

NAME: _____

CLASS: 11MTA___ or 11MTX___

CHERRYBROOK TECHNOLOGY HIGH SCHOOL



2019

YEAR 11

AP1

MATHEMATICS ADVANCED

Time allowed – 1.5 hours plus 5 minutes reading time

DIRECTIONS TO CANDIDATES:

- Write your name on the question/answer paper.
- Attempt all questions.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- For all questions show relevant mathematical reasoning and/or calculations.
- NESA approved calculators may be used.

Topic	Total
Question 1	/19
Question 2	/9
Question 3	/29
Question 4	/13
Total	/70

70 marks

Attempt All Questions

Answer the questions in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of the examination paper.

Question 1	(19 marks)	Marks
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- | | | |
|------------|-----------------------------|----------|
| (a) | Expand $(2x - 6y)(2x + 6y)$ | 1 |
|------------|-----------------------------|----------|

- | | | |
|------------|--|----------|
| (b) | Express in simplest form $(3\sqrt{2} - 2)^2$ | 2 |
|------------|--|----------|

- | | | |
|------------|--|----------|
| (c) | Simplify $\sqrt{12} + \sqrt{18} - \sqrt{48}$ | 2 |
|------------|--|----------|

- | | | |
|------------|---|----------|
| (d) | Solve $6 - 3x - 5x^2 = 0$ using the quadratic formula, correct to 3 decimal places. | 2 |
|------------|---|----------|

- (e) Solve the quadratic equation $x^2 + 4x - 10 = 0$ by completing the square.
Leave your answer in surd form.

2

- (f) Simplify

(i) $\frac{x+3}{2} - \frac{2x-1}{5}$

2

(ii) $\frac{x^2 - 9}{x^2 - 2x - 15} \div \frac{x^2 - 5x + 6}{3x^2 - 12}$

3

- (g) Simplify $\frac{(3x^2)^2 \times \sqrt[3]{x}}{3x^5}$, leaving your answer without negative and fractional indices. **2**

- (h) Expand and simplify $\sqrt{2}(2\sqrt{10} + 9\sqrt{8}) - \frac{\sqrt{5}-2}{\sqrt{5}}$ with a rational denominator. **3**

End of Question 1

Question 2 (9 marks)

Marks

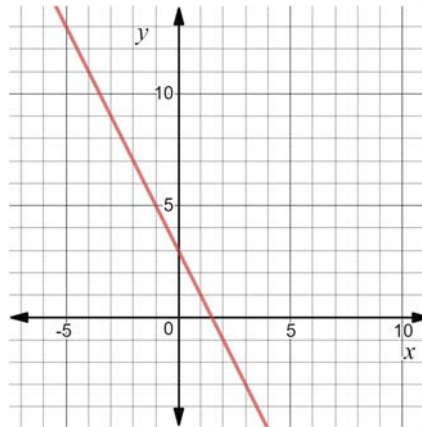
- (a) The function $f(x)$ is defined by $f(x) = \begin{cases} 4^x, & x \leq 1 \\ \frac{4}{x}, & x > 1 \end{cases}$. What is the value of $f(0 \cdot 5) + f(2)$?

- (A) 4
(B) 10
(C) 18
(D) 24

1

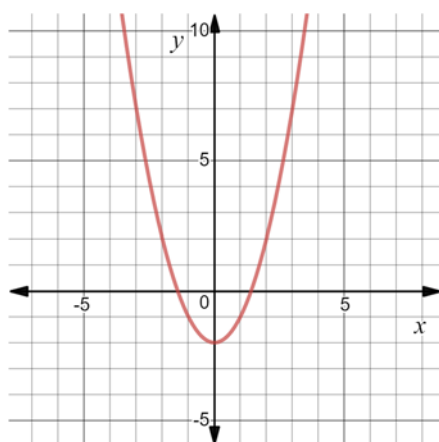
- (b) Determine **whether the following relations are also functions** and then categorise each graph as either one-to-one, one-to-many, many-to-one or many-to-many.

(i)



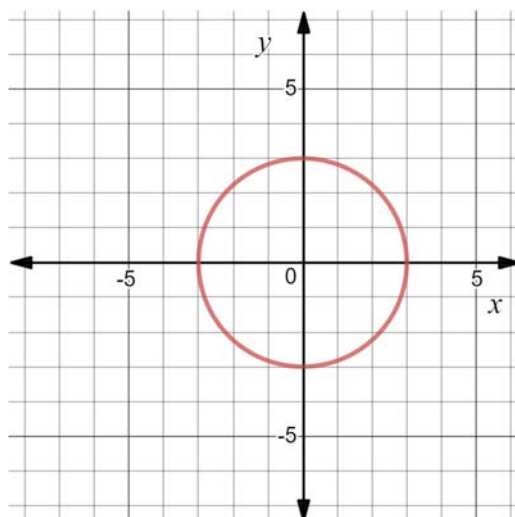
1

(ii)



1

(iii)



1

(c) Given that $f(x) = 3x^2 + 1$, find $f(-2)$

1

(d) If $A(x) = x^2 - 6$ and $B(x) = 11x - 2x^2$. Simplify

(i) $B(x) - A(x)$ **1**

(ii) $A(x)B(x)$ **1**

(iii) $A(B(x))$ **2**

End of Question 2

Question 3 (29 marks)**Marks**

(a) A line ℓ has equation $3y + 2x = 6$. What is the gradient of any line parallel to ℓ ?

(A) -2

(B) $-\frac{2}{3}$

(C) $\frac{3}{2}$

(D) 2

1

(b) What is the discriminant of $2x^2 + 3x + k = 0$?

(A) $\Delta = \sqrt{9 - 8k}$

(B) $\Delta = 9 - 8k$

(C) $\Delta = 9 - 12k$

(D) $\Delta = \sqrt{9 - 12k}$

1

(c) For $y = 2x^2 + 6x + 2$, the minimum value of y is:

(A) $-\frac{23}{2}$

(B) $-\frac{7}{2}$

(C) $-\frac{5}{2}$

(D) $\frac{31}{2}$

1

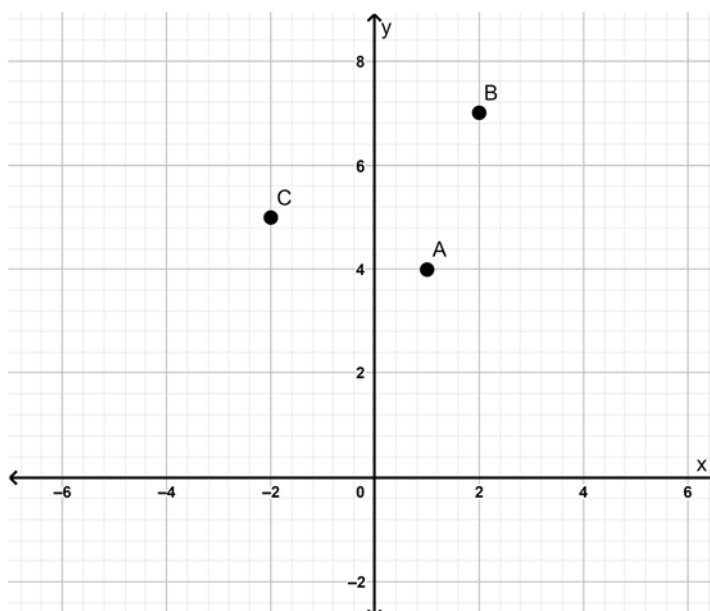
(d) Find the equation of the line in general form that passes through the points $(8,0)$ and $(0,6)$.

2

- (e) Find the values of p for which the equation $x^2 + (p - 1)x + 1 = 0$ has equal roots.

2

- (f) Three points A, B, C have coordinates $(1,4)$, $(2,7)$ and $(-2,5)$ respectively.



- (i) Calculate the length of BC , leave your answer in exact form.

1

- (ii) Show that the equation of the line BC is $x - 2y + 12 = 0$ 2

- (iii) Show that the equation of the line, ℓ , perpendicular to BC which passes through A is $2x + y - 6 = 0$. 2

- (iv) Hence, find the coordinates of P , the point where the ℓ meets the line BC . 2

- (g) For the following quadratic function $y = x^2 + 6x$.

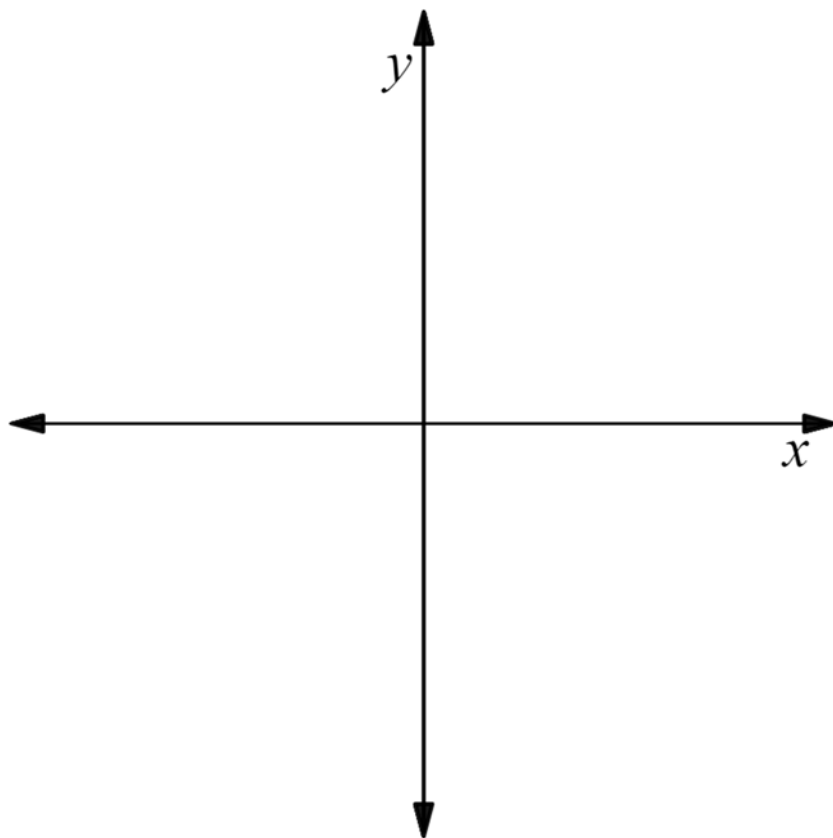
- (i) Find the x and y intercepts. 2

(ii) Find the coordinates of the vertex.

1

(iii) Neatly sketch the parabola $y = x^2 + 6x$, clearly showing all intercepts.

2



(iv) State the domain and range using interval notation.

2

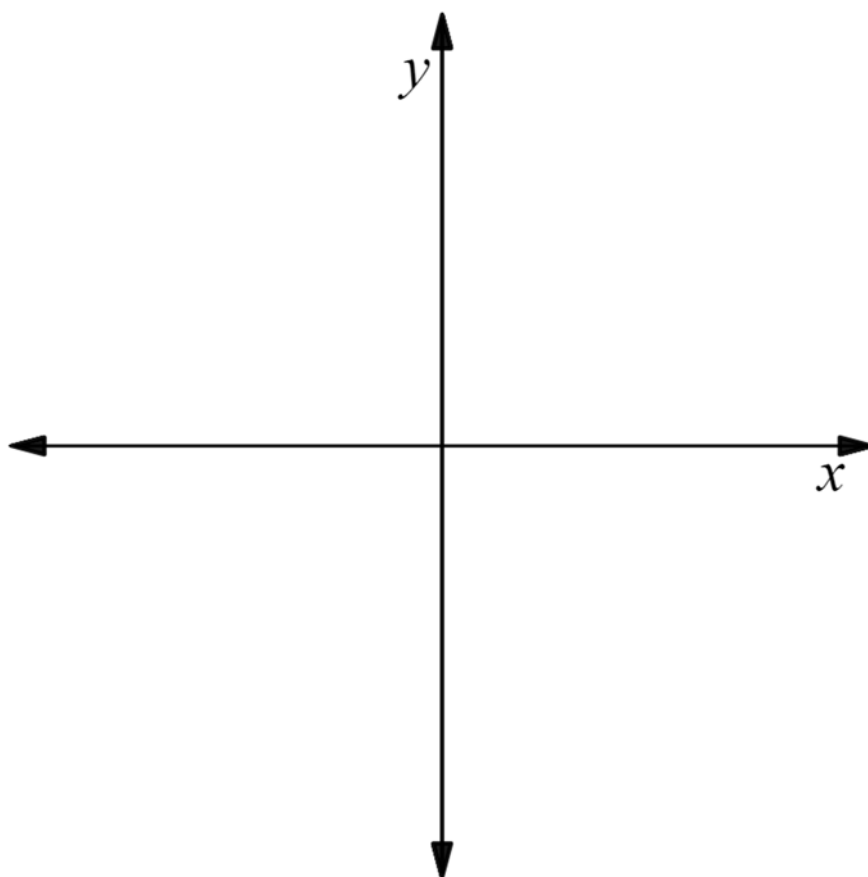
- (h) Solve the following simultaneous equations.

$$\begin{aligned}4x &= 12 + y \\ y &= x^2 - x - 12\end{aligned}$$

3

- (i) Sketch the graph of $y = -(x - 3)(x - 1)(x + 1)$, showing all essential features.

2



- (j) The cost of producing x litres of orange juice is $20 + x$ dollars. If the income from the sale of x litres of the juice is $2x^2$ dollars, calculate the number of litres that must be sold to make a profit of \$800. **3**

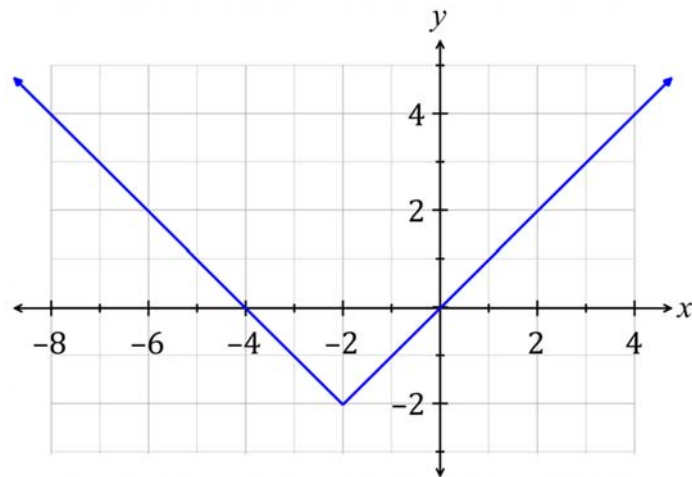
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End of Question 3

Question 4 (13 marks)

Marks

(a) What is the equation of the function below?



(A) $y = |x + 2| - 2$

(B) $y = |x - 2| + 2$

(C) $y = |2 - x| - 2$

(D) $y = |2 + x| + 2$

1

(b) For the polynomial $P(x) = 7x^6 + 5x^3 + 9x - 2$

(i) Find the degree of $P(x)$

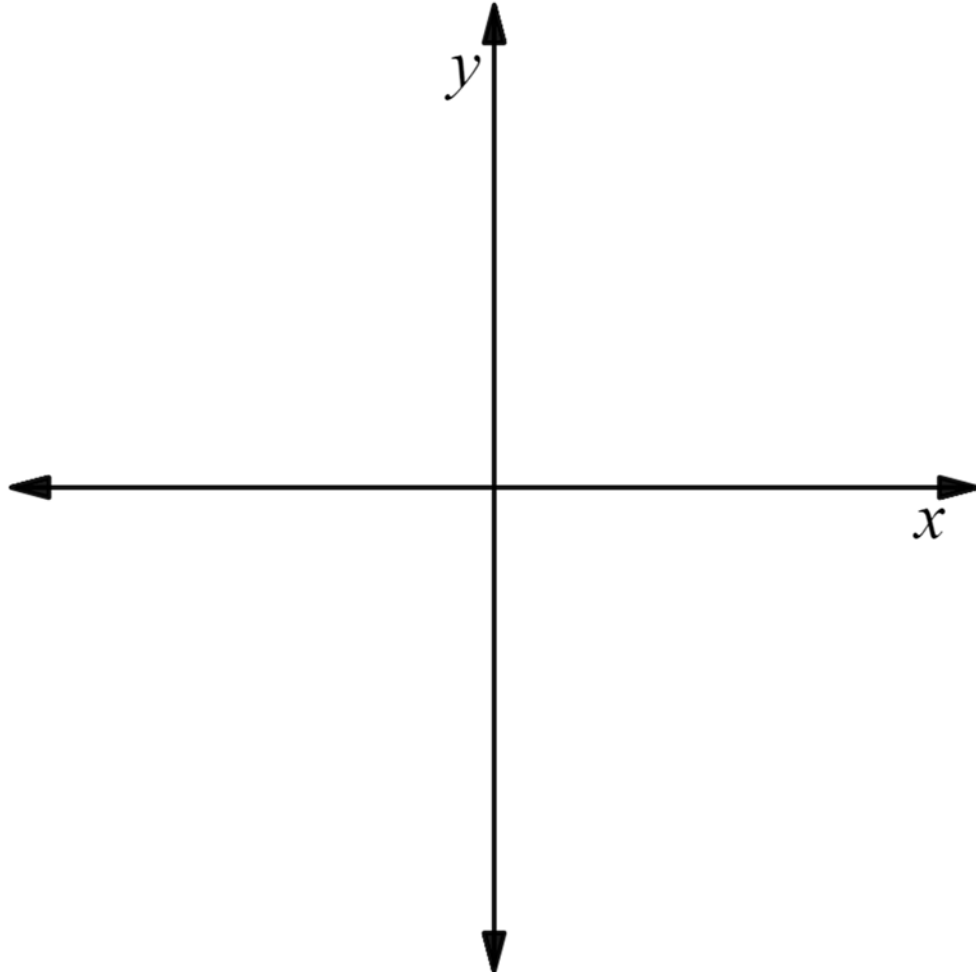
1

(ii) State the leading term

1

- (c) (i) Show that $x(x + 1)(x - 2)^2 = x^4 - 3x^3 + 4x$ 2

- (ii) Hence, sketch the graph of $P(x) = x^4 - 3x^3 + 4x$ 1



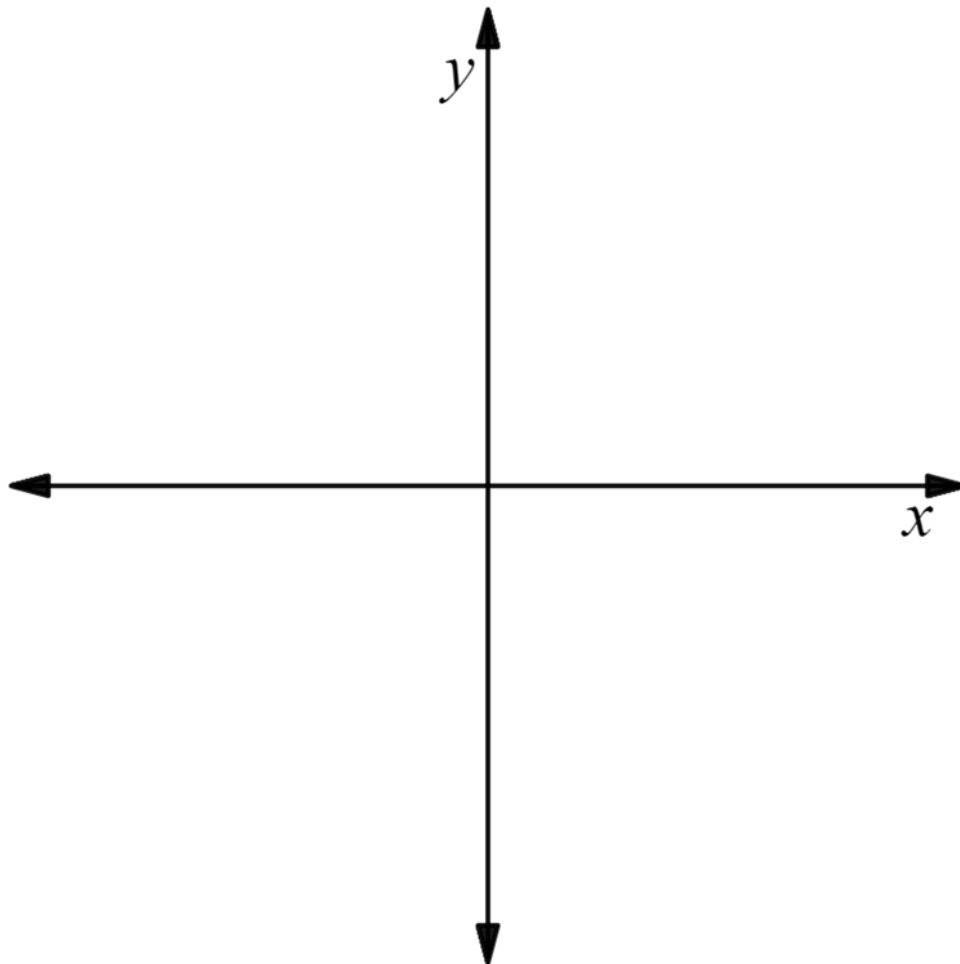
- (d) Consider the function $f(x) = \sqrt{9 - x^2}$
- (i) State the domain and range of $f(x)$, in interval notation. 2

- (ii) State whether the function $f(x)$, is odd, even or neither.
You must justify your answer algebraically.

2

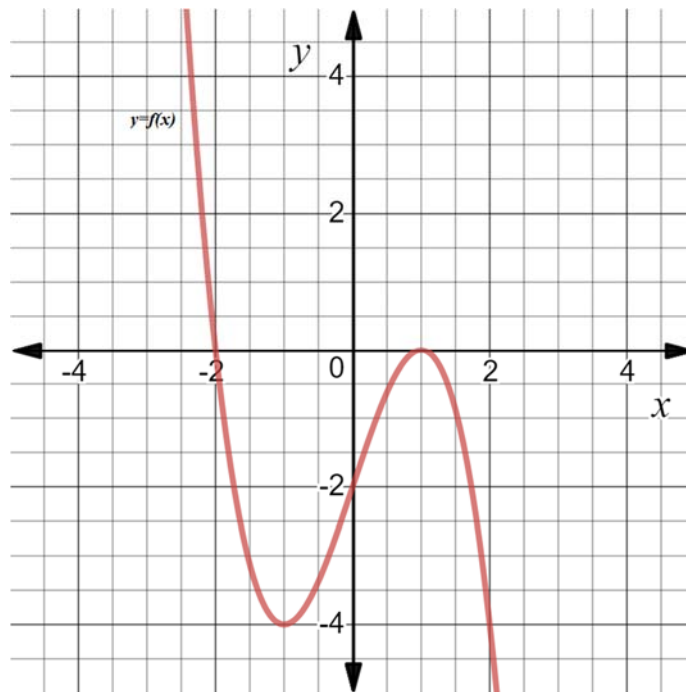
- (iii) Sketch the graph of $f(x) = \sqrt{9 - x^2}$, showing all intercepts.

1



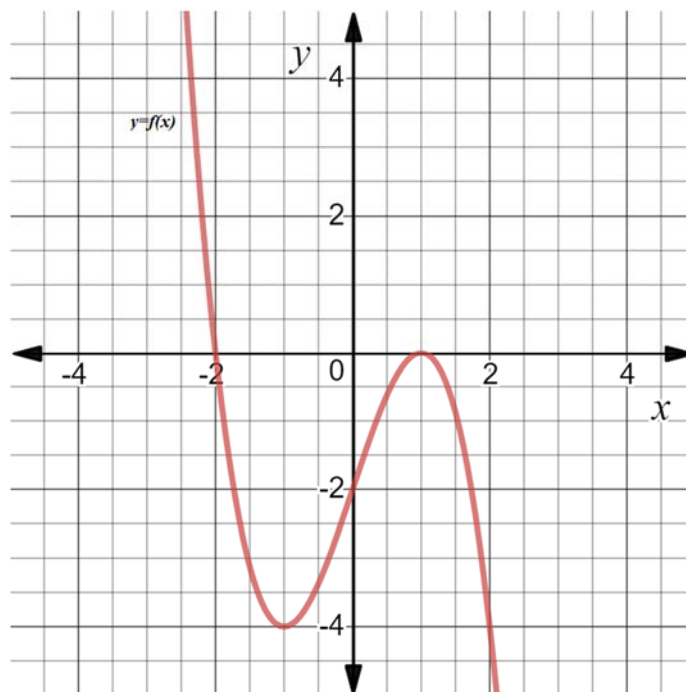
- (e) The graph of $y = f(x)$ is shown below. Sketch the graph of
- (i) $y = -f(x)$

1



- (ii) $y = -f(-x)$

1



End of Paper

Extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

[illegible]

70 marks

Attempt All Questions

Answer the questions in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of the examination paper.

Question 1 (19 marks)

Marks

- (a) Expand $(2x - 6y)(2x + 6y)$

1

$$4x^2 - 36y^2$$

- (b) Express in simplest form $(3\sqrt{2} - 2)^2$

2

$$\begin{aligned} &= (3\sqrt{2})^2 - 2 \times (3\sqrt{2}) \times 2 + 2^2 \\ &= 18 - 12\sqrt{2} + 4 \\ &= 22 - 12\sqrt{2} \end{aligned}$$

- (c) Simplify $\sqrt{12} + \sqrt{18} - \sqrt{48}$

2

$$\begin{aligned} &= 2\sqrt{3} + 3\sqrt{2} - 4\sqrt{3} \\ &= -2\sqrt{3} + 3\sqrt{2} \quad \text{or} \quad 3\sqrt{2} - 2\sqrt{3} \end{aligned}$$

- (d) Solve $6 - 3x - 5x^2 = 0$ using the quadratic formula, correct to 3 decimal places.

2

$$a = -5, b = -3, c = 6$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(-5) \times 6}}{2(-5)}$$

$$= \frac{3 \pm \sqrt{9 + 120}}{-10}$$

$$= \frac{3 \pm \sqrt{129}}{-10}$$

$$\therefore x = \frac{3 + \sqrt{129}}{-10} \quad \text{or} \quad x = \frac{3 - \sqrt{129}}{-10}$$

$$x = -1.436$$

$$x = 0.836$$

- (e) Solve the quadratic equation $x^2 + 4x - 10 = 0$ by completing the square.
Leave your answer in surd form.

2

$$x^2 + 4x = 10$$

$$x^2 + 4x + 4 = 10 + 4$$

$$(x + 2)^2 = 14$$

$$x + 2 = \pm \sqrt{14}$$

$$x = \pm \sqrt{14} - 2 \quad \text{or} \quad -2 \pm \sqrt{14}$$

- (f) Simplify

(i) $\frac{x+3}{2} - \frac{2x-1}{5}$

2

$$\frac{5(x+3)}{10} - \frac{2(2x-1)}{10}$$

$$= \frac{5x+15 - 4x+2}{10}$$

$$= \frac{x+17}{10}$$

(ii) $\frac{x^2 - 9}{x^2 - 2x - 15} \div \frac{x^2 - 5x + 6}{3x^2 - 12}$

3

$$= \frac{x^2 - 9}{x^2 - 2x - 15} \times \frac{3x^2 - 12}{x^2 - 5x + 6}$$

$$= \frac{(x-3)(x+3)}{(x-5)(x+3)} \times \frac{3(x+2)(x-2)}{(x-3)(x-2)}$$

$$= \frac{3(x+2)}{(x-5)} \quad \text{or} \quad \frac{3x+6}{x-5}$$

(g)

Simplify $\frac{(3x^2)^2 \times \sqrt[3]{x}}{3x^5}$, leaving your answer without negative and fractional indices.

2

$$= \frac{9x^4 \times x^{\frac{1}{3}}}{3x^5}$$

$$= 3x^4 \times x^{\frac{1}{3}} \times x^{-5}$$

$$= 3x^{4+\frac{1}{3}-5}$$

$$= 3x^{-\frac{2}{3}} \quad \text{or} \quad \frac{3}{x^{\frac{2}{3}}}$$

$$= \frac{3}{\sqrt[3]{x^2}} \quad \text{or} \quad \frac{3}{(\sqrt[3]{x})^2}$$

(h)

Expand and simplify $\sqrt{2}(2\sqrt{10} + 9\sqrt{8}) - \frac{\sqrt{5}-2}{\sqrt{5}}$ with a rational denominator.

3

$$= 2\sqrt{20} + 9\sqrt{16} - \frac{(\sqrt{5}-2) \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$$

$$= 4\sqrt{5} + 36 - \frac{\sqrt{5}(\sqrt{5}-2)}{5}$$

$$= 4\sqrt{5} + 36 - \frac{(5-2\sqrt{5})}{5}$$

$$= \frac{20\sqrt{5} + 180 - 5 + 2\sqrt{5}}{5}$$

$$= \frac{22\sqrt{5} + 175}{5} \quad \text{or} \quad \frac{22\sqrt{5}}{5} + 35$$

End of Question 1

- (a) The function $f(x)$ is defined by $f(x) = \begin{cases} 4^x, & x \leq 1 \\ \frac{4}{x}, & x > 1 \end{cases}$. What is the value of $f(0.5) + f(2)$?

- (A) 4
(B) 10
(C) 18
(D) 24

$$f(0.5) = 4^{0.5} = 2$$

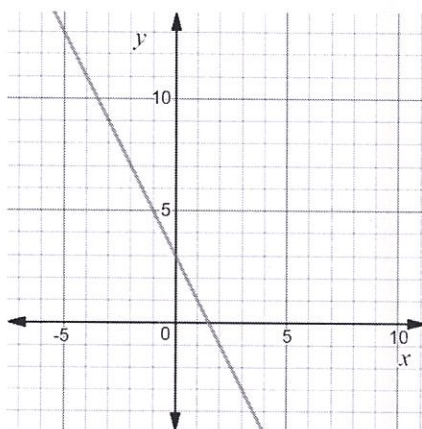
$$f(2) = \frac{4}{2} = 2$$

$$\therefore f(0.5) + f(2) = 2 + 2 = 4$$

1

- (b) Determine **whether the following relations are also functions** and then categorise each graph as either one-to-one, one-to-many, many-to-one or many-to-many.

(i)

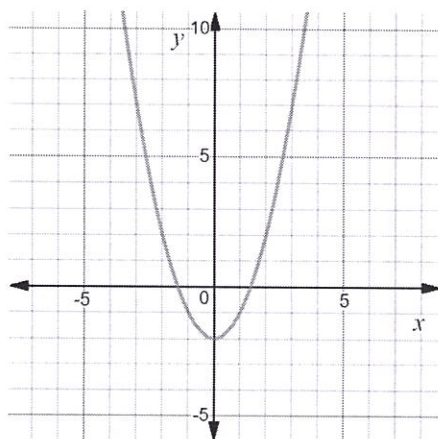


1

function

one to one

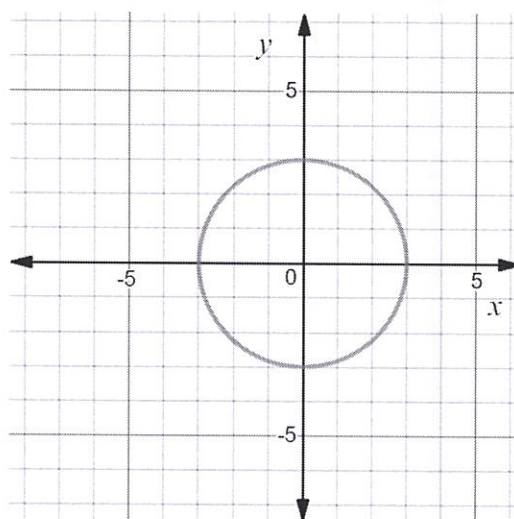
(ii)



1

function
many to one

(iii)



1

not a function - relation
many to many

(c) Given that $f(x) = 3x^2 + 1$, find $f(-2)$

1

$$\begin{aligned} f(-2) &= 3(-2)^2 + 1 \\ &= 12 + 1 \\ &= 13 \end{aligned}$$

(d) If $A(x) = x^2 - 6$ and $B(x) = 11x - 2x^2$. Simplify

(i) $B(x) - A(x)$

1

$$\begin{aligned} & 11x - 2x^2 - (x^2 - 6) \\ &= 11x - 2x^2 - x^2 + 6 \\ &= -3x^2 + 11x + 6 \end{aligned}$$

(ii) $A(x)B(x)$

1

$$\begin{aligned} & (x^2 - 6)(11x - 2x^2) \\ &= 11x^3 - 2x^4 - 66x + 12x^2 \end{aligned}$$

(iii) $A(B(x))$

2

$$\begin{aligned} & A(11x - 2x^2) \\ &= (11x - 2x^2)^2 - 6 \\ &= 121x^2 - 44x^3 + 4x^4 - 6 \\ &= 4x^4 - 44x^3 + 121x^2 - 6 \end{aligned}$$

End of Question 2

(a) A line ℓ has equation $3y + 2x = 6$. What is the gradient of any line parallel to ℓ ?

(A) -2

(B) $-\frac{2}{3}$

(C) $\frac{3}{2}$

(D) 2

$$y = -\frac{2}{3}x + 2$$

1

(b) What is the discriminant of $2x^2 + 3x + k = 0$?

(A) $\Delta = \sqrt{9 - 8k}$

(B) $\Delta = 9 - 8k$

(C) $\Delta = 9 - 12k$

(D) $\Delta = \sqrt{9 - 12k}$

$$\Delta = 9 - 8k$$

1

(c) For $y = 2x^2 + 6x + 2$, the minimum value of y is:

(A) $-\frac{23}{2}$

(B) $-\frac{7}{2}$

(C) $-\frac{5}{2}$

(D) $\frac{31}{2}$

$$\begin{aligned} x &= \frac{-b}{2a} \\ &= \frac{-6}{2 \times 2} \\ &= \frac{-6}{4} \end{aligned}$$

$$y = 2\left(\frac{-6}{4}\right)^2 + 6\left(\frac{-6}{4}\right) + 2$$

(d) Find the equation of the line in general form that passes through the points $(8,0)$ and $(0,6)$.

2

$$m = \frac{6-0}{0-8}$$

$$y - 0 = -\frac{3}{4}(x - 8)$$

$$= \frac{6}{-8}$$

$$4y = -3x + 24$$

$$= -\frac{3}{4}$$

$$3x + 4y - 24 = 0$$

- (e) Find the values of p for which the equation $x^2 + (p-1)x + 1 = 0$ has equal roots.

2

$$\Delta = 0$$

$$b^2 - 4ac = 0$$

$$(p-1)^2 - 4 \times 1 \times 1 = 0$$

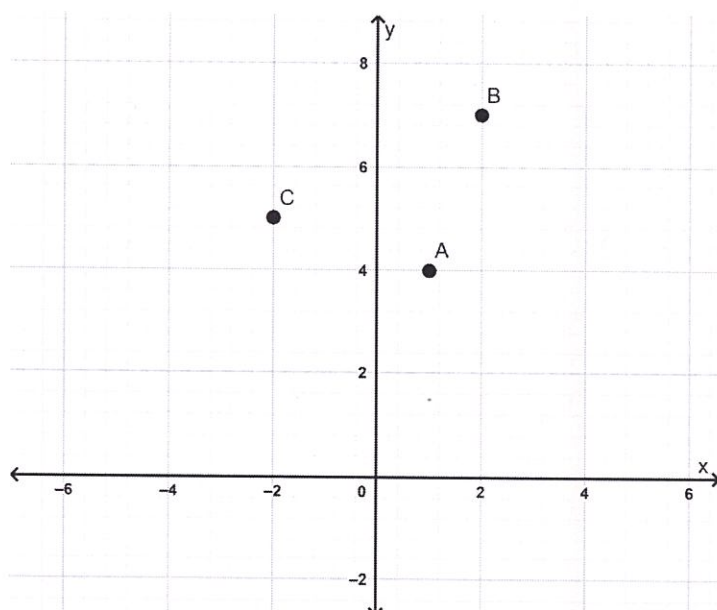
$$p^2 - 2p + 1 - 4 = 0$$

$$p^2 - 2p - 3 = 0$$

$$(p-3)(p+1) = 0$$

$$p = 3 \text{ or } p = -1$$

- (f) Three points A, B, C have coordinates $(1,4)$, $(2,7)$ and $(-2,5)$ respectively.



- (i) Calculate the length of BC , leave your answer in exact form.

1

$$B(2,7)$$

$$C(-2,5)$$

$$d = \sqrt{(2 - (-2))^2 + (7 - 5)^2}$$

$$= \sqrt{16 + 4}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

- (ii) Show that the equation of the line BC is $x - 2y + 12 = 0$

2

$$m = \frac{7-5}{2-2}$$

$$= \frac{2}{0}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

$$y - 7 = \frac{1}{2}(x - 2)$$

$$y - 7 = \frac{1}{2}x - 1$$

$$2y - 14 = x - 2$$

$$x - 2y + 12 = 0$$

- (iii) Show that the equation of the line, ℓ , perpendicular to BC which passes through A is $2x + y - 6 = 0$.

2

$$m_1 \times m_2 = -1$$

$$m = -2$$

$$y - 4 = -2(x - 1)$$

$$y - 4 = -2x + 2$$

$$2x + y - 6 = 0$$

- (iv) Hence, find the coordinates of P , the point where the ℓ meets the line BC .

2

$$x - 2y + 12 = 0 \quad \text{--- (1)}$$

$$2x + y - 6 = 0 \quad \text{--- (2)}$$

$$\text{--- } 2x - 4y + 24 = 0 \quad \text{--- (1a)}$$

$$5y - 30 = 0$$

$$5y = 30$$

$$\boxed{y = 6}$$

$$\text{sub } y = 6 \text{ into (1)}$$

$$x - 2(6) + 12 = 0$$

$$x - 12 + 12 = 0$$

$$\boxed{x = 0}$$

$$\therefore P(0, 6)$$

- (g) For the following quadratic function $y = x^2 + 6x$.

- (i) Find the x and y intercepts.

2

$$x\text{-intercept: let } y = 0$$

$$0 = x^2 + 6x$$

$$0 = x(x + 6)$$

$$x = 0 \text{ or } x = -6$$

$$y\text{-intercept: let } x = 0$$

$$y = 0^2 + 6(0)$$

$$y = 0$$

- (ii) Find the coordinates of the vertex.

1

$$y = x^2 + 6x$$

$$x\text{-coordinate } x = -3 \left(\frac{0 + -6}{2} \right)$$

sub $x = -3$ into eqn.

$$y = (-3)^2 + 6(-3)$$

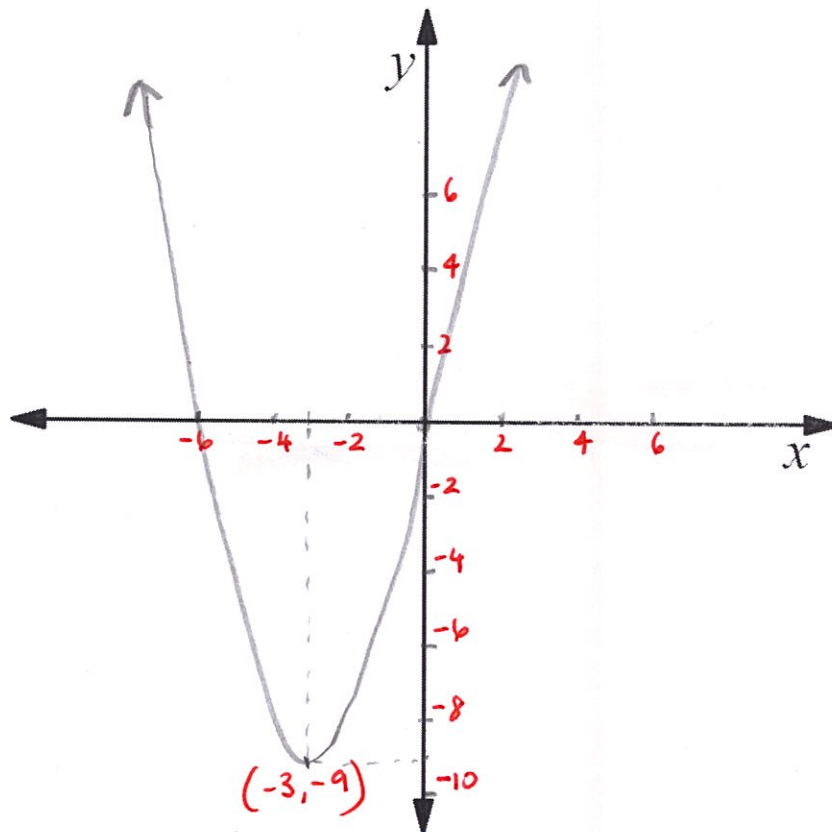
$$= 9 - 18$$

$$= -9$$

$$\therefore \text{Vertex } (-3, -9)$$

- (iii) Neatly sketch the parabola $y = x^2 + 6x$, clearly showing all intercepts.

2



- (iv) State the domain and range using interval notation.

2

$$\text{Domain: } (-\infty, \infty)$$

$$\text{Range: } [-9, \infty)$$

(h) Solve the following simultaneous equations.

$$4x = 12 + y \quad \text{--- ①}$$

$$y = x^2 - x - 12 \quad \text{--- ②}$$

Sub ② into ①

$$4x = 12 + (x^2 - x - 12)$$

$$4x = 12 + x^2 - x - 12$$

$$x^2 - 5x = 0$$

$$x(x - 5) = 0$$

$$x = 0 \text{ or } x = 5$$

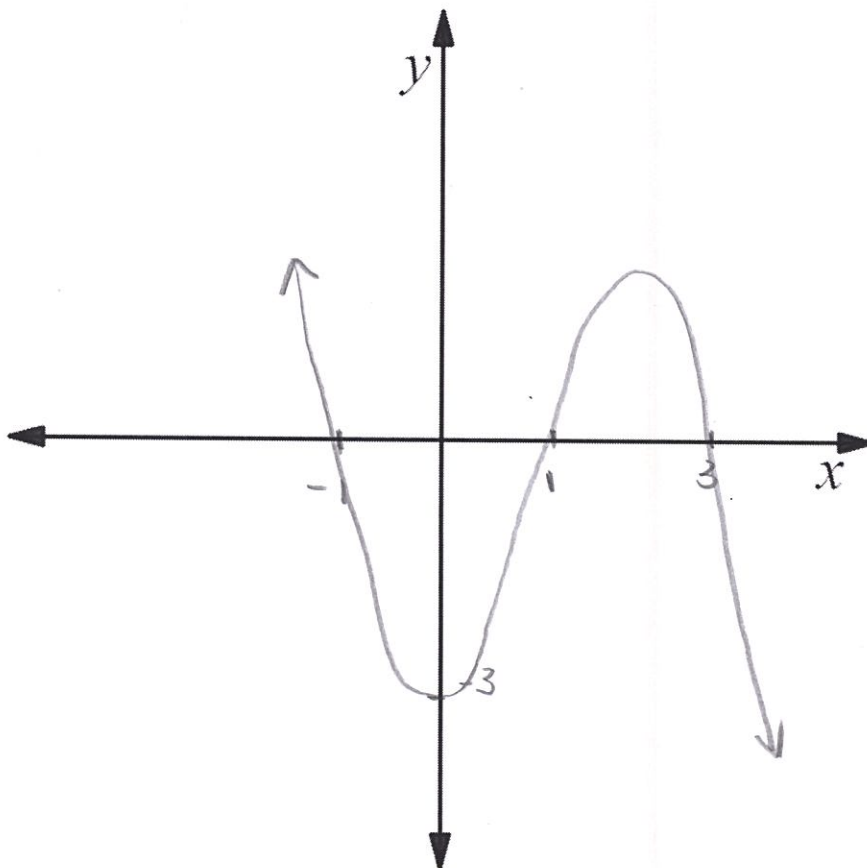
$$\text{When } x = 0, y = -12$$

$$\text{When } x = 5, y = 5^2 - 5 - 12$$

$$y = 8$$

$$\therefore (0, -12), (5, 8)$$

(i) Sketch the graph of $y = -(x - 3)(x - 1)(x + 1)$, showing all essential features.



- (j) The cost of producing x litres of orange juice is $20 + x$ dollars. If the income from the sale of x litres of the juice is $2x^2$ dollars, calculate the number of litres that must be sold to make a profit of \$800.

3

$$C = 20 + x$$

$$I = 2x^2$$

$$P = I - C$$

$$800 = 2x^2 - (20 + x)$$

$$800 = 2x^2 - 20 - x$$

$$2x^2 - x - 820 = 0$$

$$a = 2, b = -1, c = -820$$

$$x = \frac{-(-1) \pm \sqrt{1^2 - 4 \times 2 \times -820}}{2 \times 2}$$

$$= \frac{1 \pm \sqrt{1 + 6560}}{4}$$

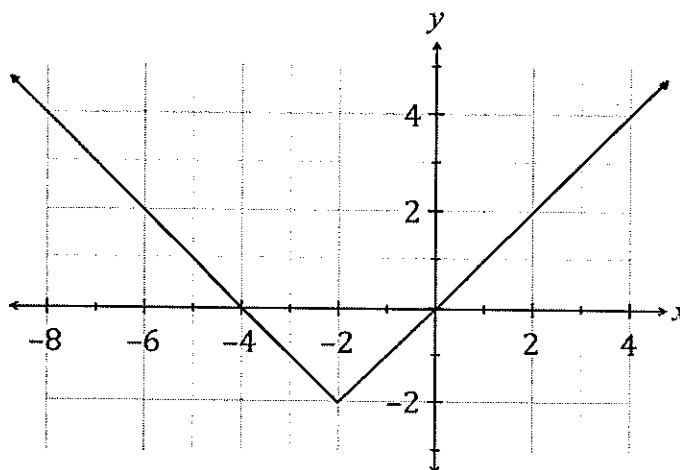
$$x = 20.5 \quad \text{or} \quad -20$$

$\therefore x$ cannot be negative.

$$\therefore x = 20.5 \text{ L.}$$

End of Question 3

(a) What is the equation of the function below?



(A) $y = |x + 2| - 2$

(B) $y = |x - 2| + 2$

(C) $y = |2 - x| - 2$

(D) $y = |2 + x| + 2$

1

(b) For the polynomial $P(x) = 7x^6 + 5x^3 + 9x - 2$

(i) Find the degree of $P(x)$

1

6

(ii) State the leading term

1

$7x^6$

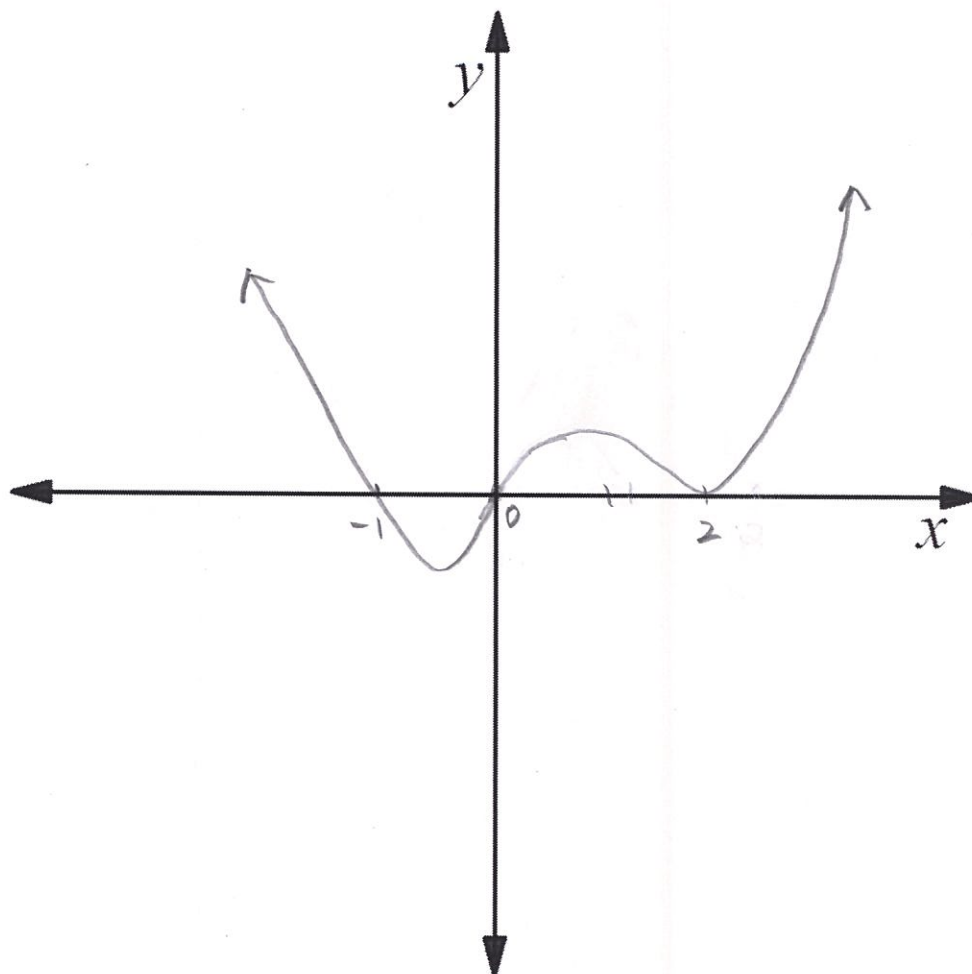
- (c) (i) Show that $x(x+1)(x-2)^2 = x^4 - 3x^3 + 4x$

2

$$\begin{aligned}
 \text{LHS} &= (x^2+x)(x^2-4x+4) \\
 &= x^4 - 4x^3 + 4x^2 + x^3 - 4x^2 + 4x \\
 &= x^4 - 3x^3 + 4x \\
 &= \text{RHS}
 \end{aligned}$$

- (ii) Hence, sketch the graph of $P(x) = x^4 - 3x^3 + 4x$

1



- (d) Consider the function $f(x) = \sqrt{9-x^2}$

- (i) State the domain and range of $f(x)$, in interval notation.

2

Domain : $[-3, 3]$

Range : $[0, 3]$

- (ii) State whether the function $f(x)$, is odd, even or neither.
You must justify your answer algebraically.

2

$$f(x) = \sqrt{9-x^2}$$

$$f(-x) = \sqrt{9-(-x)^2}$$

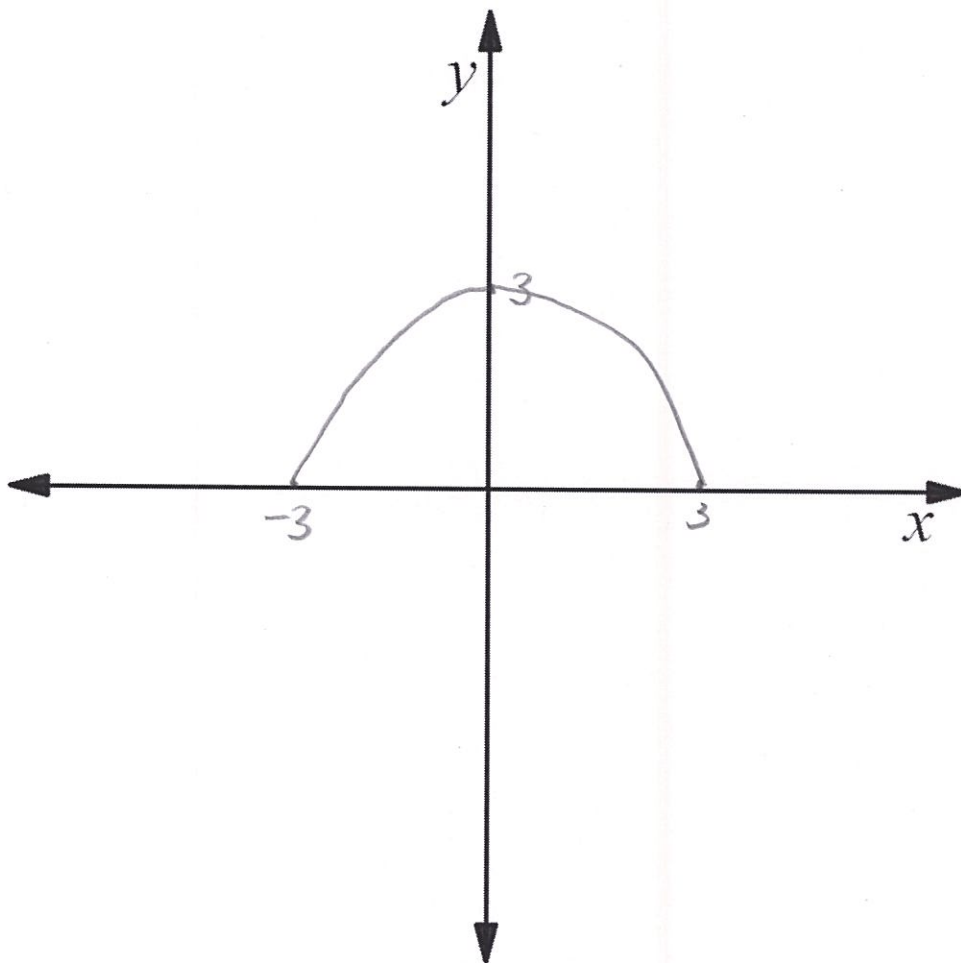
$$= \sqrt{9-x^2}$$

$$= f(x)$$

$\therefore$ even function.

- (iii) Sketch the graph of $f(x) = \sqrt{9-x^2}$, showing all intercepts.

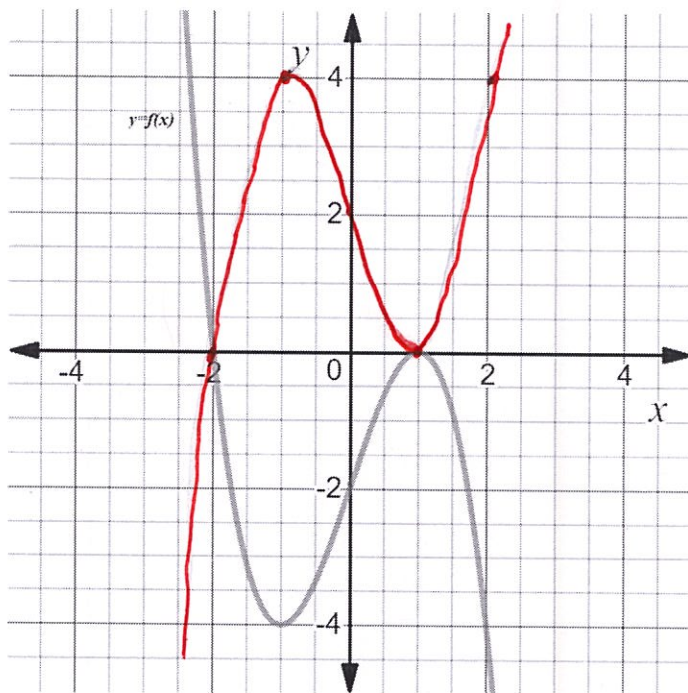
1



(e) The graph of $y = f(x)$ is shown below. Sketch the graph of

(i) $y = -f(x)$ (reflected across x -axis)

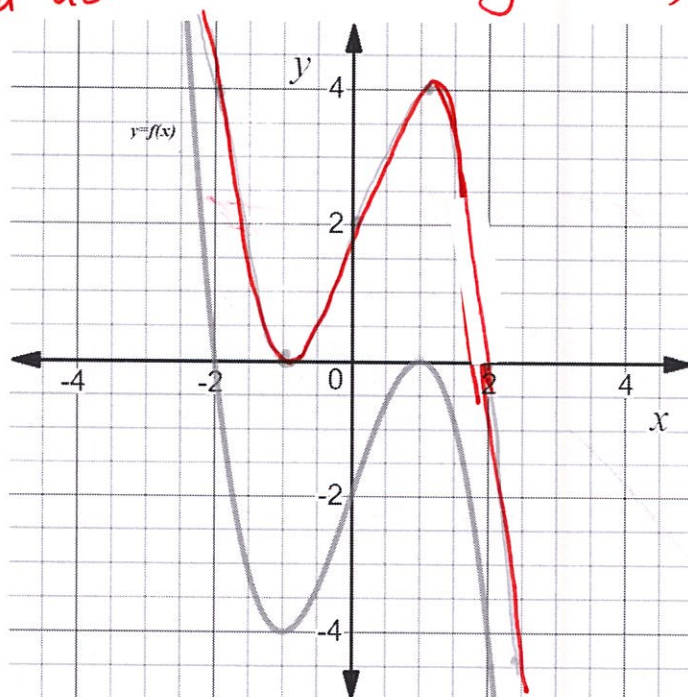
1



(ii) $y = -f(-x)$

(reflected across x -axis + y -axis)

1



End of Paper