BAULKHAM HILLS HIGH SCHOOL

HSC ASSESSMENT TASK 1 2025

MATHEMATICS ADVANCED

MARKING COVER SHEET

FINAL MARK

NESA#:_____ **Teacher:**_____

Question	Sequences and Series	Graphs and Equations	Total
MC	1,3,4 /3	2, 5 /2	**/5**
6	/2		**/2**
7		/2	**/2**
8		/4	**/4**
9	/6		**/6**
10		/7	**/7**
11	/3		**/3**
12	/2		**/2**
13		/2	**/2**
14	/3		**/3**
15		/4	**/4**
16	/2		**/2**
	/21	**/21**	**/42**

BAULKHAM HILLS HIGH SCHOOL

2025

HSC ASSESSMENT TASK 1

Mathematics Advanced

General Instructions

- Reading time – 5 minutes.
- Working time – 1 hour.
- Write using black or blue pen.
- NESA-approved calculators may be used.
- Answer all questions in the answer booklet provided.
- Show all necessary working in Questions 6-16.
- Marks may be deducted for careless or badly arranged work.
- A reference sheet is provided.

Total marks – 42
The exam consists of 16 pages.

This paper consists of TWO sections.

Section I– Pages 4 - 5 (5 marks)
Questions 1 - 5

- Attempt Questions 1 – 5
- Answer on the space provided.
- Allow about 8 minutes for this section.

Section II – Pages 6 – 16 (37 marks)

- Attempt questions 6 - 16
- Allow about 52 minutes for this section.

BAULKHAM HILLS HIGH SCHOOL

HSC ASSESSMENT TASK 1 2025

NESA#:_____

Teacher:_____

Mathematics Advanced
Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

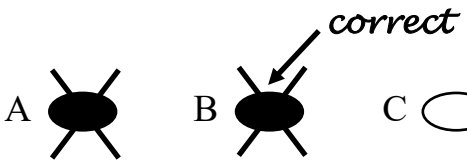
Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.



Start Here →

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

Section I - 5 marks

Use the multiple-choice answer sheet for questions 1-5.

1. Which of the following are the first three terms of the sequence $T_{n+1} = 3T_n - 1$ where $T_1 = 3$?

   (A) 3, 6, 15
   (B) 3, 8, 23
   (C) 3, 8, 26
   (D) 3, 9, 27

2 The point $P(2,-3)$ lies on the graph of a function $f(x)$. The graph of $f(x)$ is translated four units horizontally to the right and then reflected in the x-axis.

What are the coordinates of the final image of P?

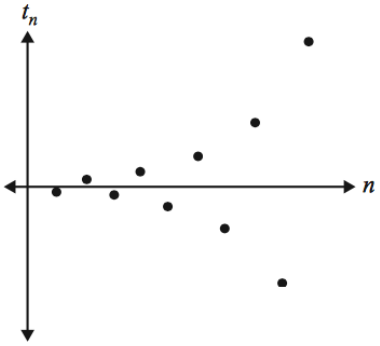
(A) $(-3,2)$
(B) $(-3,6)$
(C) $(2,-1)$
(D) (6, 3)

3. The first term of a geometric sequence is a, where $a < 0$.

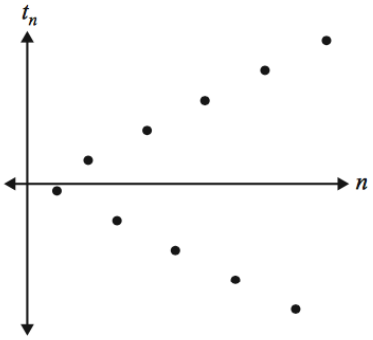
The common ratio of this sequence, r, is such that $-1 < r < 0$.

Which of the following graphs best shows the first 10 terms of this sequence?

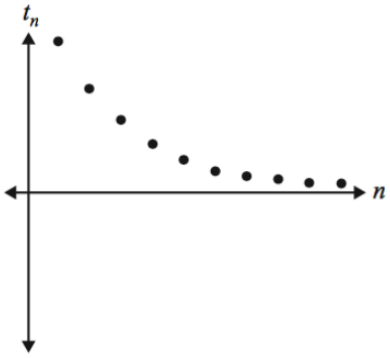
(A)



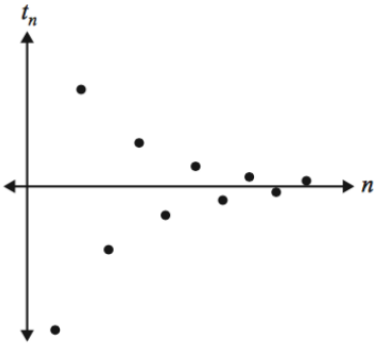
(B)



(C)



(D)



4. Mrs Darcy bought 2250 Zooper Doopers.

On the first Wednesday, she sold 50 Zooper Doopers.

On the second Wednesday, she sold 100 Zooper Doopers.

On the third Wednesday, she sold 150 Zooper Doopers.

This pattern continues until all 2250 Zooper Doopers are sold.

How many Wednesdays does it take for all of the Zooper Doopers to be sold?

(A) 7

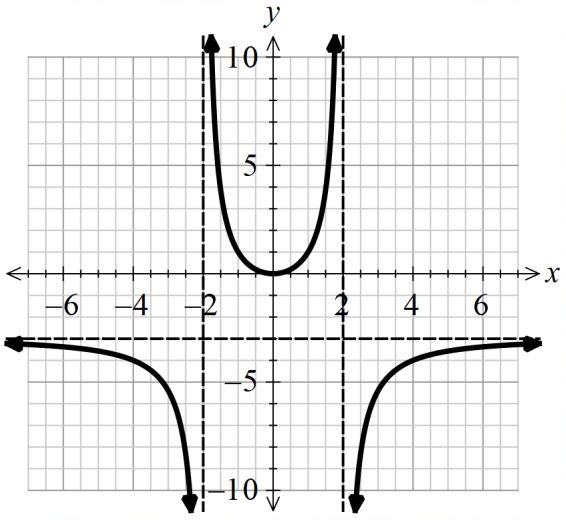
(B) 8

(C) 9

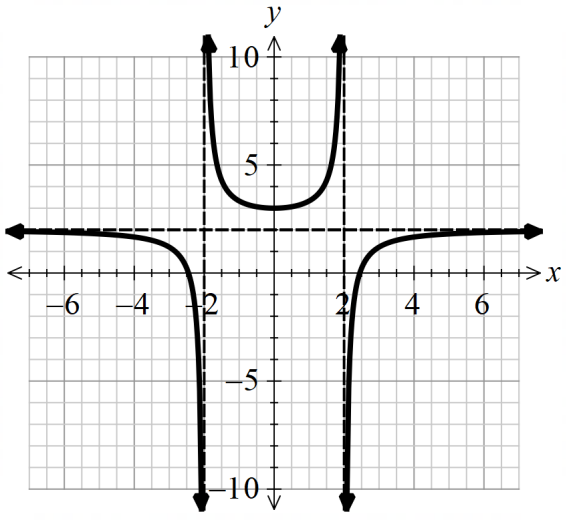
(D) 10

5. Which graph best represents the function $y = \frac{3x^2}{4 - x^2}$?

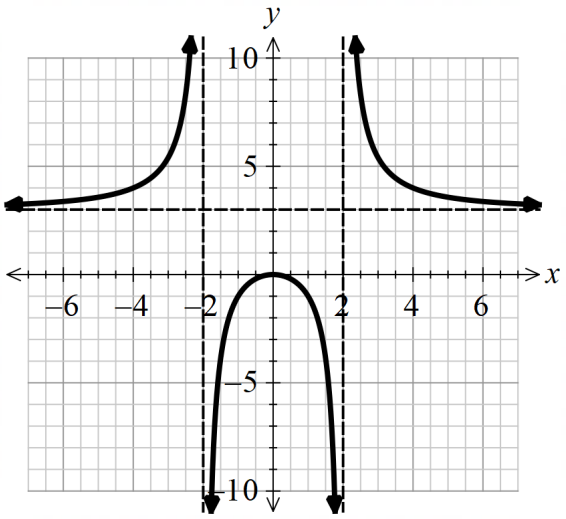
(A)



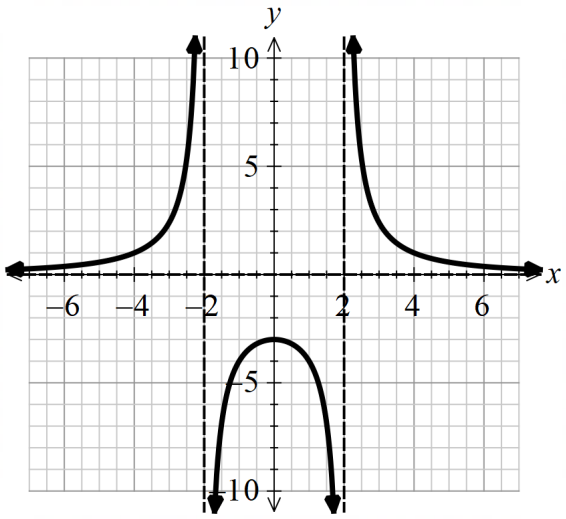
(B)



(C)



(D)



End of Section I

Section II

37 marks

Attempt Questions 6 – 16
Allow about 52 minutes for this section

Instructions:

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing paper is available. If you use extra paper, write your NESA number, your teacher's name on the paper and indicate clearly which question you are answering.

Question 6 (2 marks)

Calculate the sum of the arithmetic series $2 + 9 + 16 + \ldots + 2025$. **2**

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Question 7 (2 marks)

The graph of $y = \sqrt{4x^2 + 1}$ is horizontally dilated by a factor of 2 and translated up by 3 units. **2**

What is the equation of the new graph?

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Question 8 (4 marks)

(a) Without using calculus, sketch the graph of $y = 2 + \frac{3}{x-4}$, showing any asymptotes and intercepts. **3**

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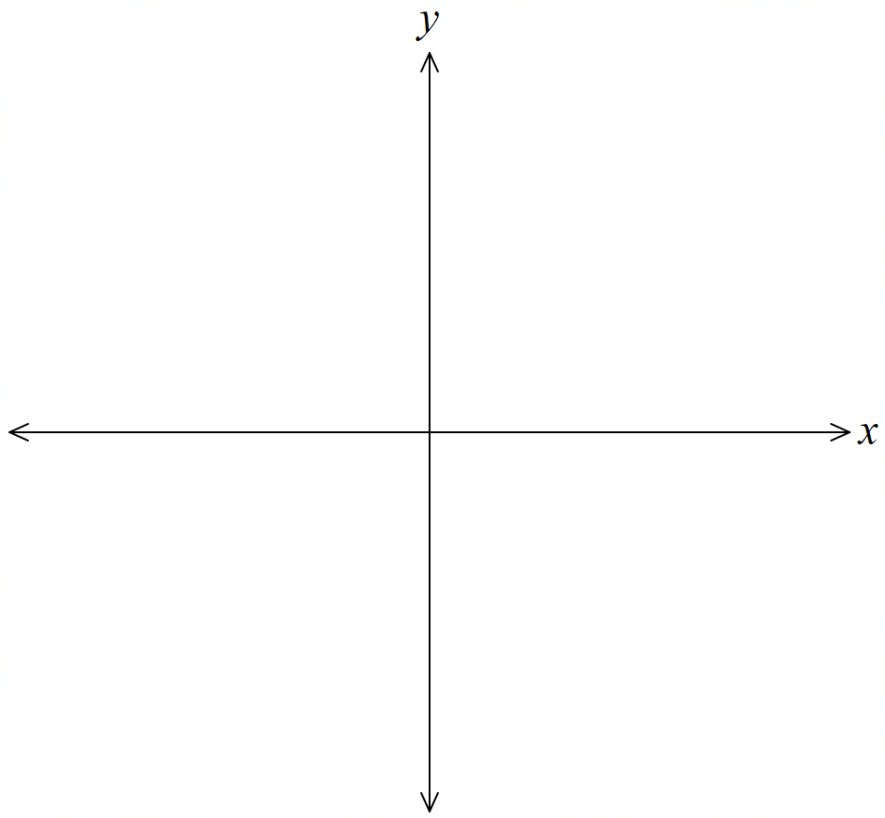

(b) Hence, or otherwise, solve $\frac{2x-5}{x-4} > \frac{5}{4}$. **1**

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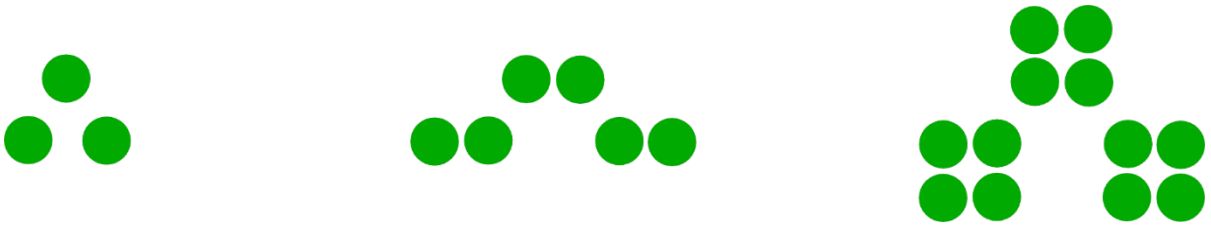
Question 9 (6 marks)

The diagram below shows the first three patterns formed by a number of circles.



(a) Show that the number of circles correspond to the terms of a geometric sequence. **2**

State the first term, a, and the common ratio, r.

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(b) Which term corresponds to a pattern that will use 768 circles? **2**

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Question 9 continues on page 9

(c) After how many terms will the total number of circles first exceed one million? **2**

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Question 10 (7 marks)

The height of the tide, h, in Salamander Bay was recorded between 8am to 8pm on a particular day and was given by the equation

$$h = 2\sin\left(\frac{\pi t}{6}\right) + 8 \text{ for } 0 \le t \le 12,$$

where h is in metres and t is the number of hours after 8am.

(a) What is the amplitude and period of the function h? **2**

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(b) What was the value of h at low tide, and at what time did low tide occur? **2**

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Question 10 continues on page 11

(c) A ship is able to enter the harbour only if the height of the tide is at least 7 metres. **3**

Find all the times between 8am and 8pm on this day during which the ship was able to enter the harbour?

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Question 11 (3 marks)

In an arithmetic series, the fourth term is 10 and the sum of the first 17 terms is -170. **3**

Find the 30^{th} term.

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Question 12 (2 marks)

When placed in a pond, the length of a fish was 18 centimetres. **2**

During its first month in the pond, the fish increased in length by 3.2 centimetres.

During its second month in the pond, the fish increased in length by 1.92 centimetres.

In each subsequent month, the fish increased in length by 60% of the previous month's growth.

Calculate the maximum length this fish can grow to.

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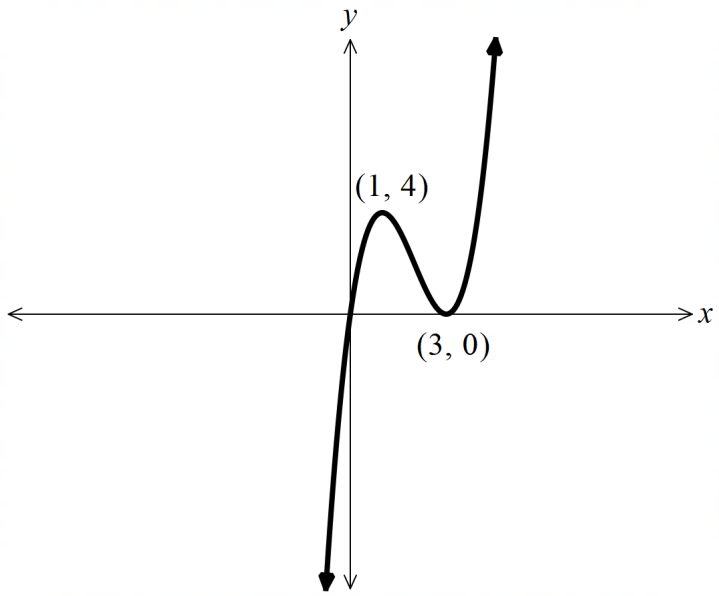
Question 13 (2 marks)

Consider the graph of $y = f(x)$ as shown. **2**



Sketch the graph of $y = f(-0.5x + 3)$ showing all intercepts and turning points.

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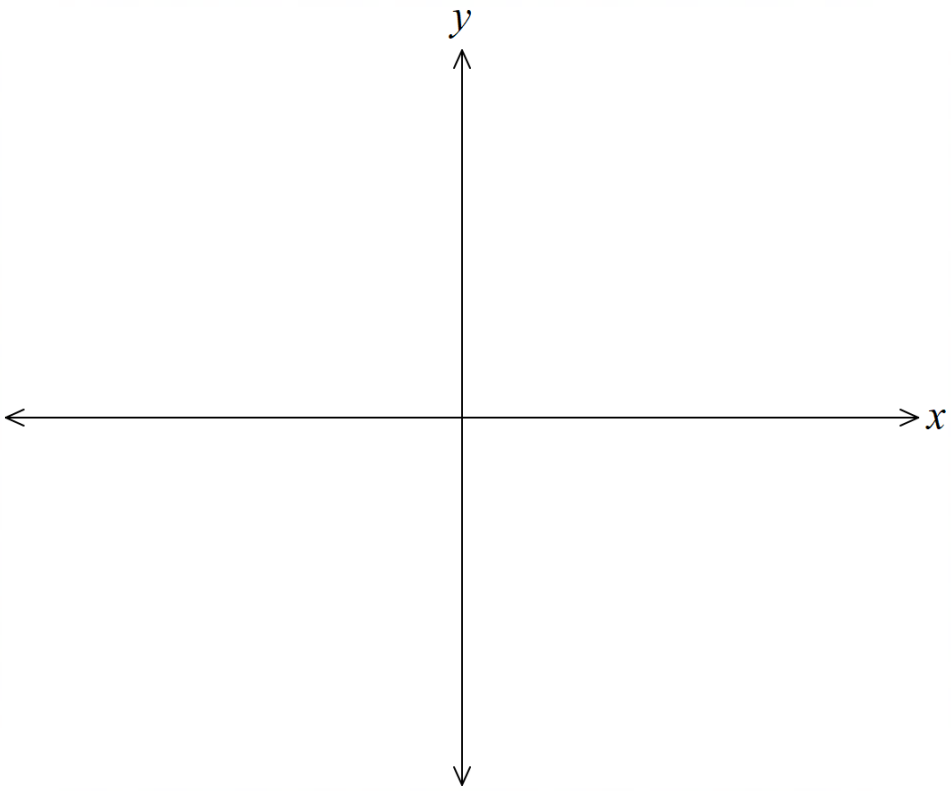


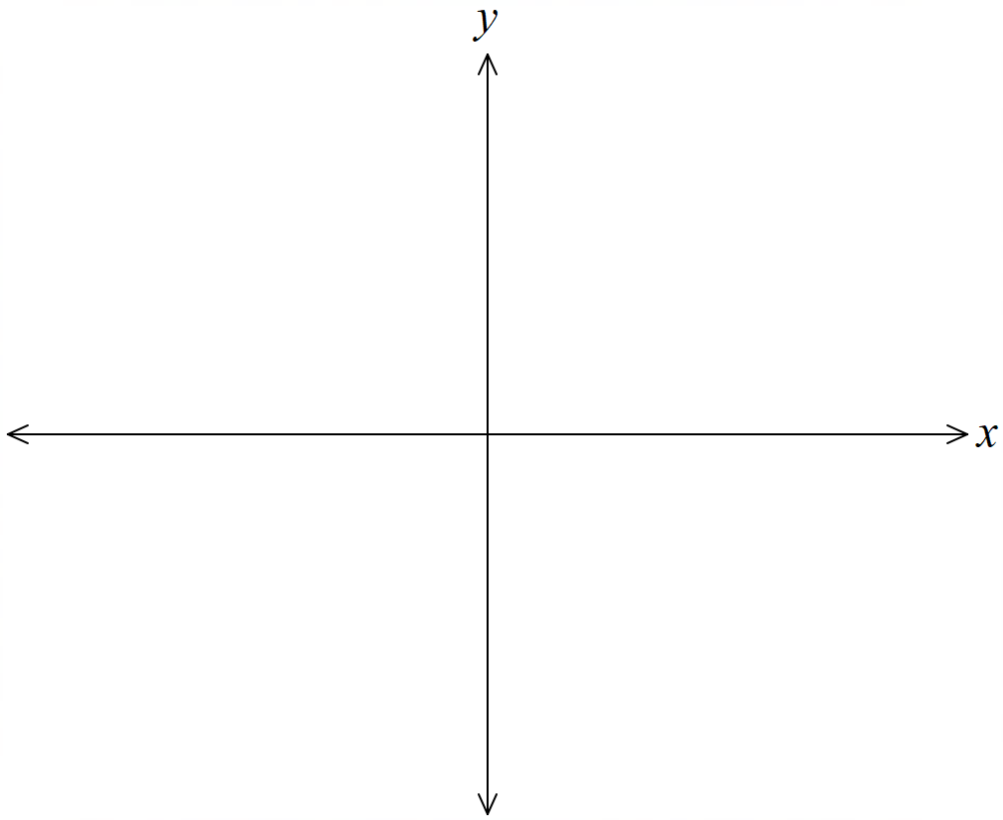
Question 14 (3 marks)

Consider the function $f(x) = \sin^2 x + \sin^4 x + \sin^6 x + ...$ **3**

For what values of x in the interval, $0 \le x \le 2\pi$, does $f(x) = \frac{1}{3}$?

Question 15 (4 marks)

(a) Sketch the graph of $y = 2|x-3|+1$. **2**



(b) Using the graph from part (i), or otherwise, find all values of m for which the equation $2|x-3|+1 = mx$ has two solutions. **2**

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Question 16 (2 marks)

Let S_n be the sum of the first n terms of an arithmetic progression with the first term a and common difference $d\,(d \neq 0)$. **2**

Suppose for two distinct positive integers m, n we have

$$\frac{S_m}{S_n} = \frac{m(2m-1)}{n(2n-1)}.$$

Prove that the m-th and n-th terms satisfy

$$T_m : T_n = (4m-3) : (4n-3).$$

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End of Exam

Extra writing space

If you use this space, clearly indicate which question you are answering.

Baulkham Hills High School

Task 1 December Examination 2025

Marking Guideline – HSC Advanced

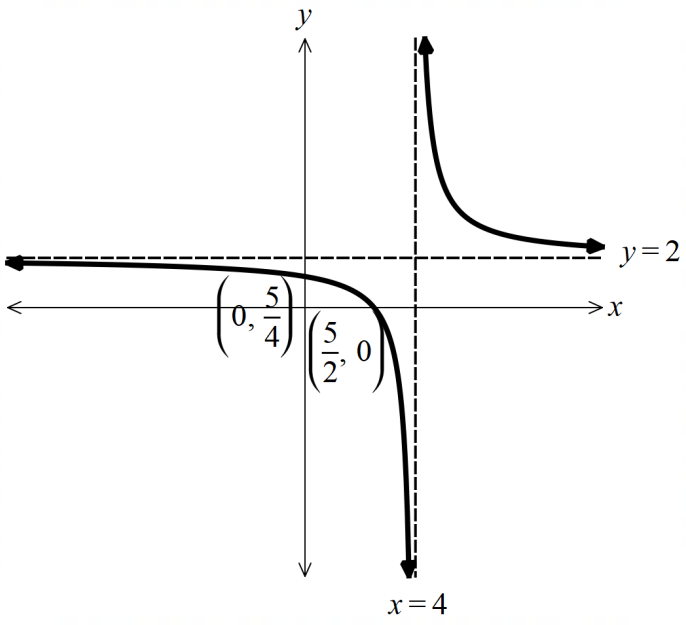
Section I (5 marks)

Award 1 mark to each correct answer.

Answers: B, D, D, C, A

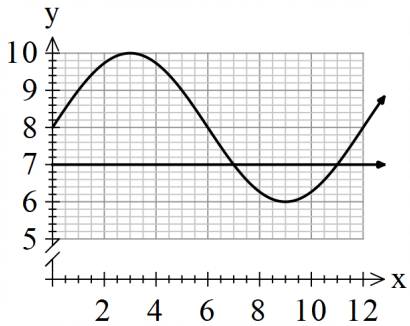
Question	Suggested solutions	Answer
1	$T_2 = 3(3) - 1$ $= 8$ $T_3 = 3(8) - 1$ $= 23$ $\therefore$ 3, 8, 23	B
2	Translated four units to the right: (6, -3) Reflected in the x-axis: (6, 3)	D
3	Given $-1 < r < 0$, the graph will converge towards 0 while alternating between positive and negative values.	D
4	$S_n = \frac{n}{2}(2a + (n-1)d)$, where $a = 50$, $d = 50$ $2250 = \frac{n}{2}(2(50) + (n-1)(50))$ $= \frac{n}{2}(100 + 50n - 50)$ $4500 = 50n + 50n^2$ $50n^2 + 50n - 4500 = 0$ $n^2 + n - 90 = 0$ $(n+10)(n-9) = 0$ $n = 9 \quad (n > 0)$	C
5	The graph must be A as the horizontal asymptote is -3 and the curve passes through (0, 0).	A

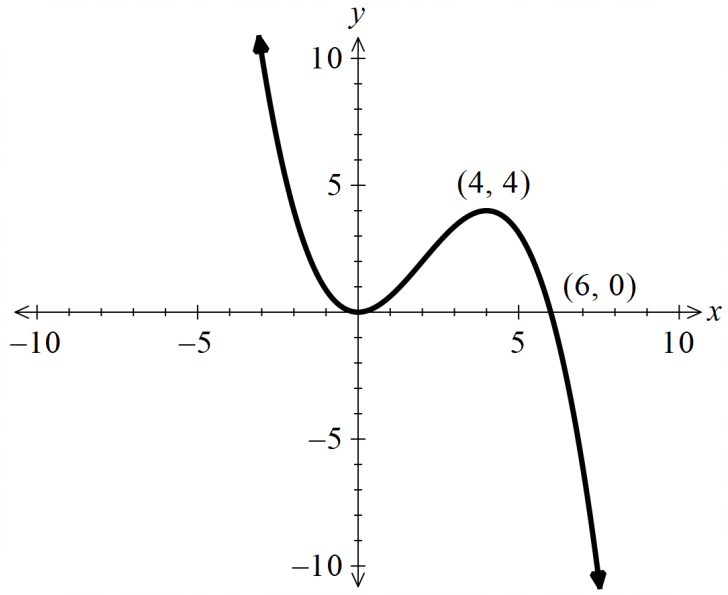
Section II (37 marks)

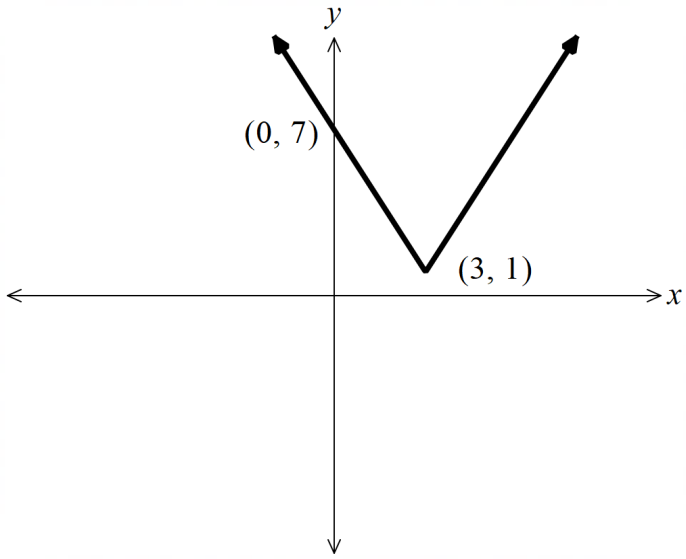
Qn	Suggested solutions	Marking Criteria
6	$a=2,\ d=7$ $2025=2+(n-1)(7)$ $2023=7n-7$ $2030=7n$ $n=290$ $S_{290}=\frac{290}{2}(2+2025)$ $=293915$	2Mk: Provides correct solution. 1Mk: Correctly applies S_n for AP.
7	$y=\sqrt{(2x)^2+1}$ Horizontally dilated by a factor of 2: $y=\sqrt{x^2+1}$ Shifted up by 3 units: $y=\sqrt{x^2+1}+3$	2Mk: Provides correct solution. 1Mk: Correctly applies one transformation.
8a	x-intercept: $0=2+\frac{3}{x-4}$ $-2=\frac{3}{x-4}$ $-2x+8=3$ $-2x=-5$ $x=\frac{5}{2}$ y-intercept: $y=2+\frac{3}{0-4}$ $=\frac{5}{4}$ Horizontal asymptote: $y=2$ Vertical asymptote: $x=4$  	3Mk: Provides correct sketch. 2Mk: Provides two correct features. 1Mk: Provides one correct feature. Features: - Shape - Intercepts - Asymptotes

8b	$\frac{2x-5}{x-4}=\frac{2x-8+8-5}{x-4}$ $=2+\frac{3}{x-4}$ $\therefore 2+\frac{3}{x-4}>\frac{5}{4}$ $\therefore x<0$ or $x>4$ from the graph.	1Mk: Provides correct answer.

9a	First three terms are: 3, 6, 12, … $\frac{T_2}{T_1}=\frac{6}{3}=2$ and $\frac{T_3}{T_2}=\frac{12}{6}=2$ Since $\frac{T_2}{T_1}=\frac{T_3}{T_2}$, the sequence is a GP. $\therefore$ The sequence is a GP, where $a=3$, $r=2$.	2Mk: Provides correct solution. 1Mk: Correct proof for GP or identifies a and r.
9b	$768=(3)(2)^{n-1}$ $256=2^{n-1}$ $n=\frac{\log_2 256}{\log_2 2}$ $=\log_2 256$ $=9$	2Mk: Provides correct solution. 1Mk: Correctly substitutes all values into the T_n formula.
9c	$\frac{3(2^n-1)}{2-1}>1000000$ $2^n>\frac{1000000}{3}+1$ $n>\frac{\ln\left(\frac{1000000}{3}+1\right)}{\ln 2}$ $n>18.3466...$ $n=19$ $\therefore$ The number of circles first exceed 1000000 after 19 terms.	2Mk: Provides correct solution 1Mk: Correctly substitutes all values into the S_n formula.

Qn	Suggested solutions	Marking Criteria
10a	Amplitude = 2 Period = $\frac{2\pi}{\left(\frac{\pi}{6}\right)}$ $=12$	2Mk: Provides correct amplitude and period. 1Mk: Provides correct amplitude or period.
10b	Low tide occurs when $h=6$. $6=2\sin\left(\frac{\pi t}{6}\right)+8$ $-1=\sin\left(\frac{\pi t}{6}\right)$ $\frac{\pi t}{6}=\frac{3\pi}{2}$ $t=9$ $\therefore$ 8am + 9hrs = 5pm	2Mk: Provides correct solution. 1Mk: Obtains the value of h at low tide.
10c	The times when the height of tide is 7m: $2\sin\left(\frac{\pi t}{6}\right)+8=7$ $\sin\left(\frac{\pi t}{6}\right)=-\frac{1}{2}$ $\frac{\pi t}{6}=\frac{7\pi}{6},\frac{11\pi}{6}$ $t=7,11$ $\therefore$ 8am + 7hrs = 3pm & 8am + 11hrs = 7pm.  $\therefore$ The times during which the ship was able to enter the harbour are 8am to 3pm and 7pm to 8pm.	3Mk: Provides correct solution. 2Mk: Correctly solves for one value of t. 1Mk: Obtains correct equation.

Qn	Suggested solutions	Marking Criteria
11	$T_4 = 10,\ S_{17} = -170$ $10 = a + 3d \quad (1)$ $-170 = \frac{17}{2}(2a + 16d)$ $-170 = 17a + 136 \quad (2)$ $(1) * 17 : 170 = 17a + 51d \quad (3)$ $(3) - (2) : 340 = 85d$ $d = -4$ $\therefore 10 = a + 3(-4)$ $a = 22$ $\therefore T_{30} = 22 + 29(-4)$ $= -94$	3Mk: Provides correct solution. 2Mk: Obtains a and d. 1Mk: Obtains the two simultaneous equations.
12	Growth: 3.2, 1.92, 1.152, … It is a GP where $a = 3.2$ & $r = 0.6$ $S_\infty = \frac{3.2}{1 - 0.6}$ $= 8$ ∴ 18 + 8 = 26cm ∴ The maximum length this fish can grow to is 26cm.	2Mk: Provides correct solution. 1Mk: Obtains a and r.
13		2Mk: Provides correct sketch. 1Mk: Obtains correct shape or turning points or intercepts.

Qn	Suggested solutions	Marking Criteria
14	$f(x)$ is a GP where $a=\sin^2 x$ & $r=\sin^2 x$. $f(x)=\dfrac{\sin^2 x}{1-\sin^2 x}$, where $x\neq\dfrac{\pi}{2},\dfrac{3\pi}{2},...$ $=\dfrac{\sin^2 x}{\cos^2 x}$ $=\tan^2 x$ $\therefore \tan^2 x=\dfrac{1}{3}$ $\tan x=\pm\dfrac{1}{\sqrt{3}}$ $x=\dfrac{\pi}{6},\dfrac{5\pi}{6},\dfrac{7\pi}{6},\dfrac{11\pi}{6}$	3Mk: Provides correct solution. 2Mk: Obtains $\tan^2 x=\dfrac{1}{3}$. 1Mk: Obtains the limiting sum.
15a		2Mk: Correct sketch. 1Mk: Correct shape or vertex.
15b	For $y=mx$ to intersect $y=2\lvert x-3\rvert+1$ twice, m could be positive or negative. If m is positive: $m=\dfrac{1-0}{3-0}$ $=\dfrac{1}{3}$ $\therefore m>\dfrac{1}{3}$ & $m<2$. $\therefore \dfrac{1}{3}<m<2$. If m is negative: there are no solutions. Therefore, the solution is: $\dfrac{1}{3}<m<2$.	2Mk: Provides correct solution. 1Mk: Obtains $m>\dfrac{1}{3}$ or $m<2$.

Qn	Suggested solutions	Marking Criteria
16	$\frac{S_m}{S_n}=\frac{m(2m-1)}{n(2n-1)}$ $\frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]}=\frac{m(2m-1)}{n(2n-1)}$ $\frac{2a+(m-1)d}{2a+(n-1)d}=\frac{(2m-1)}{(2n-1)}$ $(2n-1)(2a+(m-1)d)=(2m-1)(2a+(n-1)d)$ $4an-2a+2mnd-2nd-md+d=4am-2a+2mnd-2md-nd+d$ $4an-2nd-md=4am-2md-nd$ $4an-4am=2nd+md-2md-nd$ $4a(n-m)=nd-md$ $4a(n-m)=d(n-m)$ $4a=d \quad (\text{since } m\neq n)$ $\frac{T_m}{T_n}=\frac{a+(m-1)d}{a+(n-1)d}$ $=\frac{a+(m-1)4a}{a+(n-1)4a} \quad (\text{since } d=4a)$ $=\frac{a+4am-4a}{a+4an-4a}$ $=\frac{4am-3a}{4an-3a}$ $=\frac{a(4m-3)}{a(4n-3)}$ $=\frac{4m-3}{4n-3}$ $\therefore T_m:T_n=(4m-3):(4n-3).$	2Mk: Provides correct solution. 1Mk: Obtains $d=4a$.