

GIRRAWEE HIGH SCHOOL

Year 11 Mathematics

Test 3

June 2005

Time: 90 minutes

Instructions:

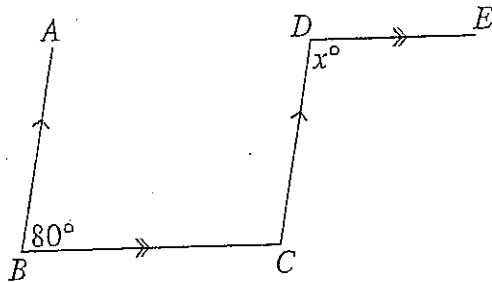
Show all necessary working.

Marks will be deducted for careless or badly arranged work.

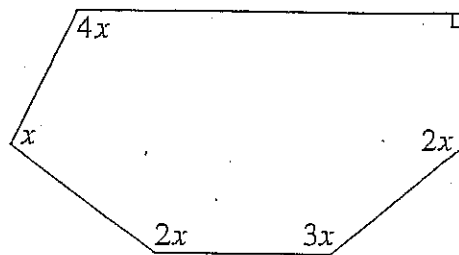
Question 1 (17 marks)

Find the value of each pronumeral, giving reasons.

(a)

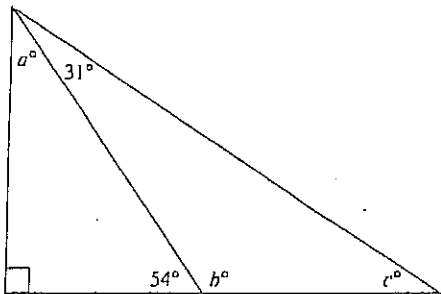


(b)

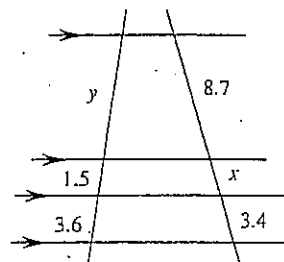


6

(c)



(d)



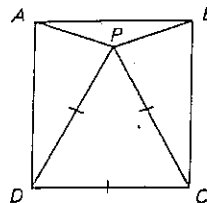
11

Question 2 (10 marks)

(a) P is a point inside a square $ABCD$ such that $\triangle PDC$ is equilateral. Prove that

(i) $\triangle APD \cong \triangle BPC$

(ii) $\triangle APB$ is isosceles.



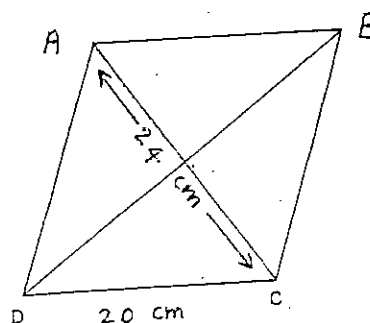
4

2

(b) The lengths of the sides of a rhombus and the length of one diagonal are respectively 20 cm, 24 cm. Calculate

(i) the length of the other diagonal.

(ii) the area of the rhombus.



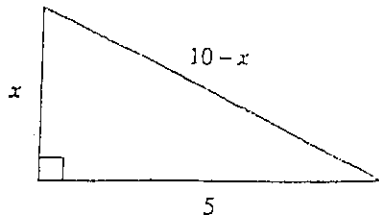
2

2

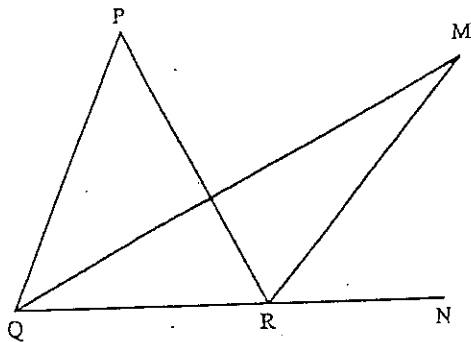
Question 3 (12 marks)

(a) Find the value of x .

3



(b) PQR is an isosceles triangle in which $PQ = PR$ and $\angle QPR = 76^\circ$. QR is produced to N . QM bisects $\angle PQR$ and RM bisects $\angle PRN$.



(i) Copy the diagram and mark on it all the given information.

2

(ii) Find the size of $\angle PQR$. Give reasons.

2

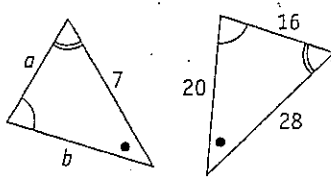
(iii) Find the size of $\angle QMR$. Give reasons.

5

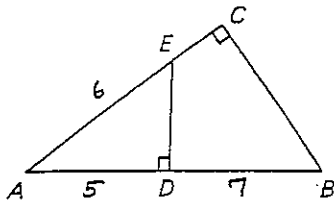
Question 4 (11 marks)

(a) Given that the two triangles are similar, find the value of each pronumeral.

4



(b)



(i) Prove $\triangle ABC \sim \triangle AED$.

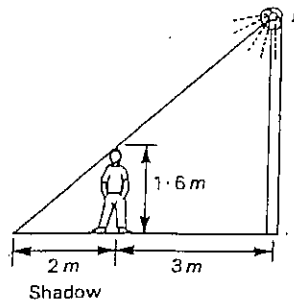
3

(ii) Calculate the length of AC

2

(c) A boy 1.6 m tall, standing 3 m from a street light casts a shadow 2 m long. 2

How high is the light above the ground?



Question 5 (13 marks)

(a) $f(x) = x^2 - 2$, find:

(i) $f(0)$

(ii) $f(2)$

(iii) $f(x^2)$

3

(b) $H(x) = x - \frac{1}{x}$. If $H(b) = 0$, find b .

2

(c) Find the centre and radius of the circle $(x+1)^2 + (y+2)^2 = 121$.

2

(d) Determine whether the following curves are functions or not.

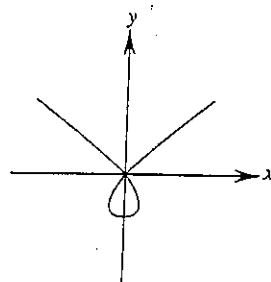
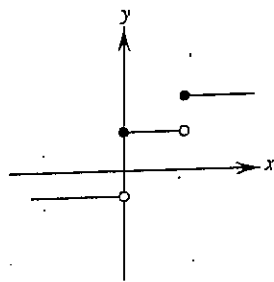
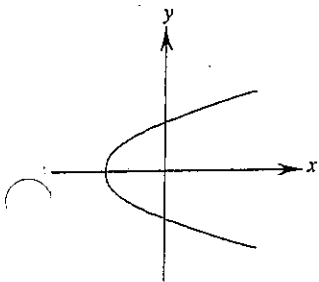
Indicate your answer by writing either yes or no.

6

(i)

(ii)

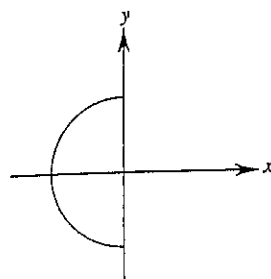
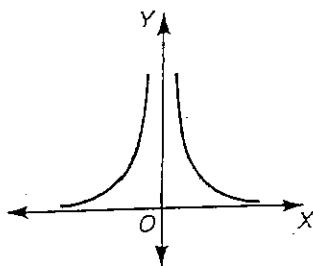
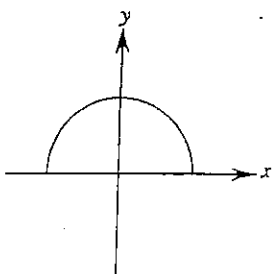
(iii)



(iv)

(v)

(vi)



Question 6 (11 marks)

(a) If $g(x) = 4 - 5x + 3x^2$ and $f(x) = 10 - 2x$ find:

(i) the value of x for which $f(x) = 2$. 2

(ii) the values of x for which $f(x) = g(x)$. 3

(b) A function is defined by the rule

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ -1 & \text{if } 0 < x < 2 \\ x & \text{if } x \geq 2 \end{cases}$$

Evaluate $f(-1) + f(1) + f(5)$. 3

(c) Show that $f(x) = 4x - x^3$ is an odd function. Hence find $f(x) + f(-x)$. 3

Question 7 (26 marks)

(a) For each of the following curves (α) , (β) , (γ) and (δ)

(i) Draw a neat sketch.

(ii) Name the type of curve.

(iii) State the domain and range.

(α) $y = (x+1)^2$ 4

(β) $y = |x| + 2$ 4

(γ) $y = 2^x$ 4

(δ) $y = \frac{1}{x-3}$ 4

(b) Find the domain of the following:

(i) $y = \sqrt{x-6}$ (ii) $y = \sqrt{25-x^2}$ 4

(c) Shade in the region on the number plane for which the inequalities hold simultaneously.

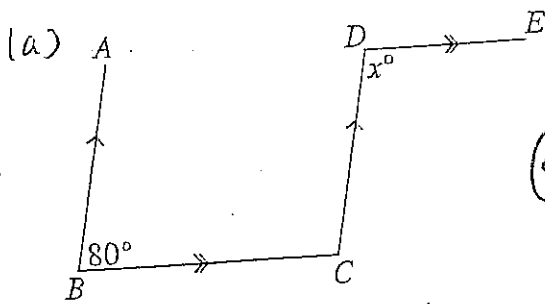
(i) $x^2 + y^2 < 16$ and $y \geq x + 2$ 3

(ii) $y \leq -\sqrt{16-x^2}$ 3

Solutions - Year 11 Mathematics, Test 3, 2005

Total marks: 100

Question 1 (17 marks)



$\angle BCD = 100^\circ$ (co-interior angles supplementary, $AB \parallel CD$)

(4)

$x = \angle BCD$ (alternate angles are equal, $BC \parallel DE$)
 $= 100^\circ$

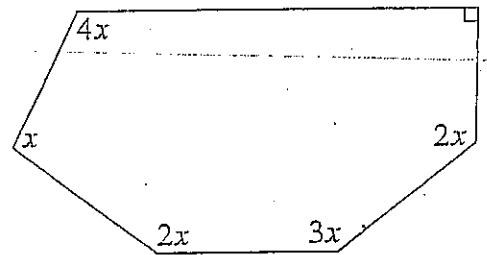
(b) Angle sum of a polygon $= (n-2) \times 180$
 $= (6-2) \times 180 = 720$

$$2x + 3x + 2x + x + 4x + 90 = 720$$

$$12x + 90 = 720$$

$$x = 52.5^\circ$$

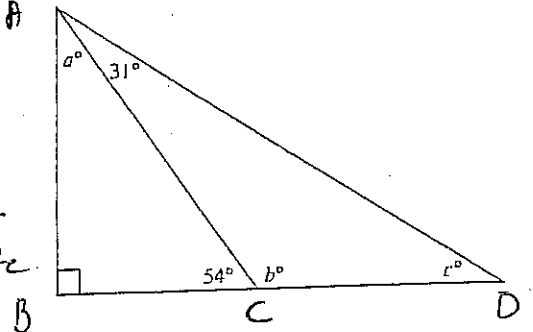
(2)



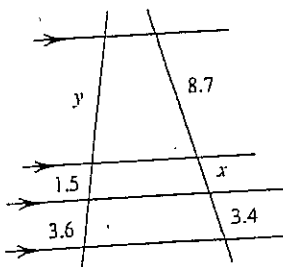
(c) $a = 180 - (90 + 54)$ (angle sum of a $\triangle ABC$ is 180°)
 $= 36$

$b = 90 + 36$ (exterior angle of a triangle is equal to the sum of opposite interior angles)
 $= 126$

$c = 180 - (31 + 126)$ (angle sum of $\triangle ACD = 180$)
 $= 23$



(d)



(1) Parallel lines present intercepts on transversals

$$\therefore \frac{y}{3.6} = \frac{8.7}{3.4}$$

$$\text{and } \frac{x}{3.4} = \frac{1.5}{3.6}$$

$$y = \frac{(8.7 \times 3.6)}{3.4}$$

$$x = \frac{(1.5 \times 3.4)}{3.6}$$

$$= 9.2$$

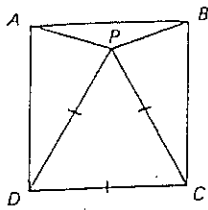
$$= 1.4$$

(2)

(2)

Question 2 (10 marks)

(a) (i)



$AD = BC$ (sides of a square)

$PD = PC$ (given)

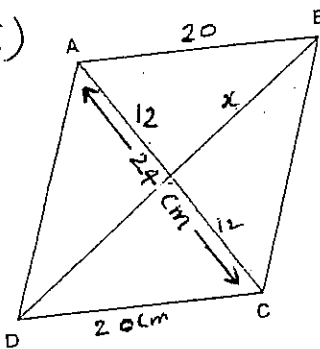
$$\angle ADP = 90 - 60 = 30 \quad ; \quad \angle PCB = 90 - 60 = 30$$

$$\therefore \angle ADP = \angle PCB$$

$\triangle APD \equiv \triangle BPC$ (SAS test)

(ii) $AP = PB$ (Corresponding sides of congruent triangles are equal)
 $\therefore \triangle APB$ is isosceles.

(b) (i)



$$x = \sqrt{20^2 - 12^2} = 16$$

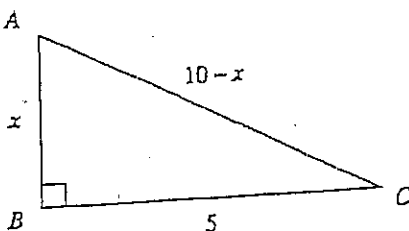
length of the diagonal = $16 \times 2 = 32 \text{ cm}$

(ii) Area $A = \frac{1}{2}xy$

$$= \frac{1}{2} \times 32 \times 24 = 384 \text{ cm}^2$$

Question 3 (12 marks)

(a)



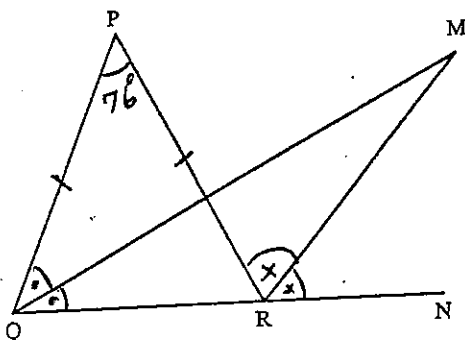
$$25 + x^2 = (10 - x)^2$$

$$25 + x^2 = 100 + x^2 - 20x$$

$$20x = 75$$

$$x = \frac{15}{4}$$

(b) (i)



(2)

(3)

$$(ii) \angle PQR + \angle PRQ = 180 - 76 \quad (\text{angle sum of } \triangle PQR) \\ = 104 \quad (3)$$

$$\angle PQR = \angle PRQ \\ = \frac{1}{2} \times 104 = 52^\circ \quad (\triangle PQR \text{ is isosceles}) \quad (2)$$

$$(iii) \angle MQR = \frac{1}{2} \times 52 = 26 \quad (QM \text{ bisects } \angle PQR)$$

$$\angle PRN = 180 - 52 = 128 \quad (ARN \text{ is a straight line}) \quad (5)$$

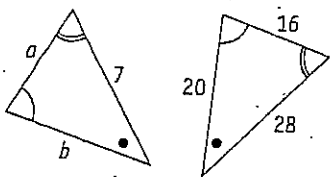
$$\angle PRM = \frac{1}{2} \times 128 = 64 \quad (RM \text{ bisects } \angle PRN)$$

$$\angle QRM = 64 + 52 = 116 \quad (\angle QRM = \angle QRP + \angle PRM)$$

$$\angle QMR = 180 - (26 + 116) \quad (\text{angle sum of } \triangle MQR) \\ = 38^\circ$$

Question 4 (ii marks)

(a)



$$\frac{a}{16} = \frac{b}{20} = \frac{7}{28}$$

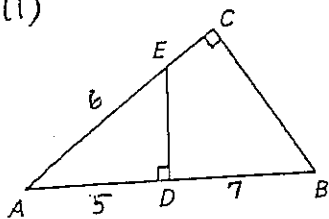
$$\frac{a}{16} = \frac{7}{28}$$

$$(2) \quad a = \frac{16 \times 7}{28} = 4$$

$$\frac{b}{20} = \frac{7}{28} \quad (2)$$

$$b = \frac{20 \times 7}{28} = 5$$

(b)(i)



$$\angle ACB = \angle ADE \quad (\text{given, equal to } 90^\circ)$$

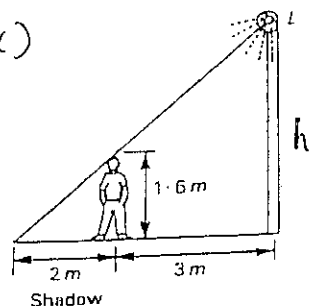
$$\angle EAD = \angle CAB \quad (\text{common angle}) \quad (3)$$

$$\therefore \triangle ABC \sim \triangle AED \quad (\text{equiangular})$$

$$(ii) \frac{AB}{AE} = \frac{AC}{AD} ; \quad \frac{12}{6} = \frac{AC}{5}$$

$$AC = \frac{12 \times 5}{6} = 10 \quad (2)$$

(c)



$$\frac{h}{1.6} = \frac{5}{2}$$

$$h = \frac{(1.6 \times 5)}{2} = 4 \text{ m}$$

(4)

(2)

Question 5 (13 marks)

(a) (i) $f(0) = -2$

(ii) $f(2) = 2^2 - 2 = 2$ (3)

(iii) $f(x^2) = (x^2)^2 - 2 = x^4 - 2$

(b) $H(b) = b - \frac{1}{b}$

$$H(b) = 0 \Rightarrow b - \frac{1}{b} = 0$$

ie $\frac{b^2 - 1}{b} = 0$ (2)

$$b^2 - 1 = 0$$

$$(b+1)(b-1) = 0 \therefore b = \pm 1$$

(c) Centre $(-1, -2)$ radius = 11 (2)

(d) (i) No (ii) Yes (iii) No (iv) Yes (v) Yes (vi) No (6)

Question 6 (11 marks)

(a) (i) $10 - 2x = 2$ (2)
 $x = 4$

(ii) $4 - 5x + 3x^2 = 10 - 2x$

$$3x^2 - 5x + 2x + 4 - 10 = 0$$

$$3x^2 - 3x - 6 = 0$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$
 (3)

$$x = 2 \text{ or } -1$$

(b) $f(-1) = 0$; $f(1) = -1$; $f(5) = 5$

$$f(-1) + f(1) + f(5) = 0 + -1 + 5 = 4$$
 (3)

(c) $f(-x) = 4(-x) - (-x)^3$

$$= -4x + x^3$$

$$= -(4x - x^3)$$

$$= -f(x)$$

ie $f(-x) = -f(x)$ (3)

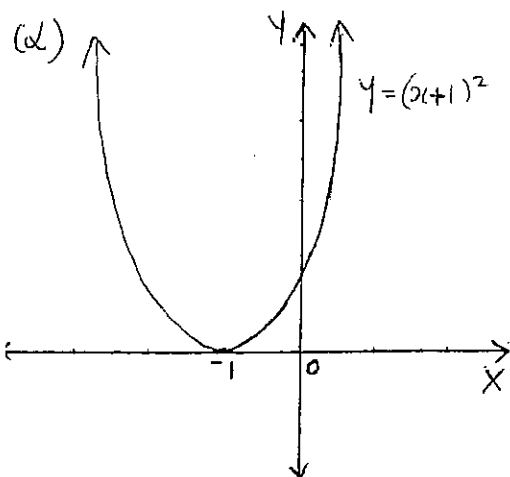
$\therefore f$ is an function

$$f(x) + f(-x)$$

$$= f(x) + -f(x)$$

$$= 0$$

Question 7 (26 marks)

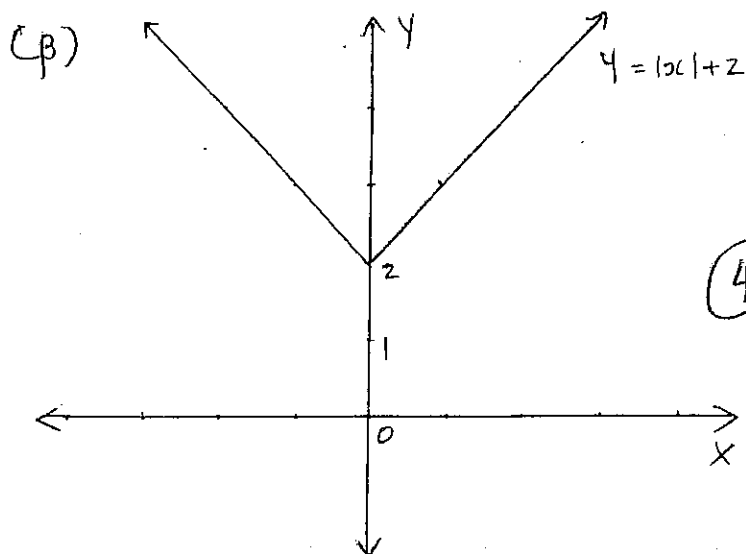


(4)

(ii) parabola

(iii) D : all reals

R : $y \geq 0$

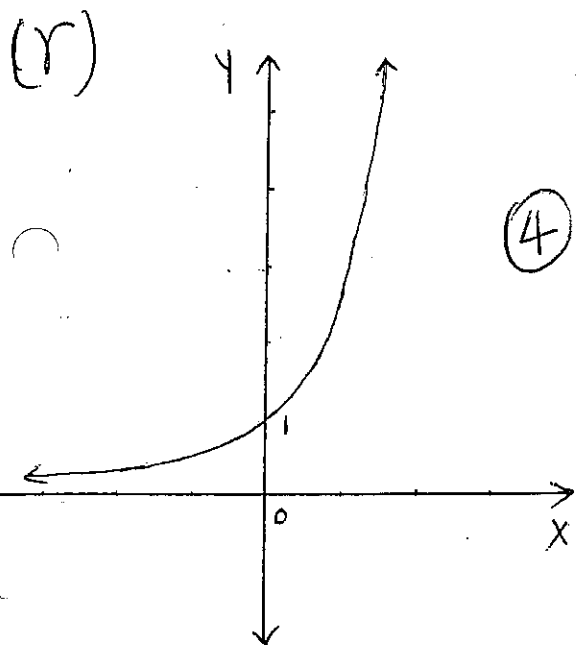


(4)

(ii) absolute value graph

(iii) D : all reals

R : $y \geq 2$

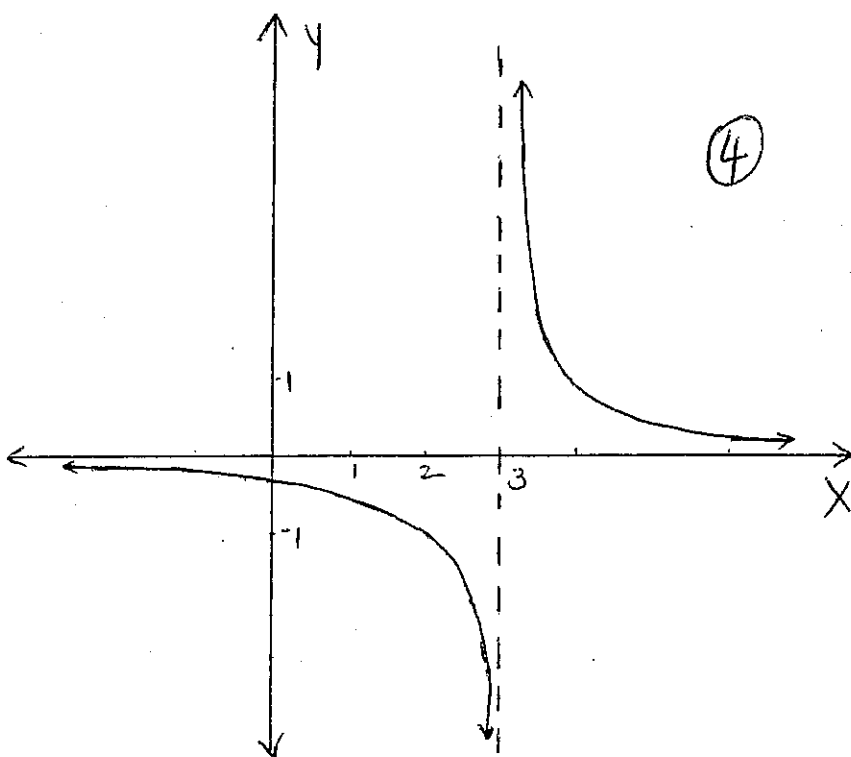


(4)

(ii) exponential graph

(iii) D : all reals

R : all reals, $y > 0$



(4)

(ii) hyperbola

(iii) D : all reals, $x \neq 3$

R : all reals, $y \neq 0$

$$(b)(i) \sqrt{x-6}$$

$$x-6 \geq 0$$

$$x \geq 6$$

D: all reals, $x \geq 6$

(2)

$$(ii) \sqrt{25-x^2}$$

$$25-x^2 \geq 0$$

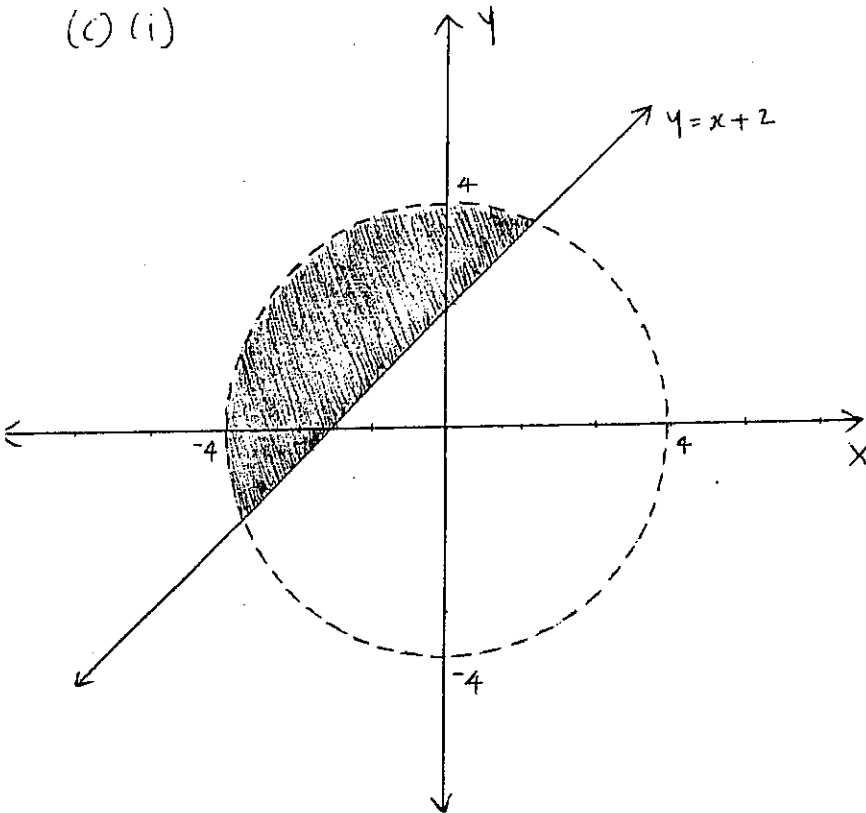
$$25 \geq x^2$$

$$-5 \leq x \leq 5$$

D: all reals, $-5 \leq x \leq 5$

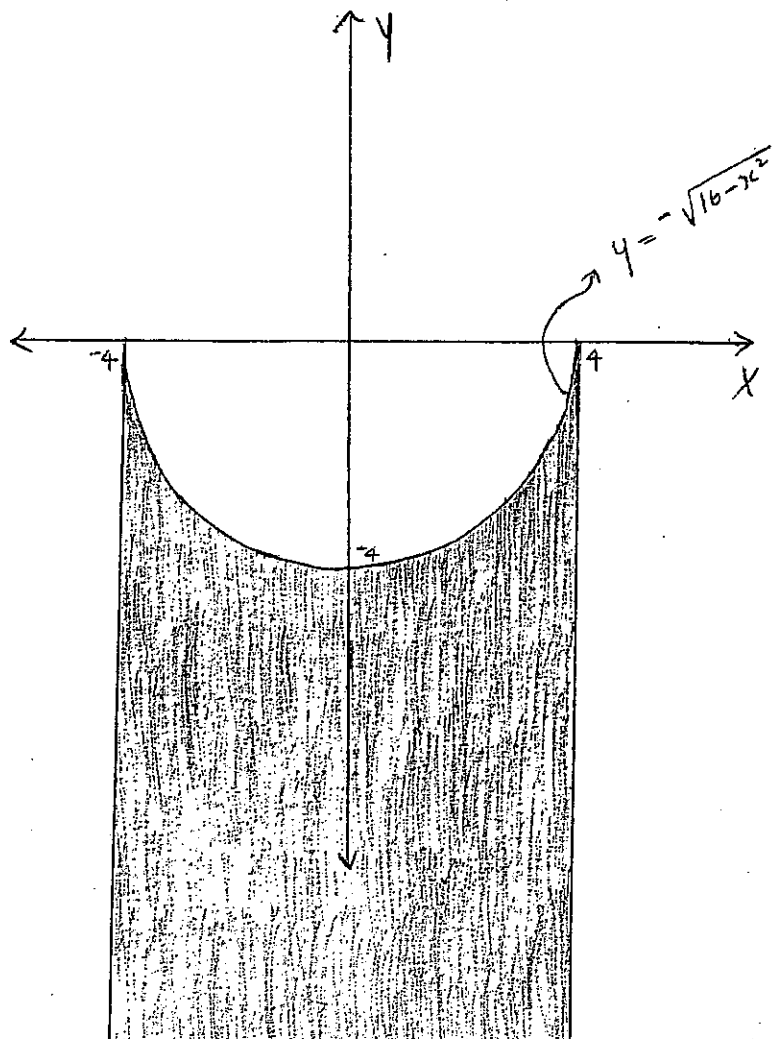
(2)

(c)(i)



(3)

(ii)



(3)