GIRRAWEEN HIGH SCHOOL

Year 11 Mathematics Test 3 June 2007

Time: 90 minutes

Instructions:

Start each question on a new page.

Show all necessary working.

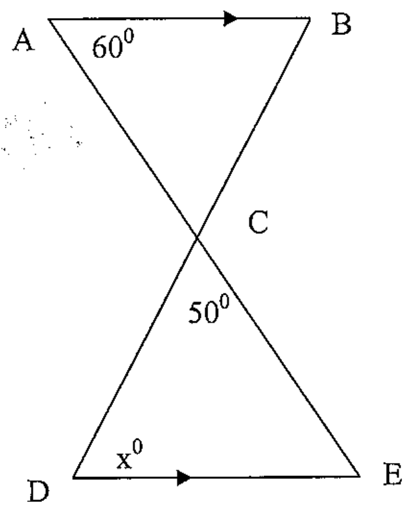
Marks will be deducted for careless or badly arranged work.

Board approved calculators may be used.

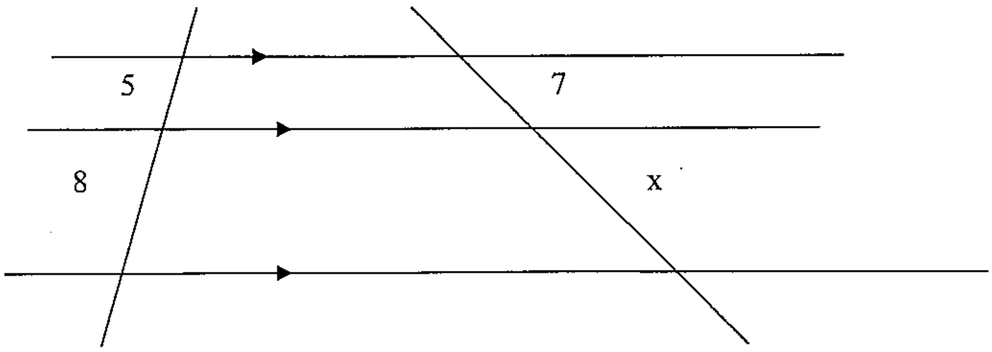
Diagrams are NOT to scale.

Question 1 (10 marks)

(a) Find x° giving reasons in: (3 marks)



(b) Find x giving reasons : (3 marks)



(c) Each exterior angle of a regular polygon is 30^0.

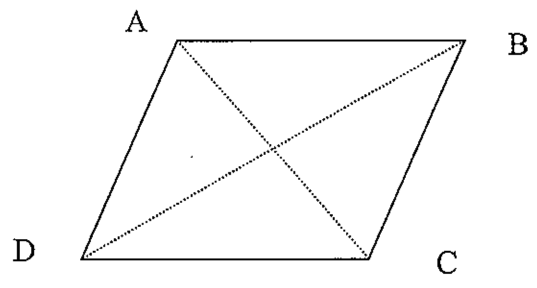
(i) Find the number of sides the polygon has. (2 marks)

(ii) Find the size of each internal angle of the polygon. (2 marks)

Question 2 (15 marks)

(a) A triangle has side lengths 7cm, 9cm and 11cm. Prove that it is NOT right-angled. (2 marks)

(b) In rhombus ABCD, AC = 8cm and DB = 15cm. Find the perimeter of the rhombus. (4 marks)

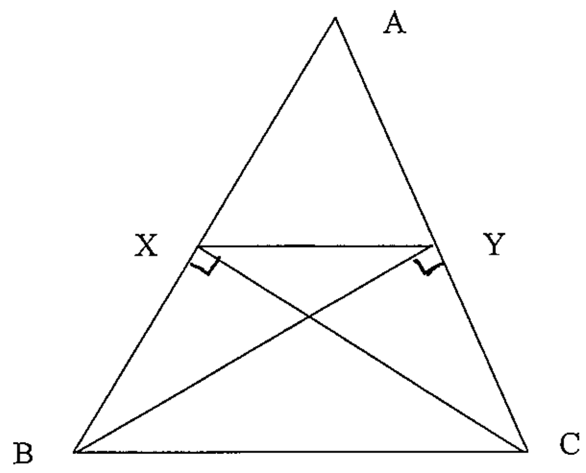


(c) Δ ABC is an isosceles triangle. AB = AC, BY $\perp$ CA and CX $\perp$ BA. Prove

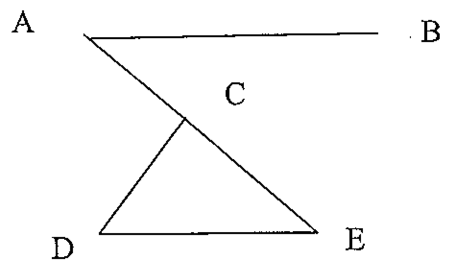
(i) $\Delta BXC \equiv \Delta CYB$ (4 marks)

(ii) AX = AY (2 marks)

(iii) XY $\parallel$ BC (3 marks)

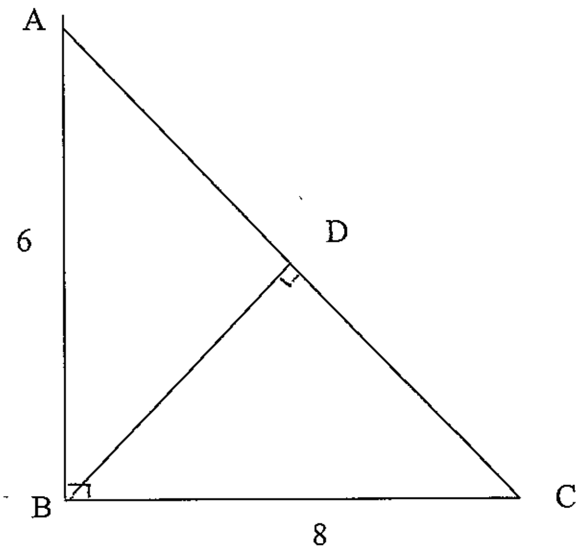


Question 3 (15 marks)
In the diagram below, AB $\parallel$ ED , Δ ECD is isosceles with EC = DC and $\angle$BAE = 30^0.

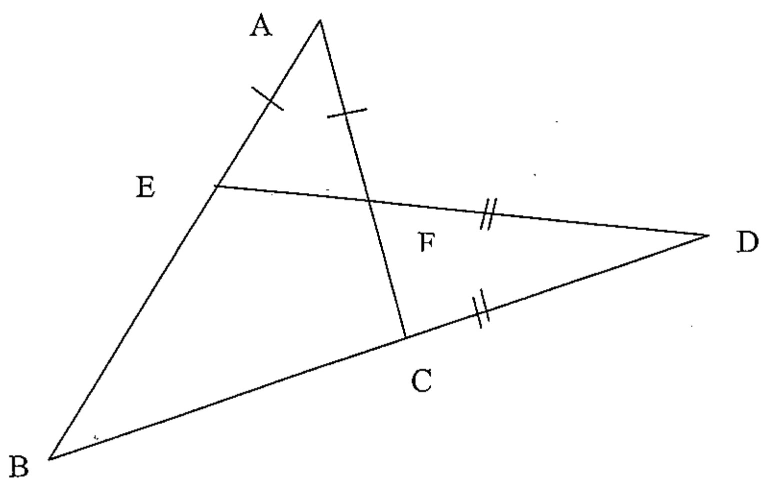


(i) Copy the diagram and mark on it all of the given information. (1 mark)
(ii) Find angle DCE giving reasons.. (3 marks)

(b) For the diagram below:
(i) Prove ΔADB $|||$ ΔABC. (3 marks)
(ii) Find AC. (2 marks)
(iii) Find BD. (3 marks)

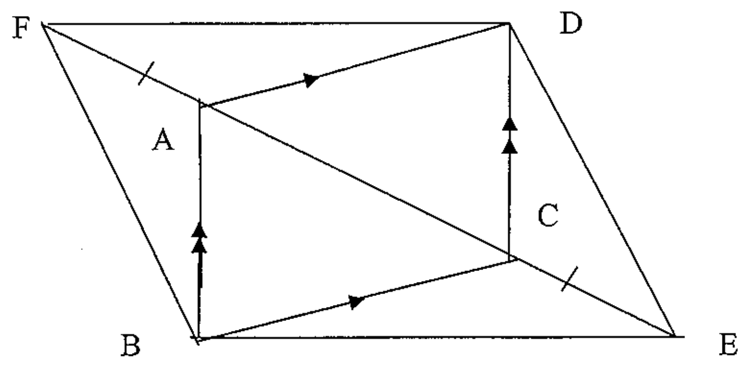


(c) If AE = AF and DC = DF in the diagram below, prove $\angle$EAF = $\angle$ FDC
(3 marks)



Question 4 (10 marks)

In the diagram below, ABCD is a parallelogram and AF = CE.



(a) Prove that $\angle$FAD = $\angle$ECB. (3 marks)

(b) Prove that $\Delta ADF \equiv \Delta CBE$ (3 marks)

(c) Prove that FBED is a parallelogram. (4 marks)

Question 5 (27 marks)

(a) For each of the following curves:

Sketch them on separate number planes (5 marks each)

State their domain and range

State whether they are functions

(i) $y = 3^x$

(ii) $y = |2x-3|$

(iii) $x^2 + y^2 = 25$

(iv) $y = x^2 + 2$

(v) $y = \dfrac{1}{x-5}$

(b) State the DOMAIN of $y = \dfrac{1}{\sqrt{3x+15}}$ (2 marks)

Question 6 (16 marks)

(a) If $f(x) = x - \frac{1}{x}$ find

(i) $f(2)$ (1 mark)

(ii) $f(a-2)$ (1 mark)

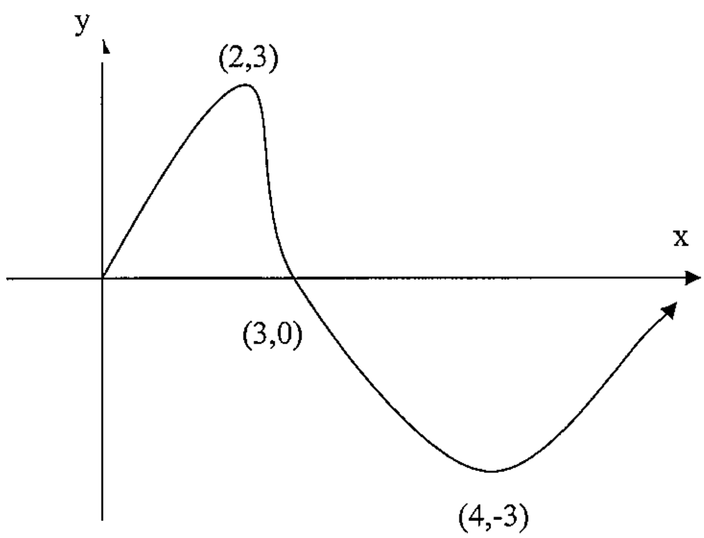
(iii) Find x if $f(x) = 0$. (3 marks)

(b) If $g(x) = \begin{cases} x-2, x<-1 \\ x^2-4, -1 \le x \le 2 \\ 3x-6, x>2 \end{cases}$

Sketch the graph of g(x) for $-2 \le x \le 3$ showing all x and y intercepts and points where the graph changes direction. (4 marks)

(c) (i) Prove that the function $f(x) = x^4 - x^2 + 1$ is an even function. (3 marks)

(ii) Copy and complete this graph for $y = h(x)$ for $-5 \le x \le 5$ if $h(x)$ is an odd function. (4 marks)



Task 3 2007 Solutions: Marks:

Q.(1)(a) $\angle AED = 60°$ [alternate $\angle$'s in $\parallel$ lines $=$]. $=$ 3

$x° = 70°$ [angle sum $\triangle DCE = 180°$].

(b) $\frac{z}{7} = \frac{8}{5}$ [$\parallel$ lines cut transversals in same ratio] 3

$\times 7$ $\times 7$

$z = 11\frac{1}{5}$ units

(c)(i) Number of sides $= 360° \div 30$ [exterior $\angle$'s of polygon add to $360°$]. 2

$= 12$

(ii) Each internal angle $= 180° - 30°$

$= 150°$ [angles in straight line supplementary] 2

OR total sum of interior angles $= 180 \times 10$

$= 1800°$.

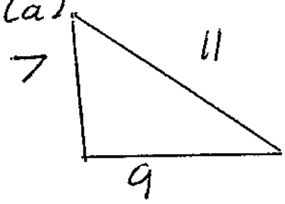
[interior angles of n sided polygon add to $180(n-2)°$].

Each angle in 12 sided polygon

$= 1800 \div 12$

$= 150°$.

Q.(2)(a)



Triangle is right-angled if, by Pythagoras' theorem, $c^2 = a^2 + b^2$, c being longest side [hypotenuse if right-angled]. 2

$11^2 = 7^2 + 9^2$

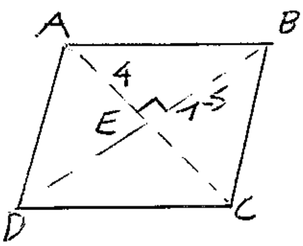
$121 = 130$.

As LHS $\neq$ RHS, triangle is NOT right-angled.

(b) $AC \perp DB$ [diagonals of rhombus $\perp$]

$AE = EC = 4$cm [diagonals of rhombus bisect each other]

$DE = EB = 7.5$cm [" " "]



Hence $(AB)^2 = 4^2 + 7.5^2$ [by Pythagoras' theorem]

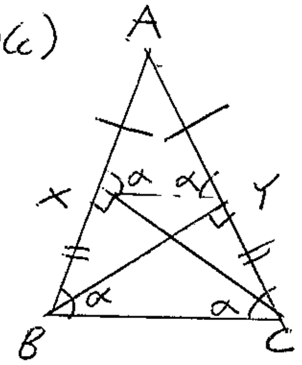
$= 72.25$

$AB = 8.5$cm.

P of rhombus $= 8.5 \times 4$ 4

$= 34$cm.

Q. (2)(c)



(i) Let $\angle ABC = \alpha$.

$\angle ACB = \angle ABC = \alpha$
[∠'s opposite = sides in isosceles $\triangle ABC$ =].

BC common 1 (4)

$\angle BXC = \angle BYC = 90°$ [data]. 1

Hence $\triangle BXC \equiv \triangle CYB$ [AAS]. 1

(ii) $AX = AB - BX$ and $AY = AC - CY$

But $AB = AC$ [data]

and $BX = CY$ [matching sides in $\equiv \triangle BXC$ and CYB]. 1 (2)

$\therefore AX = AY$. 1

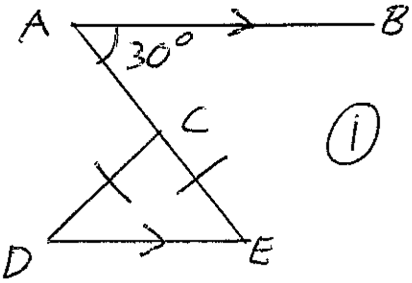
(iii) $\angle BAC = 180° - 2\alpha$ [angle sum of $\triangle ABC = 180°$]. 1

$\angle AXY = \angle AYX = \alpha$ [∠'s opposite = sides in isosceles $\triangle AXY$ =]. 1 (3)

Hence $XY \parallel BC$ [angles in corresponding positions =]. 1

Q. (3)(a)

(i)



(1)

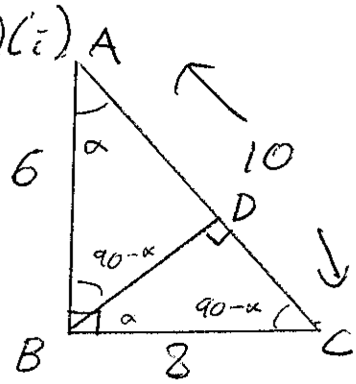
(ii) $\angle DEC = 30°$ [alternate ∠'s in ∥ lines AB and DE =]. 1

$\angle CDE = 30°$ [∠'s opposite = sides in isosceles $\triangle DEC$ =].

$\angle DCE = 120°$ [angle sum $\triangle DCE = 180°$].

(3)

(b)(i)



∠A common 1

$\angle ABC = \angle ADB = 90°$ [data] 1 (3)

Hence $\triangle ADB \parallel\!\!\mid \triangle ABC$ [all pairs of matching angles equal]. 1

(ii) $(AC)^2 = 6^2 + 8^2$ 1 (2)

$= 100$

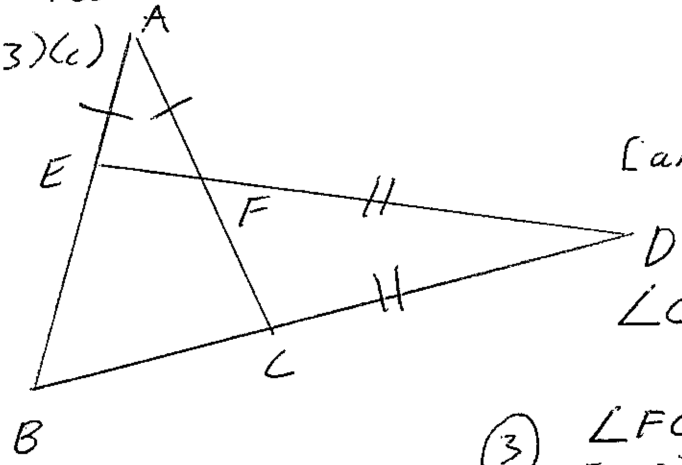
$AC = 10$ units [By Pythagoras' theorem]. 1

(iii) $\dfrac{BD}{BC} = \dfrac{AB}{AC}$ 1 [matching sides in $\parallel\!\!\mid$ $\triangle ADB$ and ABC].

$\dfrac{BD}{8} = \dfrac{6}{10}$ 1 (3)

$BD = 4\frac{4}{5}$ units 1

Q.(3)(c)



Let $\angle EAF = \alpha$

$\angle AEF = \angle AFE = 90 - \frac{1}{2}\alpha$ [angles opposite = sides in isosceles $\triangle AEF$ =]. 1

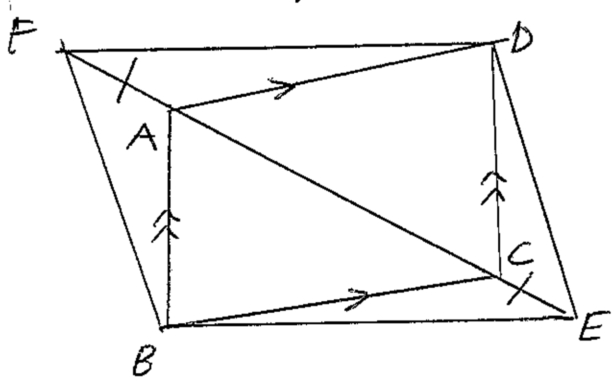
$\angle CFD = 90 - \frac{1}{2}\alpha$ [vertically opposite $\angle$'s =] 1

$\angle FCD = 90 - \frac{1}{2}\alpha$ [$\angle$'s opposite = sides in isosceles $\triangle FCD$ =].

3

$\angle FDC = \alpha$ [angle sum $\triangle FDC = 180°$].

Q.(4)



(a) Let $\angle FAD = \alpha$

$\angle CAD = 180° - \alpha$ [$\angle$'s in a straight line supplementary]. 1

$\angle ACB = 180° - \alpha$ [alternate $\angle$'s in $\parallel$ lines AD and BC =] 1

$\angle BCE = \alpha$ [$\angle$'s in a straight line supplementary]. 1

Hence $\angle BCE = \angle FAD$.

3

(b) $AF = CE$ [data] 1

$BC = AD$ [opposite sides of parallelogram =]. 1

$\angle FAD = \angle BCE$ [proven in (a)]. 1

$\therefore \triangle ADF \equiv \triangle CBE$ [SAS]. 1

3

(c) $FD = BE$ [matching sides in $\equiv \triangle ADF$ and CBE =]. 1

$\angle DFA = \angle BEC$ [matching $\angle$'s in $\equiv \triangle ADF$ and CBE =]. 1

$\therefore FD \parallel BE$ [alternate $\angle$'s =]. 1

Hence FBED is a parallelogram [one pair of opposite sides equal and parallel]. 1

4

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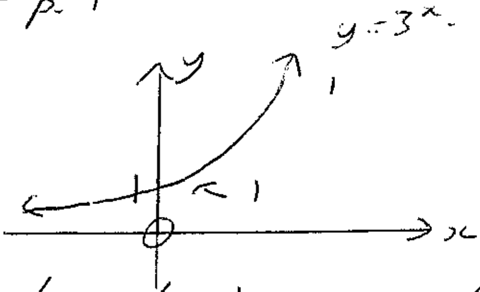
Solutions – p. 4

Q.(5)(a)(i)



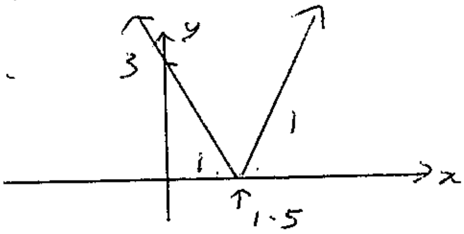
Domain: all real numbers. 1

Range: $y > 0$. 1

<u>IS</u> a function. 1

5

(ii) $y = |2x-3|$.



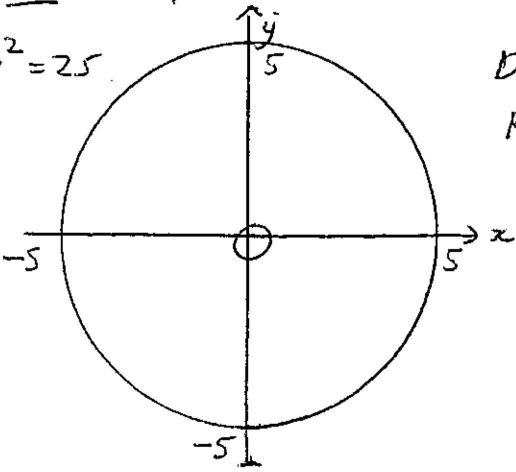
5

Domain: All real numbers. 1

Range: $y \geq 0$ 1

<u>IS</u> a function. 1

(iii) $x^2+y^2=25$



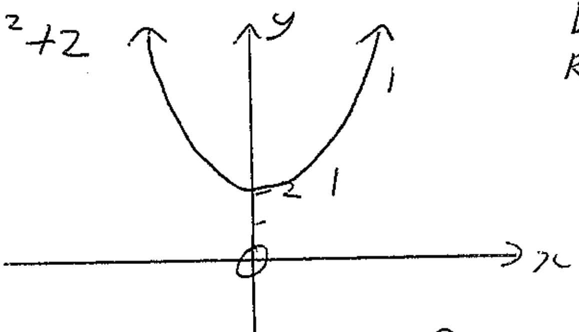
Domain: $-5 \leq x \leq 5$ 1

Range: $-5 \leq y \leq 5$. 1

5

Is <u>NOT</u> a function. 1

(iv) $y=x^2+2$



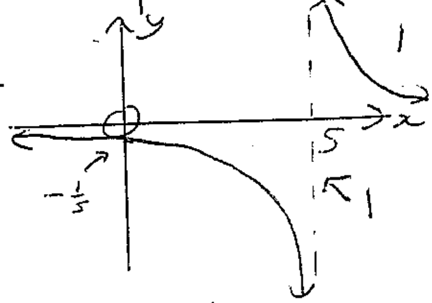
Domain: All real numbers. 1

Range: $y \geq 2$ 1

5

<u>IS</u> a function. 1

(v) $y = \dfrac{1}{x-5}$



Domain: All real numbers except 5. 1

Range: All real numbers except 0. 1

<u>IS</u> a function 5

(b) Domain of $y = \dfrac{1}{\sqrt{3x+15}}$ is where $3x+15 > 0$ 1

$3x > -15$

$\underline{x > -5.}$ 1

2

Domain: $\underline{x > -5.}$

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Solutions — p.5

Q.(6)(a) $f(x) = x - \frac{1}{x}$

(i) $f(2) = 2 - \frac{1}{2}$ ①

$= 1\frac{1}{2}$. 1

(ii) $f(a-2) = a - 2 - \frac{1}{a-2}$ ①

(iii) If $f(x) = 0$

$x - \frac{1}{x} = 0$ 1

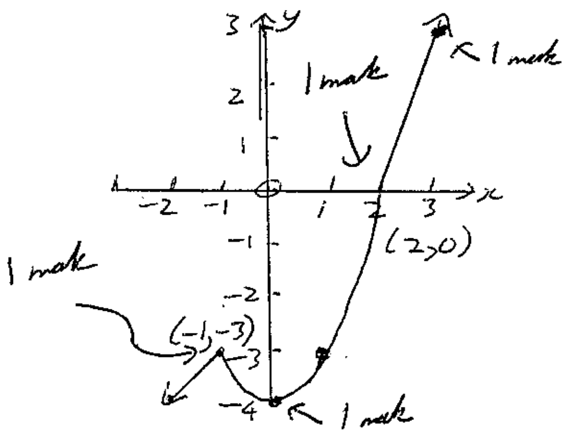
$x^2 - 1 = 0$ 1 ③

$(x-1)(x+1) = 0$

$x = \pm 1$. 1

(b)

x	-2	-1	0	1	2	3
$g(x)$	$-2-2$ $=-4$	$(-1)^2-4$ $=-3$ Check: $-1-2$ $=-3$	0^2-4 $=-4$	1^2-4 $=-3$	2^2-4 $=0$ Check: $3\times2-6$ $=0$	$3\times3-6$ $=3$



④

(c)(i) $f(x) = x^4 - x^2 + 1$ is EVEN iff $f(-a) = f(a)$ 1

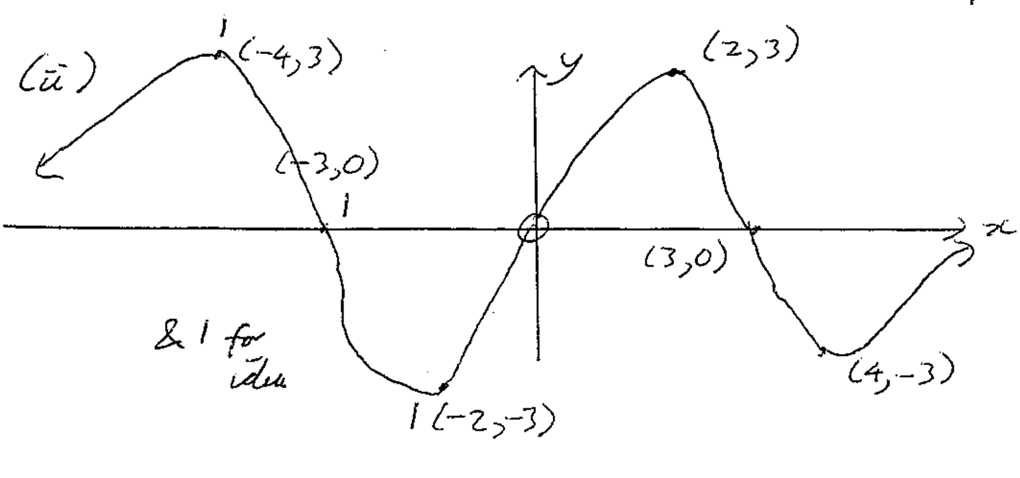
LHS: $f(-a) = (-a)^4 - (-a)^2 + 1$
$= a^4 - a^2 + 1$ 1

RHS: $f(a) = a^4 - a^2 + 1$
$=$ LHS 1

③

Hence $f(x) = x^4 - x^2 + 1$ is an even function.

(ii)



④