Year 12 Advanced Maths

Assessment Task 2, 2024

Solutions

1. (B)

Sequence is arithmetic with $a = 187$, $d = (-4)$, $n = 50$.

$$\begin{aligned} T_n &= a + (n-1)d] \\ &= 187 + (50-1)(-4) \\ &= -9 \end{aligned}$$

2. (B)

Since $y = x^3$ is an odd function, $\int_{-2}^{2} x^3 dx = 0$.

The value of $\int_{-8}^{8} y^{\frac{1}{3}} dy = 0$ since $y^{\frac{1}{3}}$ is an odd function. (B is correct)

Note: $\int_{0}^{2} x^3 dx > 0$ (area is above the x-axis)

$\therefore \int_{0}^{2} x^3 dx + \left| \int_{-2}^{0} x^3 dx \right| > 0$ (Eliminate (A))

$2\int_{0}^{2} x^3 dx > 0$ (Eliminate (C))

$$\begin{aligned} \int_{0}^{2} x^3 dx + \int_{0}^{-2} x^3 dx &= \int_{0}^{2} x^3 dx - \int_{-2}^{0} x^3 dx \\ &= 2\int_{0}^{2} x^3 dx \end{aligned}$$

(Eliminate (D))

3. (D)

$$\begin{aligned} 2x + 1 - x &= 4x - 5 - (2x + 1) \\ x + 1 &= 2x - 6 \\ x &= 7 \end{aligned}$$

4. (D)

$y = \ln(5x+7)$

$\frac{dy}{dx} = \frac{5}{5x+7}$

when $x = 3, \frac{dy}{dx} = \frac{5}{5\times 3+7}$

$m_T = \frac{5}{22}$

$\therefore m_N = \frac{-22}{5}$

5. (B)

$y = \frac{x+5}{x+1}$

$= \frac{x+1+4}{x+1}$

$= \frac{x+1}{x+1} + \frac{4}{x+1}$

$= 1 + \frac{4}{x+1}$

$\therefore$ Horizontal translation 1 unit to the left and vertical translation 1 unit up of the graph

$y = \frac{1}{x}$

6. (C)

$y = 3\cos(4x)$

$\frac{dy}{dx} = -12\sin(4x)$

7. (B)

$\int 8x \times 4^{x^2+6} dx = 4\int 2x \times 4^{x^2+6} dx$

$= \frac{4\times 4^{x^2+6}}{\ln 4} + c$

$= \frac{4^{x^2+7}}{\ln 4} + c$

8. (B)

$f(x)$ has had the following transformations to become $g(x)$

* Horizontal dilation by a factor of 2

* Reflection in the y-axis

$\therefore$ There will be a maximum turning point at (-12, 1).

9. (C)

A limiting sum exists if $|r|<1$

$$r = 2x$$
$$|2x|<1$$
$$|x|<\frac{1}{2} \text{ or}$$
$$\frac{-1}{2} < x < \frac{1}{2}$$

10. (A)

When $x = 1$, $y = 0$

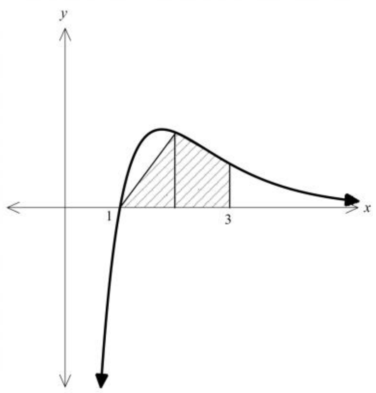
When $x = 2$, $y = 5e^{-2}\ln(2)$

When $x = 3$, $y = 5e^{-3}\ln(3)$

Note for ease of calculations – these can be stored in the memory of your calculator.

Shading shows the area estimated by the Trapezoidal rule.

$\therefore$ It is an underestimate.



Question 11(a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$y = \ln(8x+5)$$

$$\frac{dy}{dx} = \frac{8}{8x+5}$$

Question 11(b)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$y = 6^{2x-1}$$

$$\frac{dy}{dx} = \ln(6) \times 2 \times 6^{2x-1}$$

$$= 2\ln(6) \times 6^{2x-1}$$

Question 11(c)

Criteria	Marks
• Provides correct solution	2
• Correctly differentiates e^{3x} and attempts to use product rule	1

Sample answer:

$$y = 5xe^{3x}$$

let $u = 5x$ and $v = e^{3x}$

$$u' = 5 \qquad v' = 3e^{3x}$$

$$\frac{dy}{dx} = 5x \times 3e^{3x} + 5e^{3x}$$

$$= 15xe^{3x} + 5e^{3x}$$

Question 11(d)

Criteria	Marks
• Provides correct solution	2
• Correctly uses log laws to express y OR • Correctly differentiates $\sqrt{2x^2-5}$	1

Sample answer:

$$\begin{aligned} y &= \ln\sqrt{2x^2-5} \\ &= \ln(2x^2-5)^{\frac{1}{2}} \\ &= \frac{1}{2}\ln(2x^2-5) \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}\times\frac{4x}{2x^2-5} \\ &= \frac{2x}{2x^2-5} \end{aligned}$$

Question 11(e)

Criteria	Marks
• Provides correct solution	2
• Correctly differentiates cos $4x$ and attempts to use quotient rule	1

Sample answer:

$$y = \frac{3x}{\cos 4x}$$

let $u = 3x \qquad v = \cos 4x$

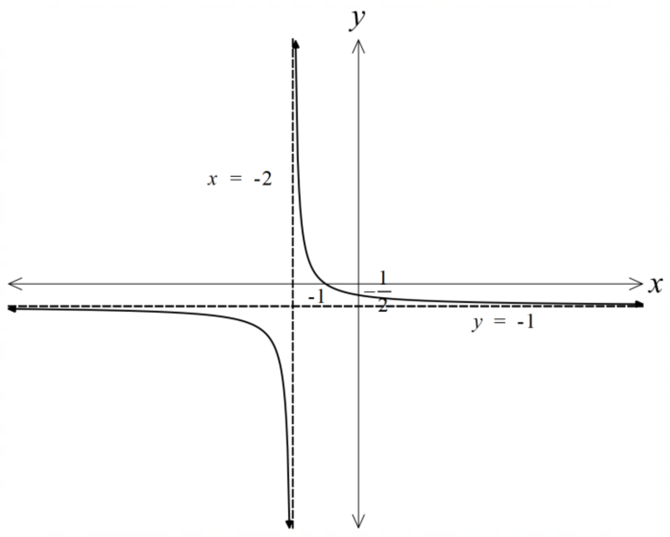
$\quad u' = 3 \qquad v' = -4\sin 4x$

$$\begin{aligned} \frac{dy}{dx} &= \frac{3\cos 4x - 3x\times -4\sin 4x}{\cos^2 4x} \\ &= \frac{3\cos 4x + 12x\sin 4x}{\cos^2 4x} \end{aligned}$$

Question 12(a)

Criteria	Marks
• Provides correct solution	2
• Correct shape and one of x or y intercepts or asymptotes clearly labelled	1

Sample answer:



Question 12(b)

Criteria	Marks
• Provides correct solution	2
• Correct solution but ignores asymptote	1

Sample answer:

$-2 < x \leq -1$

Question 13(a)

Criteria	Marks
• Provides correct solution	2
• Correctly integrates function but omits constant OR • Incorrectly integrates function but includes constant	1

Sample answer:

$$\int 5x^2 + 8x - 3\,dx = \frac{5x^3}{3} + \frac{8x^2}{2} - 3x + c$$
$$= \frac{5x^3}{3} + 4x^2 - 3x + c$$

Question 13(b)

Criteria	Marks
• Provides correct solution	2
• Correctly uses index laws on integral OR • Incorrectly uses index laws to obtain integral with negative index and correctly integrates ensuing function.	1

Sample answer:

$$\int \frac{1}{(8x-1)^3}\,dx = \int (8x-1)^{-3}\,dx$$
$$= \frac{(8x-1)^{-2}}{8\times -2} + c$$
$$= \frac{1}{-16}(8x-1)^{-2} + c$$
$$= \frac{-1}{16(8x-1)^2} + c$$

Question 13(c)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\int e^{7x-4}\,dx = \frac{1}{7}e^{7x-4} + c$$

Question 13(d)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\int \cos x \sin^5 x dx$$

$$\text{let } f(x) = \sin x$$

$$\therefore\ f'(x) = \cos x$$

$$\int \cos x \sin^5 x dx = \frac{1}{6}\sin^6 x + c$$

Question 14(a)

Criteria	Marks
• Provides correct solution	2
• Correctly integrates function	1

Sample answer:

$$\begin{aligned}\int_{-2}^{0} \frac{6}{x+3} dx &= 6\int_{-2}^{0} \frac{1}{x+3} dx \\ &= \left[6\ln|x+3|\right]_{-2}^{0} \\ &= 6\ln|0+3| - 6\ln|(-2)+3| \\ &= 6\ln 3 - 6\ln 1 \\ &= 6\ln 3\end{aligned}$$

Question 14(b)

Criteria	Marks
• Provides correct solution	2
• Correctly integrates function	1

Sample answer:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^2 3x\,dx = \left[\frac{1}{3}\tan 3x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \frac{1}{3}\tan(3\times\frac{\pi}{3}) - \frac{1}{3}\tan(3\times\frac{\pi}{4})$$

$$= \frac{1}{3}\times 0 - \frac{1}{3}\times(-1)$$

$$= \frac{1}{3}$$

Question 15

Criteria	Marks
• Provides correct solution	2
• Correctly differentiates function and equates to zero	1

Sample answer:

$y = x^3 - 3x^2 + 4$

$\frac{dy}{dx} = 3x^2 - 6x$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0$ or $x = 2$

when $x = 0$, $y = 0^3 - 3\times 0^2 + 4$

$= 4$

when $x = 2$, $y = 2^3 - 3\times 2^2 + 4$

$= 0$

$\therefore$ Stationary points are $(0,4)$ and $(2,0)$

Question 16

Criteria	Marks
• Provides correct solution	2
• Correctly finds the second derivative and equates to greater than zero	1

Sample answer:

$$y = 5x^3 - 2x^2 + 6x - 1$$

$$\frac{dy}{dx} = 15x^2 - 4x + 6$$

$$\frac{d^2y}{dx^2} = 30x - 4$$

Concave up when $\frac{d^2y}{dx^2} > 0$

$$30x - 4 > 0$$

$$30x > 4$$

$$x > \frac{4}{30}$$

$$x > \frac{2}{15}$$

Question 17

Criteria	Marks
• Provides correct solution	2
• Correctly integrates function	1

Sample answer:

$$A = \left| \int_0^2 x^3 - x^2 - 2x \, dx \right|$$

$$= \left| \left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{2x^2}{2} \right]_0^2 \right|$$

$$= \left| \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2 \right]_0^2 \right|$$

$$= \left| \frac{2^4}{4} - \frac{2^3}{3} - 2^2 - \left(\frac{0^4}{4} - \frac{0^3}{3} - 0^2 \right) \right|$$

$$= \frac{8}{3} u^2$$

Question 18

Criteria	Marks
• Provides correct solution	1

Sample answer:

when $x = 2, y = 4$ and $y = 2^x$

$$LHS = 4$$
$$\begin{aligned} RHS &= 2^2 \\ &= 4 \end{aligned}$$

$\therefore (2, 4)$ lies on $y = 2^x$

when $x = 2, y = 4$ and $y = -2x^2 + 8x - 4$;

LHS = 4

$$\begin{aligned} RHS &= -2(2)^2 + 8 \times 2 - 4 \\ &= 4 \end{aligned}$$

$\therefore LHS = RHS$

$\therefore (2, 4)$ lies on $y = -2x^2 + 8x - 4$ and is a point of intersection of $y = -2x^2 + 8x - 4$ and $y = 2^x$.

Question 18(b)

Criteria	Marks
• Provides correct solution	2
• Graphs a concave down parabola	1

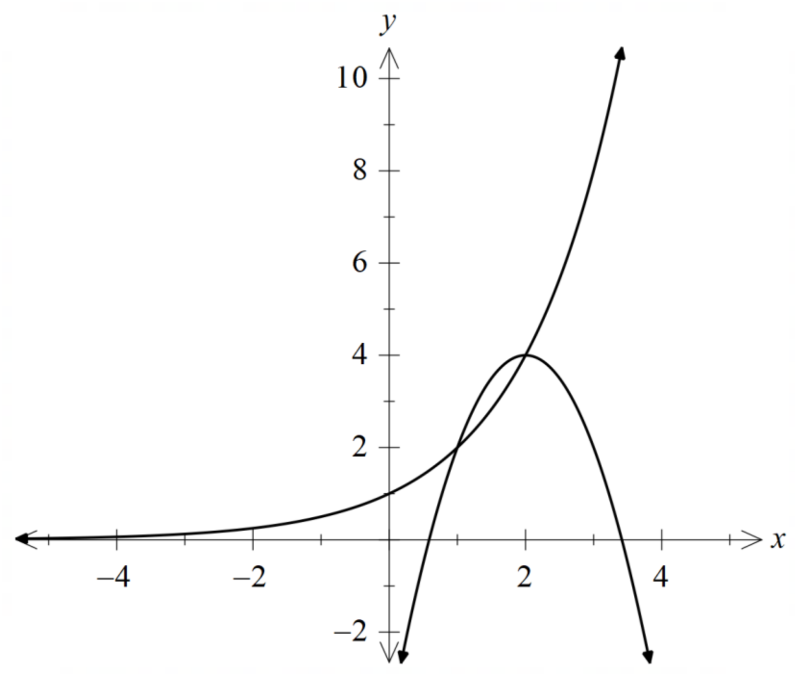
Sample answer:

$y = -2x^2 + 8x - 4$ is a concave down parabola

To find the vertex:

$$\begin{aligned} x &= \frac{-b}{2a} \\ &= \frac{-8}{2 \times -2} \\ &= 2 \end{aligned}$$

$\therefore$ Vertex is $(2, 4)$



Question 18(c)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$1 < x < 2$

Question 19(a)

Criteria	Marks
• Provides correct solution	2
• Correctly substitutes equations or eliminates y variable	1

Sample answer:

Substitute $y = 2x^2 + x - 9$ into $y = 3x - 5$

$$2x^2 + x - 9 = 3x - 5$$
$$2x^2 - 2x - 4 = 0$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$

$x = 2$ or $x = -1$

Question 19(b)

Criteria	Marks
• Provides correct solution	2
• Correctly finds $\int_{-1}^{2} -2x^2+2x+4)\,dx$	1

Sample answer:

$$A=\int_{-1}^{2} 3x-5-(2x^2+x-9)\,dx$$

$$=\int_{-1}^{2} -2x^2+2x+4)\,dx$$

$$=\left[\frac{-2x^3}{3}+\frac{2x^2}{2}+4x\right]_{-1}^{2}$$

$$=\left[\frac{-2x^3}{3}+x^2+4x\right]_{-1}^{2}$$

$$=\left(\frac{-2(2)^3}{3}+2^2+4\times 2\right)-\left(\frac{-2(-1)^3}{3}+(-1)^2+4\times(-1)\right)$$

$$=9u^2$$

Question 20(a)

Criteria	Marks
• Provides correct solution	2

Sample answer:

$$y=x\sin x$$

$u=x \qquad v=\sin x$

$u'=1 \qquad v'=\cos x$

$$\frac{dy}{dx}=x\cos x+\sin x$$

Question 20(b)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\int x\cos x + \sin x\, dx = x\sin x$$

$$\int x\cos x\, dx + \int \sin x\, dx = x\sin x$$

$$\int x\cos x\, dx - \cos x = x\sin x + c$$

$$\int x\cos x\, dx = x\sin x + \cos x + c$$

Question 21

Criteria	Marks
• Provides correct solution	4
• Correctly finds the value of the constant k and correctly differentiates to find $\frac{dA}{dt}$	3
• Correctly finds the value of the constant k	2
• Correctly finds $A_0 = 200$	1

Sample answer:

$$A = A_0 e^{kt}$$

when $t = 0, A = 200$

$$ie\ A_0 = 200$$

$$\therefore A = 200e^{kt}$$

when $t = 10, A = 16$

$$16 = 200e^{10k}$$

$$\frac{16}{200} = e^{10k}$$

$$\ln\frac{16}{200} = 10k$$

$$k = \frac{1}{10}\ln\frac{16}{200}$$

$$\frac{dA}{dt} = 200ke^{kt} \quad \text{where } k = \frac{1}{10}\ln\frac{16}{200}$$

when $t = 5$,

$$\frac{dA}{dt} = 200ke^{kt}$$

$$= -14.28767882$$

$\therefore$ Rate of decay is $14.29 grams/day$

Question 22(a)

Criteria	Marks
• Provides correct solution	2
• Correctly uses a log law OR • Finds $T_2 - T_1$ and $T_3 - T_2$	

Sample answer:

$$\begin{aligned} &\log 3, \log 9, \log 27, \log 81, \ldots. \\ &= \log 3, \log 3^2, \log 3^3, \log 3^4, \ldots \\ &= \log 3, 2\log 3, 3\log 3, 4\log 3 \\ T_2 - T_1 &= 2\log 3 - \log 3 \\ &= \log 3 \\ T_3 - T_2 &= 3\log 3 - 2\log 3 \\ &= \log 3 \\ &\therefore \text{AP where } a = \log 3, d = \log 3, n = 10 \end{aligned}$$

Question 22(b)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\begin{aligned} S_{10} &= \frac{10}{2}\left[2\log 3 + (10-1)\log 3\right] \\ &= 5(2\log 3 + 9\log 3) \\ &= 5(11\log 3) \\ &= 55\log 3 \end{aligned}$$

Question 23(a)

Criteria	Marks
• Provides correct solution	4
• Finds the y values of the stationary points	3
• Solves the trigonometric equation to find x coordinates of the stationary point or correctly finds one pair of coordinates correctly	2
• Finds the derivative	1

Sample answer:

$$y = \sin\left(2x + \frac{\pi}{3}\right)$$

$$\frac{dy}{dx} = 2\cos\left(2x + \frac{\pi}{3}\right)$$

$$2\cos\left(2x + \frac{\pi}{3}\right) = 0$$

$$\cos\left(2x + \frac{\pi}{3}\right) = 0$$

$$\left(2x + \frac{\pi}{3}\right) = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$2x = \frac{\pi}{6}, \frac{7\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{7\pi}{12}$$

$$\text{when } x = \frac{\pi}{12}, y = \sin\left(2\times\frac{\pi}{12} + \frac{\pi}{3}\right)$$

$$= 1$$

$$\text{when } x = \frac{7\pi}{12}, y = \sin\left(2\times\frac{7\pi}{12} + \frac{\pi}{3}\right)$$

$$= -1$$

$$\frac{d^2y}{dx^2} = -4\sin\left(2x + \frac{\pi}{3}\right)$$

$$\text{when } x = \frac{\pi}{12}, \frac{d^2y}{dx^2} = -4\sin\left(2\times\frac{\pi}{12} + \frac{\pi}{3}\right)$$

$$= -4 \rightarrow \left(\frac{\pi}{12}, 1\right) \text{ is a relative maximum turning point}$$

$$\text{when } x = \frac{7\pi}{12}, \frac{d^2y}{dx^2} = -4\sin\left(2\times\frac{7\pi}{12} + \frac{\pi}{3}\right)$$

$$= 4 \rightarrow \left(\frac{7\pi}{12}, -1\right) \text{ is a relative minimum turning point}$$

Question 23(b)

Criteria	Marks
• Provides correct solution	2
• Finds all x values of the points of inflection OR • Finds the coordinates of one point of inflection	1

Sample answer:

$$\frac{d^2y}{dx^2} = -4\sin\left(2x + \frac{\pi}{3}\right)$$

$$-4\sin\left(2x + \frac{\pi}{3}\right) = 0$$

$$\sin\left(2x + \frac{\pi}{3}\right) = 0$$

$$\left(2x + \frac{\pi}{3}\right) = 0, \pi, 2\pi$$

$$2x = \pi - \frac{\pi}{3} \; or \; 2\pi - \frac{\pi}{3}$$

$$2x = \frac{2\pi}{3} \; or \; \frac{5\pi}{3}$$

$$x = \frac{2\pi}{6}\left(= \frac{\pi}{3}\right) or \; \frac{5\pi}{6}$$

$$\text{when } x = \frac{\pi}{3}, y = \sin\left(2 \times \frac{\pi}{3} + \frac{\pi}{3}\right)$$

$$= 0$$

$$\text{when } x = \frac{5\pi}{6}, y = \sin\left(2 \times \frac{5\pi}{6} + \frac{\pi}{3}\right)$$

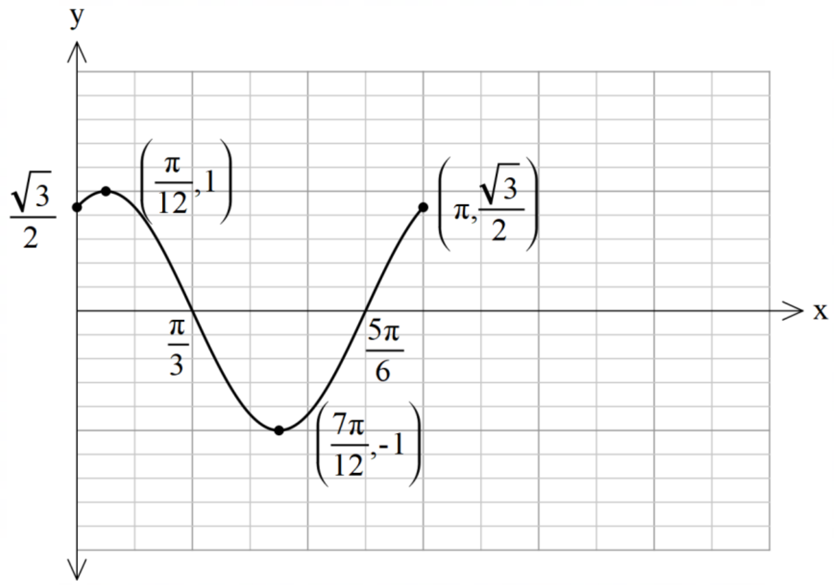
$$= 0$$

$$\therefore \text{ Points of inflection are } \left(\frac{\pi}{3}, 0\right) \text{and } \left(\frac{5\pi}{6}, 0\right)$$

Question 23(c)

Criteria	Marks
• Provides correct solution	2
• Correct shape with period and amplitude correct	1

Sample answer:



Question 24(a)

Criteria	Marks
• Provides correct solution	3
• Correctly substitutes into the sum of an arithmetic or geometric series formula	2
• Identifies a combination of series or correctly calculates sum without using arithmetic and geometric sequence methods	1

Sample answer:

$$\sum_{n=1}^{20} 5^n - 5n = 5^1 - 5\times 1 + 5^2 - 5\times 2 + 5^3 - 5\times 3 + ... + 5^{20} - 5\times 20$$

$$= 5^1 + 5^2 + 5^3 + ... + 5^{20} - (5\times 1 + 5\times 2 + 5\times 3 + ... + 5\times 20)$$

Geometric series where $a = 5, r = 5, n = 20$

Arithmetic series where $a = 5, d = 5, n = 20$

$$S_{20} = \frac{5(5^{20}-1)}{5-1} - \frac{20}{2}\left(2\times 5 + (20-1)\times 5\right)$$

$$= \frac{5(5^{20}-1)}{4} - 1050$$

Question 25 (a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$c^2 = a^2 + b^2$$

$d^2 = 0.8^2 + x^2$ where d = Feifei's rowing distance

$$d^2 = 0.64 + x^2$$

$$d = \sqrt{0.64 + x^2}$$

$$T = \frac{D}{S}$$

$$t = \frac{2.5 - x}{7} + \frac{\sqrt{0.64 + x^2}}{5}$$

Question 25 (b)

Criteria	Marks
• Provides correct solution	4
• Tests for maximum/minimum	3
• Correctly solves $\frac{dt}{dx} = 0$	2
• Correctly differentiates t	1

Sample answer:

$$t = \frac{2.5 - x}{7} + \frac{\sqrt{0.64 + x^2}}{5}$$

$$t = \frac{2.5 - x}{7} + \frac{1}{5}(0.64 + x^2)^{\frac{1}{2}}$$

$$\frac{dt}{dx} = -\frac{1}{7} + \frac{1}{5} \times \frac{1}{2} \times 2x(0.64 + x^2)^{-\frac{1}{2}}$$

$$= -\frac{1}{7} + \frac{x}{5\sqrt{0.64 + x^2}}$$

Stationary points occur when $\frac{dt}{dx} = 0$

$$-\frac{1}{7} + \frac{x}{5\sqrt{0.64 + x^2}} = 0$$

$$\frac{x}{5\sqrt{0.64 + x^2}} = \frac{1}{7}$$

$$\frac{x}{5} = \frac{\sqrt{0.64 + x^2}}{7}$$

$$\frac{7x}{5} = \sqrt{0.64 + x^2}$$

$$\frac{49x^2}{25} = x^2 + 0.64$$

$$49x^2 = 25x^2 + 16$$

$$24x^2 = 16$$

$$x^2 = \frac{16}{24}$$

$$x = \pm 0.8164965809$$

$$= 0.82 \ \text{(2dp)} \quad \text{since } x > 0$$

x	0	0.82	1
$\frac{dt}{dx}$	$-\frac{1}{7}$	0	0.0133…

$\therefore$ when $x = 0.816...$ the minimum amount of time is taken for Feifei to reach Airi.

Feifei must run $2.5 - 0.816... = 1.683503419$

$= 1.68\,\text{km}\,(2\,\text{d.p.})$

BAULKHAM HILLS HIGH SCHOOL 2024
YEAR 12 ASSESSMENT TASK 2
MATHEMATICS ADVANCED

MARKING COVER SHEET

FINAL MARK

NESA#:_______________________________ **Teacher:**_______________________________

Question	Curve Sketching using the derivative	Exponential and logarithmic functions	Trigonometric functions	Integration	Sequences and series	Graphs and Equations	Total
Multiple Choice		4, 7	6	2, 10	1, 3, 9	5, 8	**10**
11		a, b, c, d 6	e 2				**8**
12						4	**4**
13		c 1	d 1	a, b, 4			**6**
14		a 2	b 2				**4**
15, 16	4						**4**
17				2			**2**
18						4	**4**
19				4			**4**
20			2				**2**
21		4					**4**
22					3		**3**
23	c 2		a, b 6				**8**
24					3		**3**
25	5						**5**
TOTAL	**11**	**15**	**14**	**12**	**9**	**10**	**71**

BAULKHAM HILLS HIGH SCHOOL
YEAR 12 ASSESSMENT TASK 1 2024
MATHEMATICS ADVANCED

NESA#: ____________________________

Teacher: ____________________________

Mathematics Advanced

General Instructions	• Reading time - 5 minutes • Working time - 2 hours • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • In Questions 11 - 25, show relevant mathematical reasoning and/or calculations
Total marks: 72	**Section I - 10 marks** (pages 3 - 8) • Attempt Questions 1 - 10 • Allow about 15 minutes for this section **Section II - 62 marks** (pages 9 - 26) • Attempt Questions 11 - 25 • Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1 Consider the sequence 187, 183, 179, … What is the 50th term?

(A) -13 (B) -9 (C) 9 (D) 13

2 Which of the following is equivalent to $\int_{-2}^{2} x^3 dx$?

(A) $\int_0^2 x^3 dx + \left|\int_{-2}^0 x^3 dx\right|$

(B) $\int_{-8}^{8} y^{\frac{1}{3}} dy$

(C) $2\int_0^2 x^3 dx$

(D) $\int_0^2 x^3 dx + \int_0^{-2} x^3 dx$

3 The arithmetic sequence $x, 2x+1, 4x-5$, is formed. What is the value of x?

(A) $\frac{-1}{9}$

(B) $\frac{5}{3}$

(C) 5

(D) 7

4 What is the gradient of the normal to the curve $y = \ln(5x+7)$ at the point where $x = 3$?

(A) $\frac{-1}{5}$

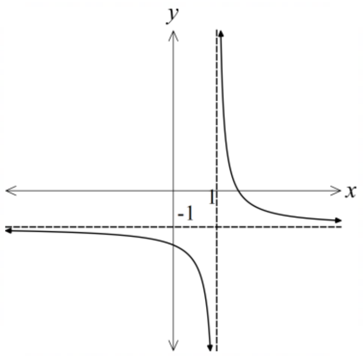
(B) 5

(C) $\frac{5}{22}$

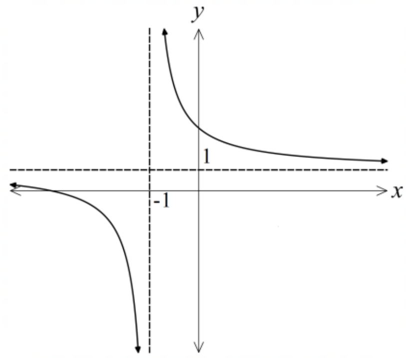
(D) $\frac{-22}{5}$

5 Which of the following is the graph of $y = \dfrac{x+5}{x+1}$?

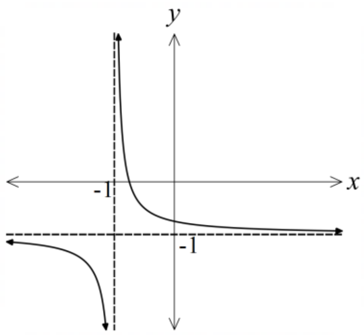
(A)



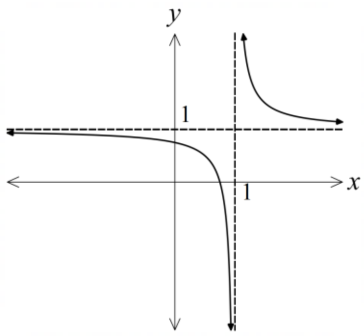
(B)



(C)



(D)



6 What is the derivative of $y=3\cos(4x)$?

(A) $3\sin(4x)$

(B) $-3\sin(4x)$

(C) $-12\sin(4x)$

(D) $12\sin(4x)$

7 What is the primitive function of $8x\times 4^{x^2+6}$?

(A) $\dfrac{4^{x^2+6}}{\ln 4}+c$

(B) $\dfrac{4^{x^2+7}}{\ln 4}+c$

(C) $\dfrac{4^{x^2+6}}{4\ln 4}+c$

(D) $\dfrac{4^{x^2+6}}{\ln 16}+c$

8 The function $f(x)$ has a maximum turning point at (6, 1). The function $g(x) = f\left(\frac{-1}{2}x\right)$. Which of the following about $g(x)$ is true?

(A) There is a maximum turning point at (12, 1).

(B) There is a maximum turning point at $(-12, 1)$.

(C) There is a minimum turning point at (12, 1).

(D) There is a minimum turning point at $(-12, 1)$.

9 It is known that the following geometric series has a limiting sum.

$$2x, 4x^2, 8x^3, 16x^4, \ldots\ldots\ldots$$

What could be the value of x?

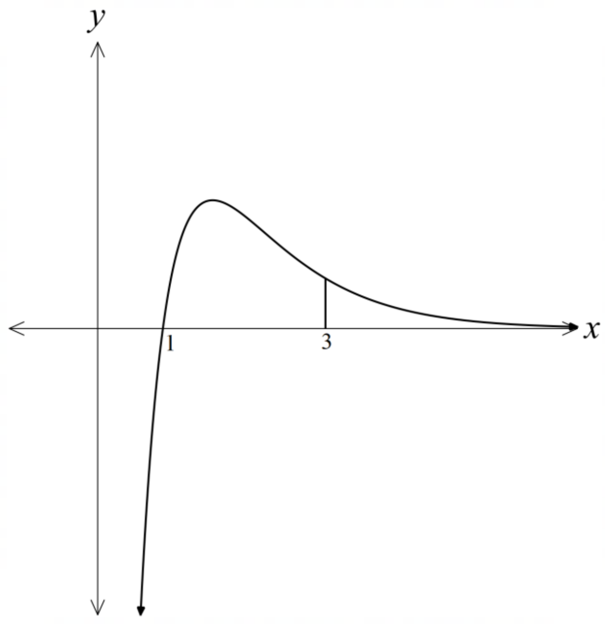
(A) -2

(B) $\frac{-1}{2}$

(C) $\frac{1}{4}$

(D) 4

10 The graph of $y = 5e^{-x} \ln x$ is shown below. The area is estimated by using the Trapezoidal Rule and 3 function values, from $x = 1$ to $x = 3$ (inclusive). Which of the following statements is true?



(A) The area is approximately 0.606u^2 and is an underestimate.

(B) The area is approximately 0.606u^2 and is an overestimate.

(C) The area is approximately 0.371u^2 and is an underestimate.

(D) The area is approximately 0.371u^2 and is an overestimate.

Section II

62 marks
Attempt Questions 11 - 25
Allow about 1 hour and 45 minutes for this section
Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning.

Question 11 (8 marks)

Differentiate each of the following:

a) $y = \ln(8x+5)$ **1**

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b) $y = 6^{2x-1}$ **1**

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c) $y = 5xe^{3x}$ **2**

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d) $y = \ln\sqrt{2x^2 - 5}$ **2**

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e) $y = \dfrac{3x}{\cos 4x}$ **2**

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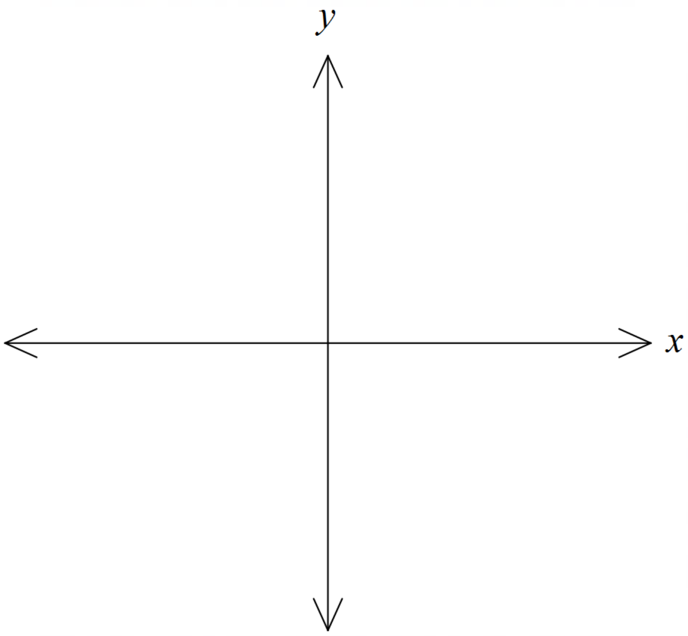
Question 12 (4 marks)

a) On the number plane below, graph the function $y = \frac{1}{x+2} - 1$. Clearly label all intercepts and asymptotes. **2**



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b) Hence or otherwise solve $\frac{1}{x+2} - 1 \geq 0$. **2**

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Question 13 (6 marks)

Find:

a) $\int 5x^2 + 8x - 3\,dx$ **2**

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b) $\int \frac{1}{(8x-1)^3}\,dx$ **2**

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c) $\int e^{7x-4}\,dx$ **1**

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d) $\int \cos x \sin^5 x\,dx$ **1**

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Question 14 (4 marks)

Calculate the following definite integrals. Leave your answers in exact form where necessary.

a) $\int_{-2}^{0} \frac{6}{x+3}\,dx$ **2**

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b) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^2 3x\,dx$ **2**

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Question 15 (2 marks)

Find the coordinates of the stationary points of $y = x^3 - 3x^2 + 4$. **2**

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Question 16 (2 marks)

For which values of x is the curve $y = 5x^3 - 2x^2 + 6x - 1$ concave up? **2**

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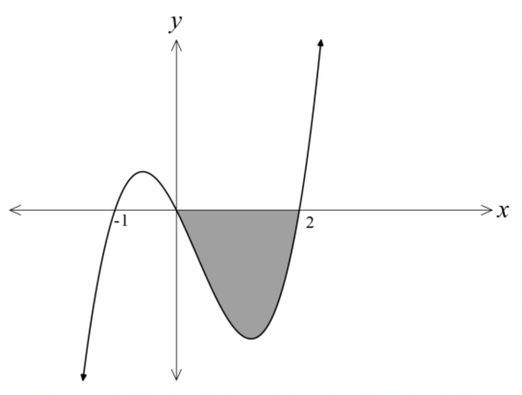
Question 17 (2 marks)

The graph of $y = x^3 - x^2 - 2x$ is shown on the number plane below. **2**



Calculate the area between the curve and the x axis between $x = 0$ and $x = 2$.

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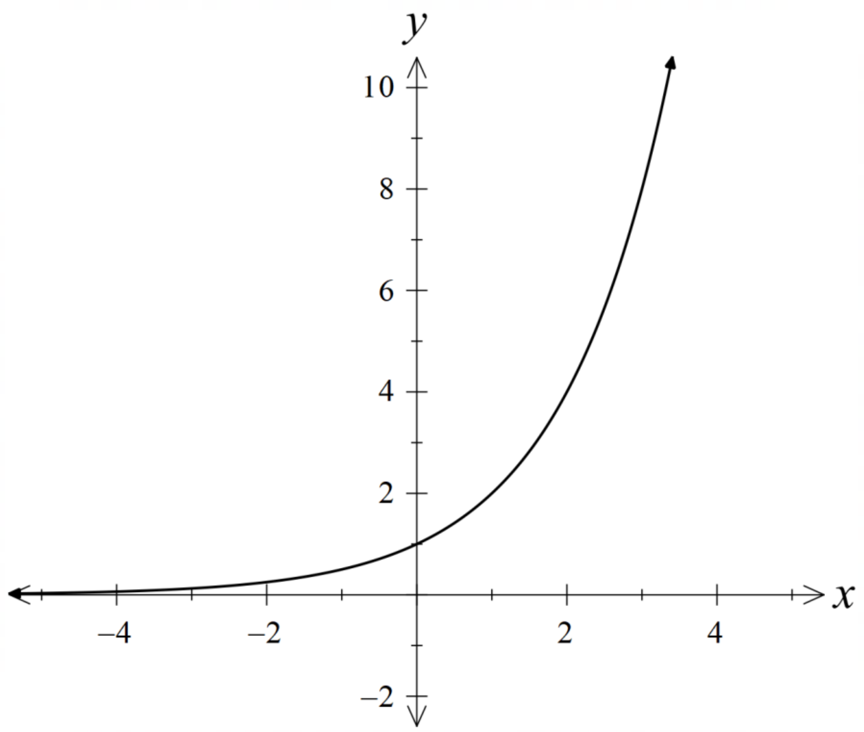
Question 18 (4 marks)

On the number plane below, the graph of $y = 2^x$ is shown.



a) One of the points of intersection of $y = 2^x$ and $y = -2x^2 + 8x - 4$ is $(1, 2)$. Show that the other point of intersection is $(2, 4)$. **1**

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b) On the number plane on page 16, sketch the graph of $y = -2x^2 + 8x - 4$. Do not find the x - intercepts. **2**

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c) Hence or otherwise solve $-2x^2 + 8x - 4 > 2^x$. **1**

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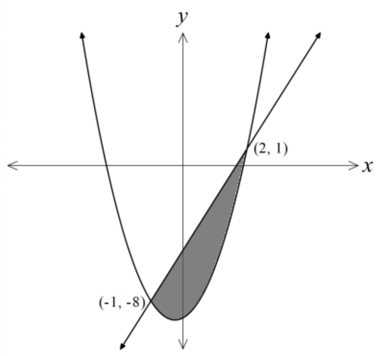
Question 19 (4 marks)

The graph below shows the functions $y = 2x^2 + x - 9$ and $y = 3x - 5$.



a) Show that the x coordinates of the points of intersection are $x = 2$ and $x = -1$. **2**

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b) Hence or otherwise, find the area of the shaded region between the line $y = 3x - 5$ **2**

and the parabola $y = 2x^2 + x - 9$.

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Question 20 (2 marks)

a) Differentiate $y = x\sin x$. **1**

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b) Hence or otherwise, find $\int x\cos x\,dx$ **1**

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Question 21 (4 marks)

A certain radioactive isotope element decays exponentially according to the model $A = A_0 e^{kt}$ where A is the number of grams of the isotope at the end of t days and A_0 is the number of grams present initially. **4**

Initially there are 200 grams of this isotope. After 10 days, 16 grams of this substance remain. What is the rate of decay after 5 days? Give your answer correct to 2 decimal places.

Question 22 (3 marks)

a) Show that the sequence $\log 3, \log 9, \log 27, \log 81, ...$ is an arithmetic sequence. **2**

b) Hence or otherwise find the sum to 10 terms of the sequence **1**

$\log 3, \log 9, \log 27, \log 81, ...$ Leave your answer in simplest exact form.

Question 23 (8 marks)

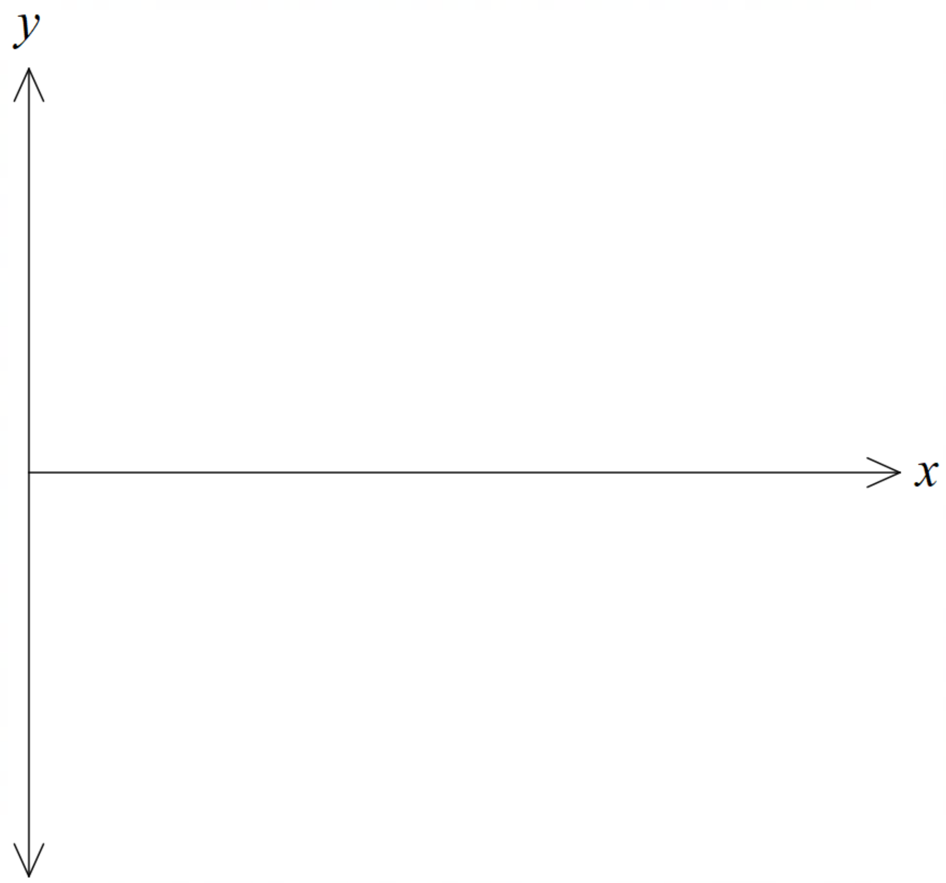
Consider the function $f(x) = \sin\left(2x + \frac{\pi}{3}\right)$ in the domain $0 \le x \le \pi$.

a) Find the coordinates of the stationary points of $y = f(x)$ and determine their nature. **4**

b) Find the coordinates of the points of inflection. **2**

c) Hence, or otherwise graph the function $f(x)=\sin\left(2x+\frac{\pi}{3}\right)$ on the number plane **2**

below in the domain $0 \le x \le \pi$.

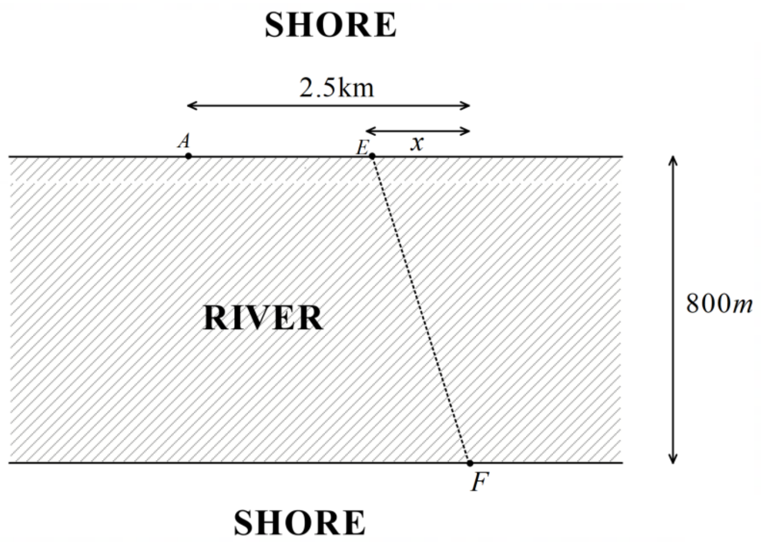


Question 24 (3 marks)

A sequence is defined by the rule $T_n = 5^n - 5n$ where $n \geq 1$. Find the sum of the first 20 terms. Leave your answer in simplest index form. **3**

Question 25 (5 marks)

Feifei (F) and Airi (A) are on opposing sides of a river bank. Feifei can run at a rate of 7km/hr and row at a rate of 5km/hr. The river bank is 800 metres wide and there would be 2.5 kilometres between Feifei and Airi if they were on the same side of the shore. Feifei wishes to meet Airi. Let x be the distance from directly opposite from Feifei's starting point to her exit point (E) from the water and let t be the total time taken for Feifei to meet Airi.



a) Show that the time (t) taken for Feifei to row and then run to Airi can be given by **1**

the equation $t = \dfrac{2.5 - x}{7} + \dfrac{\sqrt{0.64 + x^2}}{5}$.

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b) Hence, or otherwise, find how far Feifei must run along the shore in order to minimise the time taken to reach Airi. Give your answer correct to 2 decimal places. **4**

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End of Exam

Baulkham Hills High School

Year 12 - Assessment Task 2

2024

NESA#:______________________________ **Teacher:**______________________________

Mathematics
Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

Start Here →

1.	A ○	B ○	C ○	D ○	**6.**	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○	**7.**	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○	**8.**	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○	**9.**	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○	**10.**	A ○	B ○	C ○	D ○