
Student Number

Teacher ____________________

Cheltenham Girls High School

2020 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced

Assessment Task 2

General Instructions

- Working time – 45 minutes
- Write using black pen only
- Calculators approved by NESA may be used
- Use the multiple-choice answer sheet attached for Question 1-5
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 30	**Section I – 5 marks** (pages 3 – 4) • Attempt Questions 1– 5 • Allow about 7 minutes for this section **Section II – 25 marks** (pages 5 - 9) • Attempt Questions 6–12 • Allow about 38 minutes for this section

Multiple Choice Q1-5	Q6-9	Q10	Q11-12	Total
/5	/11	/8	/6	/30

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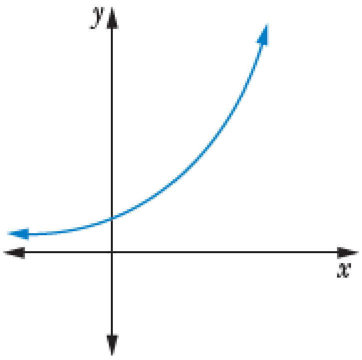
Section I – Multiple Choice (5 marks)

1 From the curve shown, which inequalities are correct?



A $\frac{dy}{dx} > 0, \frac{d^2y}{dx^2} < 0$

B $\frac{dy}{dx} > 0, \frac{d^2y}{dx^2} > 0$

C $\frac{dy}{dx} < 0, \frac{d^2y}{dx^2} < 0$

D $\frac{dy}{dx} < 0, \frac{d^2y}{dx^2} > 0$

2 The transformation of $y = f(x)$ to $y = 2f(x-3)$ is:

A Vertical dilation scale factor 2 and a horizontal shift of right 3 units

B Horizontal dilation scale factor 2 and a horizontal shift of right 3 units

C Vertical dilation scale factor 0.5 and a horizontal shift of right 3 units

D Horizontal dilation scale factor 0.5 and a horizontal shift of right 3 units

3 The derivative of $y = 5\ln(5-x)$ is:

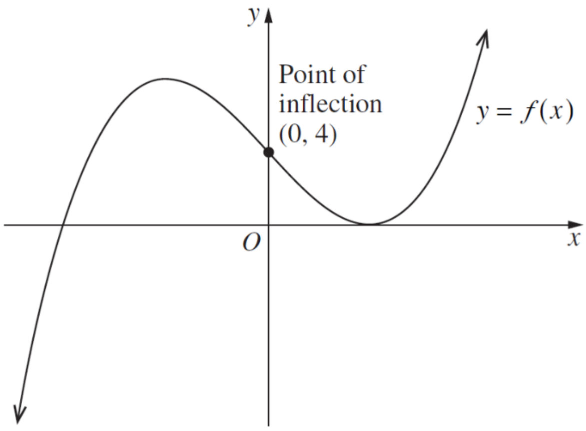
A $y' = -5\ln(5-x)$

B $y' = \frac{-1}{1-x}$

C $y' = \frac{-1}{5-x}$

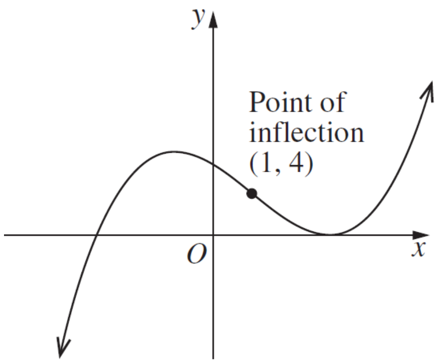
D $y' = \frac{-5}{5-x}$

4 The diagram shows the graph of $y = f(x)$.



Which of these graphs represents $y = f(x + 1)$?

A



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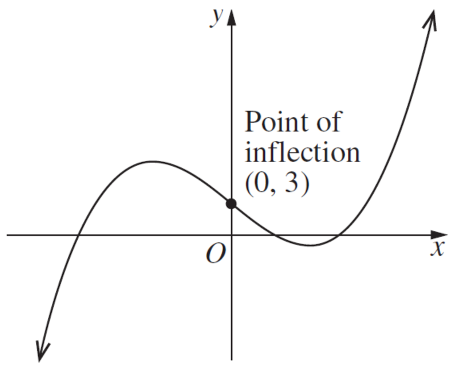
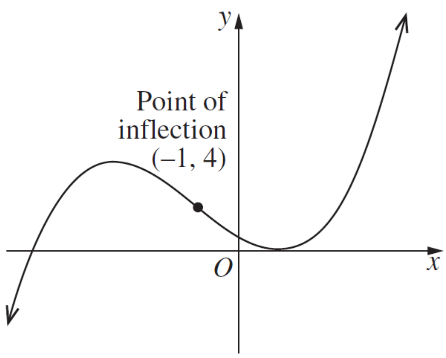
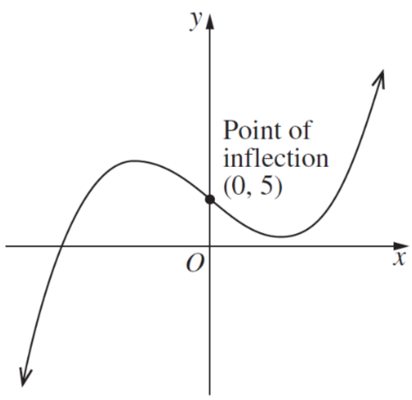


C



D



5 An object is attached to the end of a vertical spring. The object is released at time $t = 0$ and its position at time t is given by $f(t) = 5\cos t$. By examining the velocity and acceleration of the object you can describe its motion at 1.5 seconds as:

A Moving right, speeding up.

B Moving left, slowing down.

C Moving right, slowing down.

D Moving left, speeding up.

Section 2 – Written Response (25 marks) **Mark**

6 Differentiate the following:

a) $f(x) = \sin(3x + 5)$ 1

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b) $f(t) = e^{-0.5t}(t^2 + 1)$ 2

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7 By considering the sign of the first derivative, show that the function $f(x) = \frac{1}{3x-2}$ is decreasing throughout its domain. 2

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8 a) Sketch the graph of $y = (x-2)^2 - 4$. 2

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b) From the graph solve the inequality of $(x-2)^2 - 4 < 0$. 1

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9 If $y = \ln\left(\frac{x}{x-1}\right)$ show that $y'' = \frac{1}{(x-1)^2} - \frac{1}{x^2}$ 3

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Questions 6-9 are worth 11 marks in total

10 Consider the curve $y = \frac{1}{4}x^4 - x^3$

(a) Determine the nature of any stationary points and find any points of inflection. 4

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Question 10 continues on the next page

Question 10 continued 3

(b) Sketch the curve for $-1.5 \leq x \leq 4.5$, indicating where the curve crosses the x-axis.

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(c) What is the maximum value within the given domain? 1

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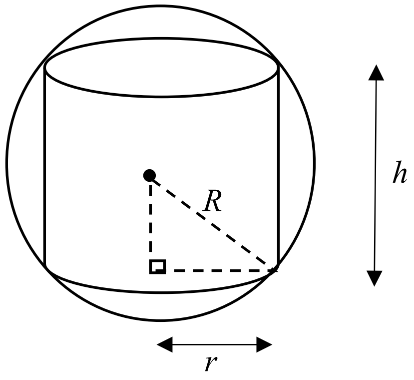
11 A cylinder of height h and radius r inscribed in a sphere of radius 20 cm, as shown in the diagram below.



(a) Show that the volume, V, of the cylinder is given by $V = \frac{1}{4}\pi(1600h - h^3)$. 2

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b) Find the value of h for which the volume of the cylinder is a maximum. Give a reason why this value of h gives the maximum volume.

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12 The coordinates of the image point of the vertex (x, y) of a parabola are (-3, 5) when $y = f(x)$ is transformed to $y = -2f(3x + 6) - 1$. Find the coordinates of the original point (x, y). 2

End of Test

Section II extra writing space

If you use this space, Clearly indicate which question you are answering

Student Number

Student Name

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Mathematics Advanced

Assessment Task 2

Section I - Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ✗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ✗ B ✗ C ○ D ○

Start Here →

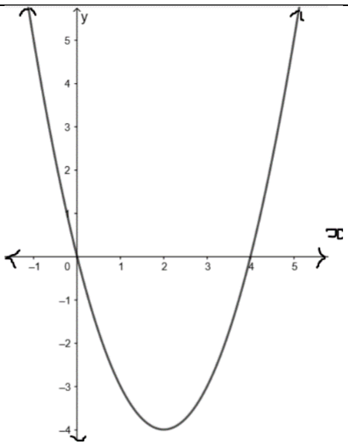
1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

Solutions	**Marking Criteria**
1. B 2. A 3. D 4. C 5. D	1 mark for each correct solution
6. $a)\ f'(x) = 3cos(3x+5)$	1 mark for correct solution
6. b) $f'(x) = vu' + uv'$ $= (t^2+1) \times -0.5e^{-0.5t} + e^{-0.5t}(2t)$ $= e^{-0.5t}(-0.5t^2 + 2t - 0.5)$	1 mark for some progress towards solution 2 marks for correct solution
7. $f'(x) = -3 \times (3x-2)^{-2}$ $= \frac{-3}{(3x-2)^2}$ $\therefore as\ \ (3x-2)^2 > 0\ for\ all\ x,$ $f'(x)\ will\ always\ be\ a\ negative\ for\ all\ values\ of\ x.$ Therefore the function is decreasing throughout its domain.	1 mark for finding correct derivative or connecting the negative gradient to decreasing function 2 marks for the correct solution.
8. a) 	1 mark for some of the correct features 2 marks for the correct sketch
b) $0 < x < 4$	1 mark for correct answer
9. $y = \ln(x) - \ln(x-1)$ $y' = \frac{1}{x} - \frac{1}{x-1}$ $y'' = -1x^{-2} - (-1)(x-1)^{-2}$ $y'' = \frac{-1}{x^2} + \frac{1}{(x-1)^2}$ $\therefore y'' = \frac{1}{(x-1)^2} - \frac{1}{x^2}\ as\ required$	1 mark for splitting expression 2 marks for making some progress 3 marks for the correct solution

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10. a) $y = \frac{1}{4}x^4 - x^3$

Stationary points $y' = 0$

$y' = x^3 - 3x^2$

$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$\therefore x = 0\,, y = 0\ and$

$x = 3, y = -6.75$

Nature:

$y'' = 3x^2 - 6x$

Sub $x = 0, y'' = 0$

Sub $x = 3, y'' = 9$

Therefore (0,0) is a possible point of inflection and (3,-6.75) is a minimum turning point.

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Points of inflection $y'' = 0$

$0 = 3x^2 - 6x$

$0 = 3x(x-2)$

$\therefore x = 0, y = 0\ and\ x = 2, y = -4$

Check concavity

<table>
<tr><td>x</td><td>-0.1</td><td>0</td><td>0.1</td></tr>
<tr><td>y''</td><td>0.63</td><td>0</td><td>-0.57</td></tr>
</table>

<table>
<tr><td>x</td><td>1.9</td><td>2</td><td>2.1</td></tr>
<tr><td>y''</td><td>-0.57</td><td>0</td><td>0.63</td></tr>
</table>

Therefore as concavity changes (0,0) is a horizontal point of inflection and (2,-4) is a point of inflection.

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1 mark for find the derivative

2 marks for making some progress

3 marks for making substantial progress

4 marks for the correct solution

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b) x-intercepts

sub in y=0

x=0 or x=4

y-intercepts

sub in x=0

y=0

Endpoints

Sub x=-1.5 y =4.6 (to 1 dec. pl.)

Sub x=4.5 y = 11.4 (to 1 dec.pl.)

If no scale on axes must mark endpoints and turning points

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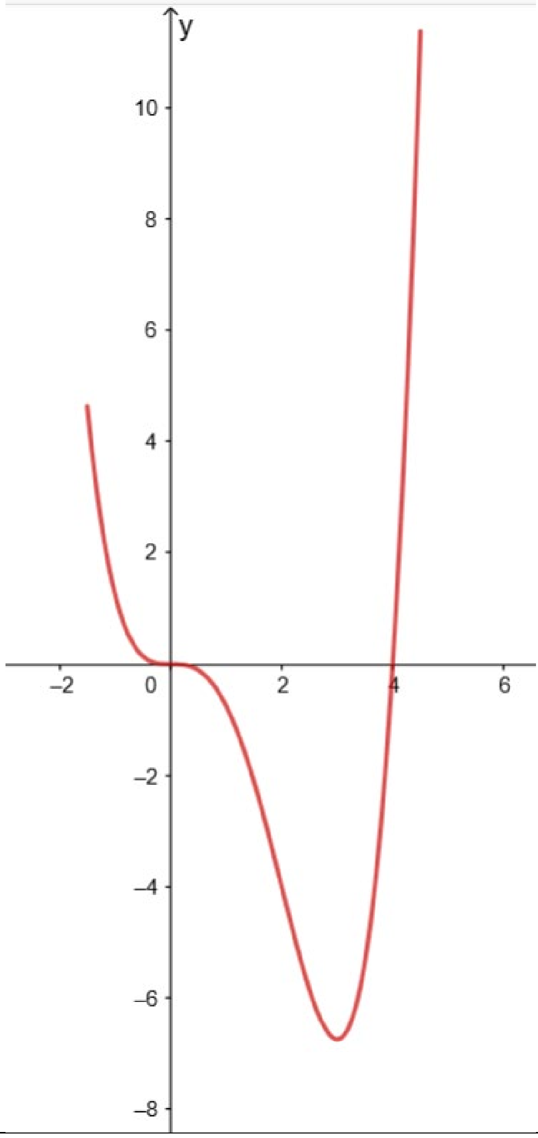

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1 mark for correct shape or intercepts or correct end points

2 marks for substantial progress

3 marks for correct solution

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c) from the graph, 11.4 (to 1 decimal place)

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1 mark for correct answer

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11. a) $Using\ Pythagoras'theorem\ R^2 = \left(\frac{h}{2}\right)^2 + r^2$

$20^2 = \frac{h^2}{4} + r^2$

$r^2 = 400 - \frac{h^2}{4}$

$V = \pi r^2 h$

$V = \pi(400 - \frac{h^2}{4}) \times h$

$\therefore V = \frac{1}{4}\pi\left(1600h - h^3\right)$ as required

</td>
<td>1 mark for making some progress

2 marks for the correct solution</td>
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$b)\ \frac{dV}{dh} = \frac{1}{4}\pi\left(1600 - 3h^2\right)$

$0 = \frac{1}{4}\pi\left(1600 - 3h^2\right)$

$0 = \left(1600 - 3h^2\right)$

$3h^2 = 1600$

$\therefore h = \frac{40}{\sqrt{3}}$

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$\frac{d^2V}{dh^2} = \frac{\pi}{4} \times -6h = \frac{-3\pi h}{2}$

Sub in $h = \frac{40}{\sqrt{3}}$

$\frac{d^2V}{dh^2} = -108.8$ (to 1 dec. pl.)

Therefore the volume of the cylinder is a maximum when $h = \frac{40}{\sqrt{3}}$. This value of h gives a maximum value for the volume as it occurs graphically at a maximum turning point as the second derivative is a positive value.

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<td>1 mark for making some progress

2 marks for the correct solution</td>
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12. let (x,y) be the original point of the vertex.

Then order of transformation is HD of a scale factor of 1/3 then HT of 2 units left, then VD of a scale factor -2 and lastly VT of down 1 unit.

$(\frac{1}{3}x - 2$, -2y-1) = (-3, 5)

$\frac{1}{3}x - 2 = -3$ and -2y-1=5

x = -3 y = -3

Therefore original vertex is (-3,-3)

</td>
<td>1 mark for making some progress

2 marks for the correct solution</td>
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Markers Comments

- Question 6: Generally well done. Please check for careless errors by checking your answer reflects the correct rule on the reference sheet.
- Question 7: Poorly done. Careless errors with finding the derivative as using chain rule must times by the $f'(x)$. The most common error was the proof behind proving it is a negative function. Needed more detail <u>if a fraction</u> as must prove the denominator is always positive then with a negative numerator will always have a negative function.
- Question 8: Generally well done. Graph of parabola was particularly well done by most students. Careless errors with forgetting to mark important features ie turning point and axes intercepts still occurring.

  Part (b) not as well done. Review of how to read OFF a graph to solve inequalities is needed. Be careful to note if less than sign means when y<0 and do not introduce equal sign anywhere in solution.
- Question 9: Poorly done due to students forgetting the logarithm rules and how to apply them in a deriving of logarithm problem. Students that used the quotient rule method are making the question way more complicated than necessary and generally could not get the algebra to the correct solution required. Reflect on the method shown in the solution, practice this method as it will save time and prevent errors in the future.
- Question 10 (a) was generally poorly done. Many students did not find the second derivative, and therefore did not realise that there is a point of inflection at (2, -4). Of those who did identify the point of inflection, a number did not test for a change in concavity. Whilst many recognised that there is a stationary point at (0,0), they stated that it was point of inflection rather than a horizontal point of inflection.
- Students made a number of errors in Question 10(b) including, not taking into account the domain and drawing the graph beyond the two end values or not indicating the end values, not labelling or incorrectly identifying the x-intercepts, failing to flatten the graph at the horizontal point of inflection (0,0) or making it flat at the point of inflection (2, -4).
- In question 10 (c), many students stated that either (4.5, 11.39) or 4.5 was the maximum value rather than 11.39.
- Question 11 (a) was generally poorly done. Students made many assumptions about the relationship of h, r and R. Despite the diagram, we cannot assume that the height is equal to two times the radius of the sphere. The clues to help you with this question was given by the right angle triangle, whose height will be half the cylinders, and that we need to find a value for the radius in terms of the height from the "Show that" formula. Please remember with these questions, you cannot start with the answer as part of your proof and if you are completely stuck with what to do you can just use the formula in the next question.
- Question 11 (b) was equally poorly done as part (a). The question explicitly says to give reasons for why you have a maximum. To do this you need to either test a value on either side using the first derivative or sub the value into second derivative. Another common mistake was to expand the brackets as $\frac{1}{4}\pi$ is a constant.
- In Question 12, most students were able to find at least one of the original points.