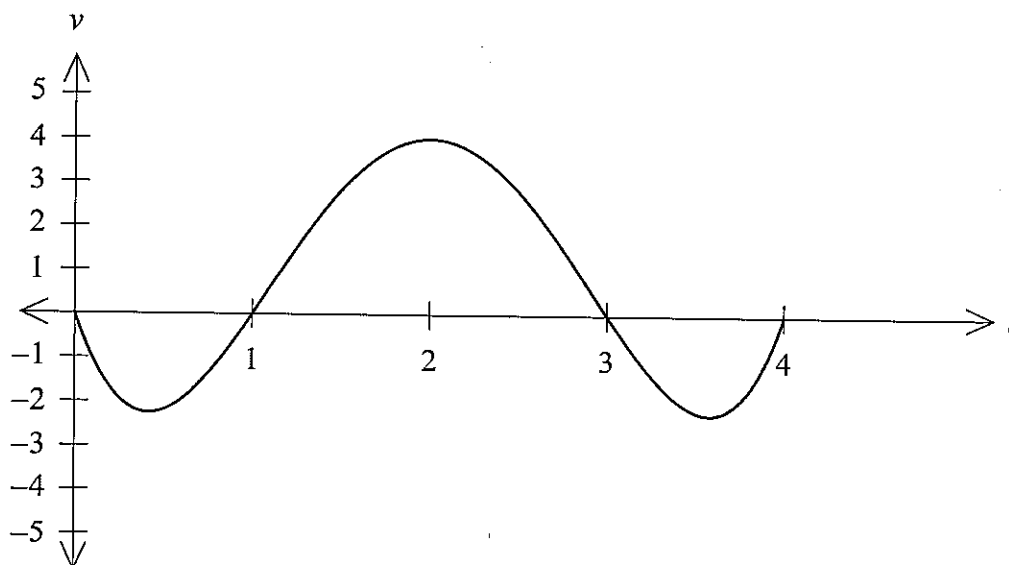
3. The diagram below shows the displacement,  $x$  metres, of a moving object at time  $t$  seconds.



Which of the following statements describes the motion of the object at point  $A$ .

- ☒ (A) Velocity is negative and acceleration is positive.
- (B) Velocity is negative and acceleration is negative.
- (C) Velocity is positive and acceleration is negative.
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4. What amount to the nearest dollar must be invested now at 6% per annum, compounded monthly, so that in four years time it will have grown to \$50,000?
- ☒ (A) \$39,605
- ☒ (B) \$39,355
- (C) \$47,095
- (D) \$49,012

5. A particle moves along a straight line. Its velocity  $v$  at time  $t$  is shown in the graph below.



For what value of  $t$  is the displacement of the particle a maximum?

- (A)  $t = 0$ .
- (B)  $t = 1$ .
- ☒ (C)  $t = 3$ .
- (D)  $t = 4$ .

**End of Section I**

## Section II

41 marks

Attempt Questions 6 - 13

Allow about 52 minutes for this section

### Instructions:

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
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### Question 6 (6 marks)

A particle is moving in a straight line with velocity  $v = 2e^t + 4e^{-t}$ ,  $t$  is in seconds and  $v$  m/s. Initially, the particle is at the origin  $O$ .

- (a) What is its initial velocity?

1

$v(0) = 2e^0 + 4e^0$   
 $= 6$   
1 - correct solution

- (b) Is the particle ever at rest? Give reasons.

1

Since  $2e^t > 0$  and  $4e^{-t} > 0$  for all values of  $t$  must  
then  $2e^t + 4e^{-t} > 0$  for all values of  $t$  } state both  
 $\therefore$  the particle will never be at rest.

If proving  $v=0$  then must show  $e^{2t} = -2$ .

- (c) Find the displacement function,  $x$ , of the particle, at time  $t$  seconds.

2

$$x = \int (2e^t + 4e^{-t}) dt$$
$$= 2e^t - 4e^{-t} + C$$

$$t=0, x=0$$

$$0 = 2e^0 - 4e^0 + C$$

$$= 2 - 4 + C$$

$$\therefore C = 2$$

$$x = 2e^t - 4e^{-t} + 2$$

2 - correct solution

1 - obtain

$$x = 2e^t - 4e^{-t} + C$$

- (d) Find the time when the particle is at  $x = 4$ . (Give your answer in exact form)

2

$$x=4, 2e^t - 4e^{-t} + 2 = 4$$

$$2(e^t - 2e^{-t} + 1) = 0$$

$$\therefore e^t - \frac{2}{e^t} - 1 = 0$$

$$e^{2t} - e^t - 2 = 0$$

$$(e^t - 2)(e^t + 1) = 0 \quad e^t + 1 > 0$$

$$\therefore e^t - 2 = 0$$

$$e^t = 2$$

$$t = \log_e 2$$

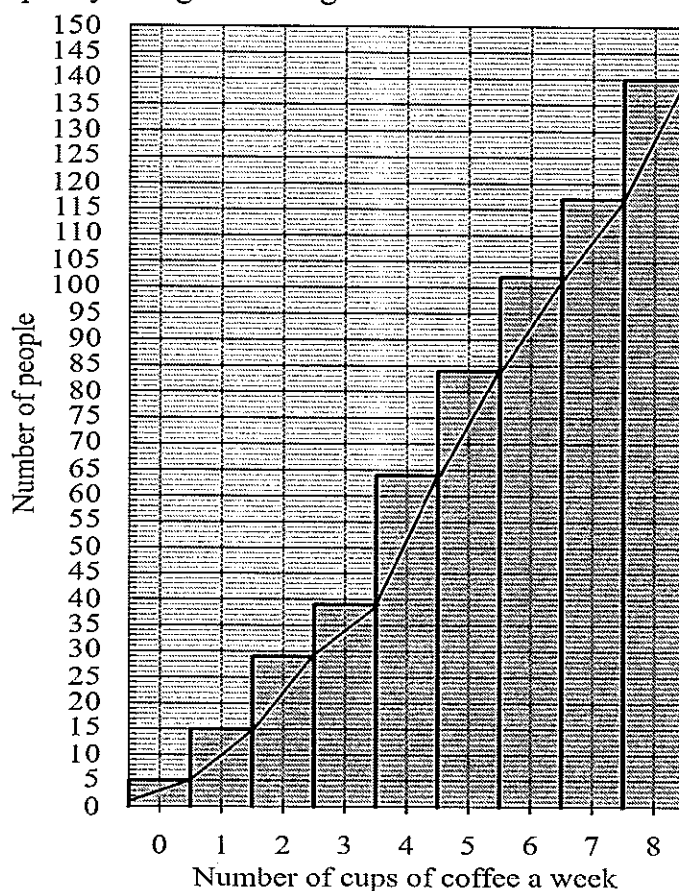
2 - correct solution

1 - obtain

$$e^{2t} - e^t - 2 = 0$$

**Question 7 (4 marks)**

Minh is doing research on coffee consumption by university students. He asks his course mates to indicate how many cups of coffee they typically consume each week. The data collected is displayed below in a cumulative frequency histogram and ogive.



- (a) Find the lower quartile, median and upper quartile.

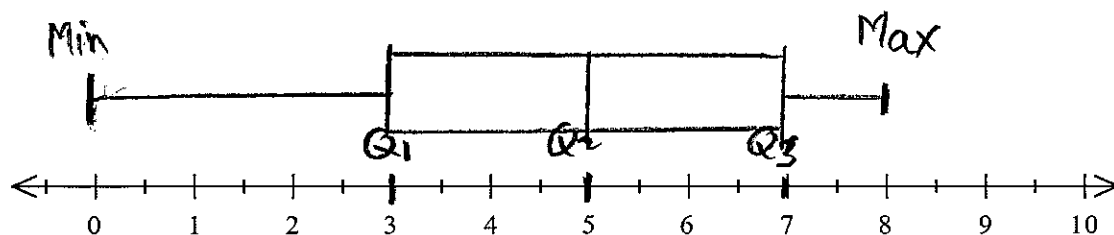
2

$Q_1 = 3$   
 $Q_2 = 5$   
 $Q_3 = 7$

2 - obtain all three  $Q_1, Q_2, Q_3$  correctly.  
 1 - obtain any two  $Q$ 's correctly  
 - obtain correct positions.

- (b) Construct a box-plot to represent the data.

2 - Box-plot above axis with correct extremes.  
 1 - plotted box or extremes.



**Question 8** (4 marks)

When Fred was born, his parents opened a new savings account and made a deposit of \$300. At the start of each subsequent six-month period, his parents make a deposit of \$300. The account earns an interest rate of 6% per annum, compounded half-yearly.

Let  $A_n$  be the amount (in dollars) in the account at the end of  $n^{\text{th}}$  half-year.

- (a) Show that  $A_2 = 300 \times (1.03)^2 + 300 \times (1.03)$ .

1

$$\begin{aligned} A_1 &= 300(1.03) \\ A_2 &= (A_1 + 300) \times 1.03 \\ &= (300 \times 1.03 + 300) \times 1.03 \\ &= 300 \times 1.03^2 + 300 \times 1.03 \end{aligned}$$

$r = \frac{6\%}{2} = 3\% \text{ per HY}$

- (b) Show that  $A_n = 10300(1.03^n - 1)$ .

2

$$\begin{aligned} A_1 &= 300(1.03) \\ A_2 &= 300 \times 1.03^2 + 300 \times 1.03 \\ &\vdots \\ A_n &= 300 \times 1.03^n + 300 \times 1.03^{n-1} + \dots + 300 \times 1.03 \\ &= 300(1.03^n + 1.03^{n-1} + \dots + 1.03) \\ &= 300 \times 1.03 \frac{(1.03^n - 1)}{1.03 - 1} \\ &= 10300(1.03^n - 1) \end{aligned}$$

2 - correct solution  
1 - write an expression for  $A_n$   
or uses sum of a GP formula without establishing  $A_n$

- (c) Show that the amount available for withdrawal at the end of the 18<sup>th</sup> year will be \$19552 to the nearest dollar. (Assume that no withdrawals are made from the fund during this time.)

1

$$\begin{aligned} A_{36} &= 10300(1.03^{36} - 1) \\ &= 19552 \text{ (nearest dollar)} \end{aligned}$$

**Question 9** (4 marks)

Caitlin is a scientist studying the growth of a particular type of giraffe over several years. The data she recorded is shown in the table below:

Note: She did not record the height of the giraffe in the 5<sup>th</sup> year.

|                               |     |     |     |     |   |     |     |     |     |     |
|-------------------------------|-----|-----|-----|-----|---|-----|-----|-----|-----|-----|
| Years since born, $t$         | 1   | 2   | 3   | 4   | 5 | 6   | 7   | 8   | 9   | 10  |
| Height of giraffe, $h$ metres | 0.6 | 1.5 | 2.1 | 2.7 | * | 3.1 | 3.6 | 4.2 | 4.7 | 5.0 |

- (a) Calculate the Pearson's correlation coefficient. Give your answer correct to 3 decimal places.

1

$$r = 0.989$$

- (b) Find the equation of the least-squares line of best fit, expressing the height ( $h$ ) in terms of years ( $t$ ). Give your answer correct to 2 decimal places.

1

$$a = 0.5135135135$$

$$b = 0.4575675676$$

$$h = 0.46t + 0.51$$

- (c) Use the equation of the least-squares line of best fit to predict the giraffe's height in the fifth year. Give your answer correct to 1 decimal place.

1

$$h = 0.46 \times 5 + 0.51$$

$$= 2.8 \text{ m}$$

- (d) Estimate how many years it will take for the giraffe to reach a height of 6.1 metres. Give your answer correct to 1 decimal place.

1

$$h = 0.46t + 0.51$$

$$6.1 = 0.46t + 0.51$$

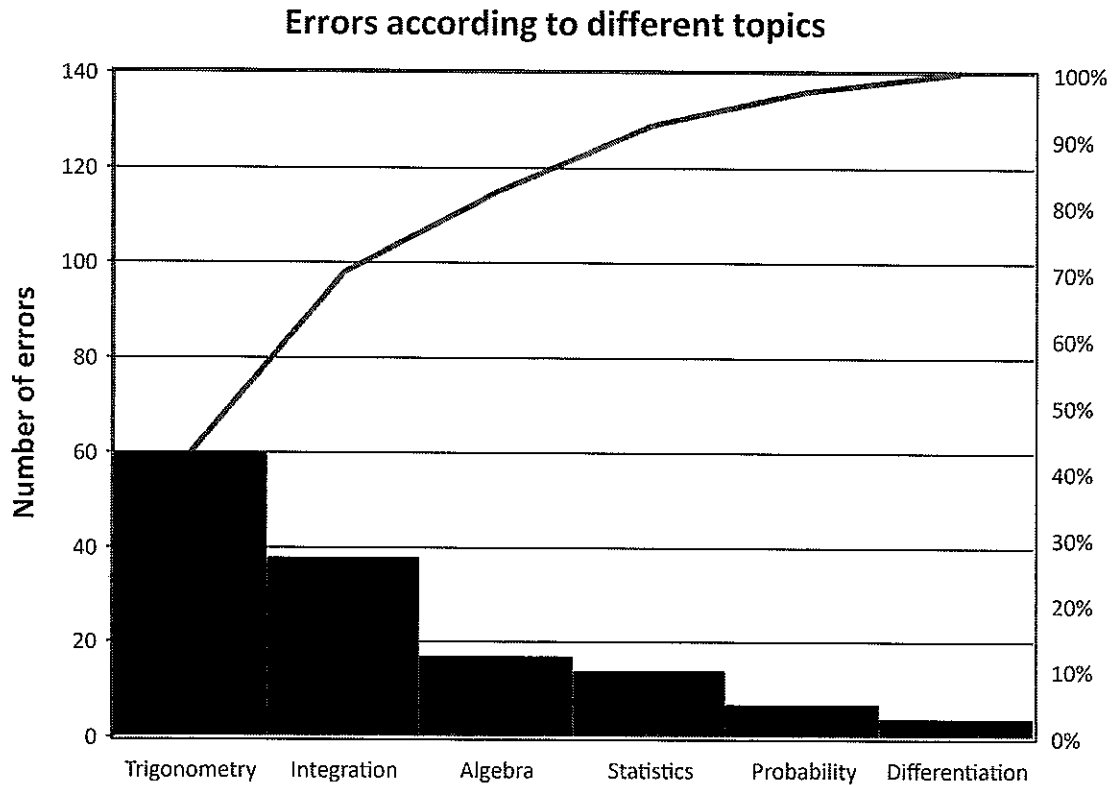
$$t = \frac{6.1 - 0.51}{0.46}$$

$$= 12.2 \text{ yrs}$$

**Question 10 (2 marks)**

A student analyses errors according to different topics on his recent mathematics examinations.

The data is shown in the following Pareto chart:



- (a) Approximately what percentage of errors were from the Integration topic?

1

$\approx 27\%$

accept 27%–28%

- (b) The student wishes to focus on as few topics as possible that address 80% of errors in preparation for their next examination. What topics should be the focus of his revision?

1

either : Trigonometry ( $\approx 43\%$ ) + Integration ( $\approx 27\%$ ) + Algebra ( $\approx 12\%$ ) = 82%

or : Trigonometry ( $\approx 43\%$ ) + Integration ( $\approx 27\%$ ) + stat ( $\approx 10\%$ ) = 80%

Question 11 (8 marks)

An object that was initially at the origin moves with velocity ( $v$  m/s) given by  $v = 24 + 10t - t^2$ .

- (a) Find the maximum speed of the object.

$$v = -t^2 + 10t + 24$$

2

The maximum speed is at the turning point

$$t = -\frac{b}{2a}$$
$$= -\frac{10}{2 \times -1}$$
$$= 5$$

2 - correct solution

1 - obtain  $t = 5$

$$v_{\max} = 24 + 10 \times 5 - 5^2$$
$$= 49 \text{ m/s}$$

- (b) Find the acceleration when the object is at rest.

3

The object is at rest when  $v = 0$

$$-t^2 + 10t + 24 = 0$$

$$t^2 - 10t - 24 = 0$$

3 - correct solution

$$(t - 12)(t + 2) = 0$$

2 - obtain  $t = 12$  and  $a = -2t + 10$

$$t = 12 \quad t \geq 0$$

1 - obtain  $-t^2 + 10t + 24 = 0$

$$a = \frac{dv}{dt}$$

or  $t = 12$

$$= -2t + 10$$

$$= -2 \times 12 + 10$$

$$= -14 \text{ m/s}^2$$

(c) Find the total distance travelled by the object during the first 15 seconds.

3

$$x = \int v dt$$
$$= \int (-t^2 + 10t + 24) dt$$

$$= -\frac{t^3}{3} + 5t^2 + 24t + C$$

$$t=0, x=0$$

$$0 = 0 + 0 + 0 + C$$

$$\therefore C = 0$$

$$x = -\frac{t^3}{3} + 5t^2 + 24t$$

$$t=0, x=0$$

$$t=12, x = -\frac{12^3}{3} + 5 \times 12^2 + 24 \times 12$$

$$= 432$$

(the object is at rest at  $t=12$  in b))

$$t=15, x = -\frac{15^3}{3} + 5 \times 15^2 + 24 \times 15$$
$$= 360$$

$$\text{Distance travelled} = 432 + (432 - 360)$$
$$= 504 \text{ m}$$

3 - correct solution

2 - correct displacement at  $t=12$   
and at  $t=15$

1 - correct displacement at  $t=15$   
or  $t=12$

**Question 12** (7 marks)

Arjun borrows \$1,000,000 for a period of 10 years at an interest rate of 0.5% per month, compounded monthly. At the end of each month, a repayment of \$  $M$  is made.

Let \$  $A_n$  be the amount owing after the  $n^{\text{th}}$  repayment has been made.

- (a) Show that  $A_2 = 1000000(1.005)^2 - M(1.005) - M$ .  $r = 0.5\% / \text{month}$  1

$$A_1 = 1000000 \times 1.005 - M$$

$$A_2 = A_1 \times 1.005 - M$$

$$= (1000000 \times 1.005 - M) \times 1.005 - M$$

$$= 1000000 \times 1.005^2 - M \times 1.005 - M$$

- (b) Show that  $A_n = (1000000 - 200M)(1.005)^n + 200M$ . 2

$$A_2 = 1000000 \times 1.005^2 - M \times 1.005 - M$$

$$\vdots$$

$$A_n = 1000000 \times 1.005^n - M \times 1.005^{n-1} - M \times 1.005^{n-2} - \dots - M$$

$$= 1000000 \times 1.005^n - M(1.005^{n-1} + 1.005^{n-2} + \dots + 1)$$

$$= 1000000 \times 1.005^n - M \frac{(1.005^n - 1)}{1.005 - 1} \quad \text{2 - correct solution}$$

$$= 1000000 \times 1.005^n - 200M(1.005^n - 1) \quad \text{1 - substantially program to obtain correct solution}$$

$$= (1000000 - 200M) \times 1.005^n + 200M$$

or 1 - obtain  $A_n$  with GP.

- (c) Hence show that the monthly instalment  $M$  is given by:  $M = \frac{5000(1.005)^{120}}{1.005^{120} - 1}$ . 2

After  $n$  instalment  $A_n = 0$

$$(1000000 - 200M) \times 1.005^n + 200M = 0 \quad n = 10 \text{ Yrs}$$

$$1000000 \times 1.005^n - 200M \times 1.005^n + 200M = 0 \quad = 120 \text{ months}$$

$$M = \frac{1000000 \times 1.005^{120}}{200 \times (1.005^{120} - 1)} \quad \text{2 - correct solution}$$

$$= \frac{5000(1.005)^{120}}{1.005^{120} - 1} \quad \text{1 - obtain } A_n = 0$$

- (d) Find the amount still owing to the bank after 5 years, correct to two decimal places.

2

$$\begin{aligned} 5 \text{ yrs} &= 5 \times 12 \\ &= 60 \text{ months} \end{aligned}$$

$$M = \frac{5000 \times 1.005^{120}}{1.005^{120} - 1}$$

$$= 11102.05 \text{ (2dp)}$$

$$A_n = (1000000 - 200M) \times (1.005)^n + 200M$$

$$\begin{aligned} A_{60} &= (1000000 - 200 \times 11102.05) \times 1.005^{60} + 200 \times 11102.05 \\ &= 574259.79 \text{ (2dp)} \end{aligned}$$

2 - correct solution

1 - obtain correct  $M = 11102.05$

**Question 13 (6 marks)**

The acceleration ( $a \text{ m/s}^2$ ) of an object at time  $t$  seconds is given by  $a = \frac{-1}{(5t+1)^2}$ . The object is initially 3 metres to the left of the origin travelling with velocity 1 m/s.

- (a) Find an expression for the object's velocity ( $v$ ) at time  $t$ .

2

$$a = \frac{-1}{(5t+1)^2}$$

$$v = \int a dt$$

$$= \int -(5t+1)^{-2} dt$$

$$= \frac{-(5t+1)^{-2+1}}{(-2+1) \times 5} + C$$

2 - correct solution

1 - obtain  $v = \frac{1}{5(5t+1)} + C$

$$= \frac{1}{5(5t+1)} + C$$

when  $t=0$ ,  $v=1$

$$1 = \frac{1}{5(5 \times 0 + 1)} + C$$

$$C = 0.8$$

$$v = \frac{1}{5(5t+1)} + 0.8$$

- (b) Find the limiting speed of the object.

1

$$v = \frac{1}{5(5t+1)} + 0.8$$

$$\text{as } t \rightarrow \infty, \frac{1}{5(5t+1)} \rightarrow 0$$

$$v \rightarrow 0.8$$

Limiting speed of the object is 0.8 m/s

- (c) Find the position of the object at the end of the 10<sup>th</sup> second. (Answer to the nearest metre)

3

$$v = \frac{1}{5(5t+1)} + 0.8$$

$$x = \int v dt$$

$$= \int \left( \frac{1}{5(5t+1)} + 0.8 \right) dt$$

$$= \frac{1}{5} \times \frac{\log_e(5t+1)}{5} + 0.8t + C$$

$$= \frac{\log_e(5t+1)}{25} + 0.8t + C$$

when  $t=0$ ,  $x=-3$

$$-3 = \frac{\log_e(5 \times 0 + 1)}{25} + 0.8 \times 0 + C$$

$$\therefore C = -3$$

$$x = \frac{1}{25} \times \log_e(5t+1) + 0.8t - 3$$

when  $t=10$

$$x = \frac{1}{25} \log_e(5 \times 10 + 1) + 0.8 \times 10 - 3$$

$$= 5.157 \text{ m}$$

$$\approx 5 \text{ m to the right of the origin}$$

3 - correct solution

2 - obtain

$$x = \frac{1}{25} \log_e(5t+1) + 0.8t - 3$$

1 - obtain

$$x = \frac{1}{25} \log_e(5t+1) + 0.8t + C$$

End of Exam