FINAL MARK

GIRRAWEEN HIGH SCHOOL
PRELIMINARY ASSESSMENT TASK 3
ANSWERS COVER SHEET

Name: ______________________________

QUESTION	MARK	P2	P3	P4
1	/19	✓	✓	✓
2	/20	✓	✓	✓
3	/20	✓	✓	✓
4	/31		✓	✓
TOTAL	/90	/59	/90	/90

GIRRAWEEN HIGH SCHOOL

Year 11 Mathematics | Task 3 | June 2008

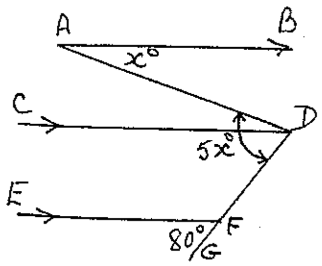
Time: 90 minutes

Instructions:

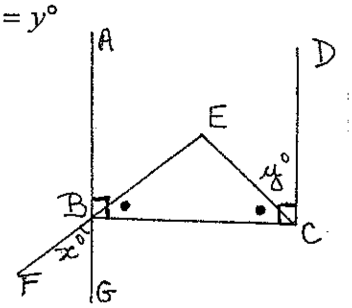
- Start each question on a new page.
- Show all necessary working.
- Marks will be deducted for careless or badly arranged work.
- Board approved calculators may be used.
- Diagrams are NOT to scale.

QUESTION 1 (19 marks)

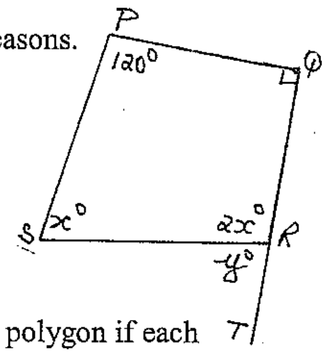
(a) Find the value of x, giving reasons. (4 marks)



(b) Given AB $\perp$ BC and DC $\perp$ BC and $\angle EBC = \angle ECB$. (4 marks)
Prove $x° = y°$

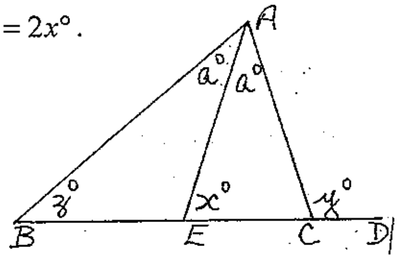


(c) Find the values of x and y, giving reasons. (4 marks)



(d) Find the number of sides of a regular polygon if each interior angle is $165°$. (2 marks)

(e) Prove that $y^\circ + z^\circ = 2x^\circ$. (5 marks)

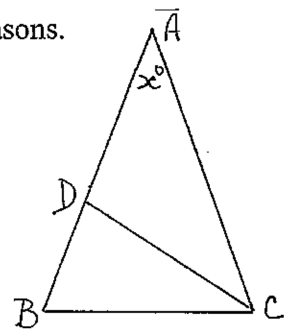


QUESTION 2 (20 marks)

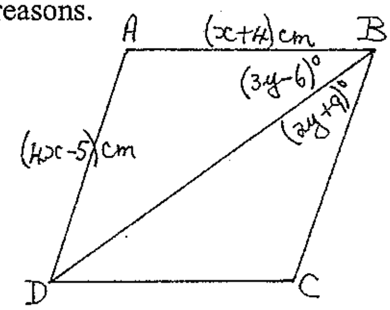
(a) Given Δ ABC with AB = AC. Also AD = DC = CB

(i) Copy the diagram and mark on it all the given information. (1 mark)

(ii) Find the value of x, giving reasons. (6 marks)



(b) ABCD is a rhombus. (6 marks)
Find the values of x and y, giving reasons.



(c) A triangle has sides 4 cm, 7.5 cm and 8.5 cm. (2 marks)
Prove if it is right-angled or not.

(d) A rhombus has sides of 52 cm. The shorter diagonal is 40 cm.

(i) Find the length of the longer diagonal. (3 marks)

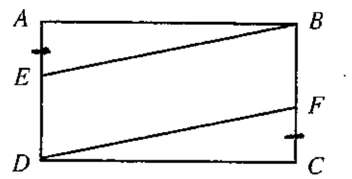
(ii) Find the area of the rhombus. (2 marks)

QUESTION 3 (20 marks)

(a) ABCD is a rectangle and AE = CF.

(i) Show that Δ AEB ≡ Δ CFD. (4 marks)

(ii) Hence, or otherwise, prove that EBFD is a parallelogram. (4 marks)

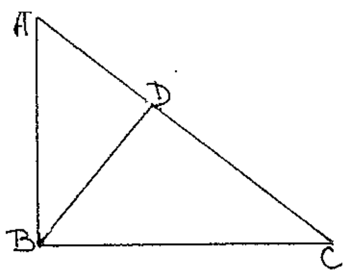


(b) $\angle ABC = 90°$ and $\angle ADB = 90°$. AD = 4 cm and DC = 9 cm.

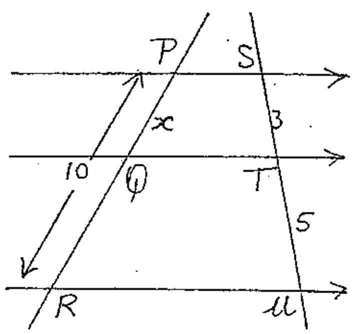
(i) Copy the diagram and mark on it all the given information. (1 mark)

(ii) Prove that Δ ABD|||Δ BCD. (5 marks)

(iii) Find the length of BD, giving reasons. (3 marks)

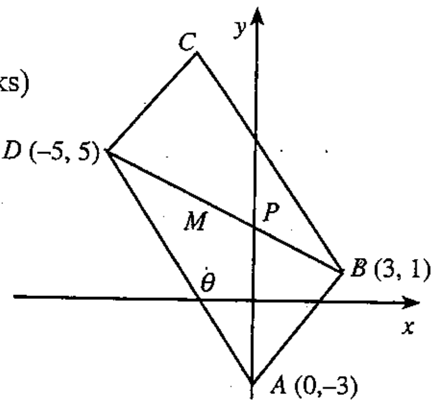


(c) Find the value of x, giving a reason. (3 marks)



QUESTION 4 (31 marks)

(a)



ABCD is a parallelogram. Find:

(i) θ to the nearest degree. (2 marks)

(ii) the coordinates of M, the midpoint of BD. (2 marks)

(iii) the coordinates of the point C. (2 marks)

(iv) the equation of the line AB, in general form. (3 marks)

(v) the perpendicular distance between D and the line AB. (3 marks)

(vi) the area of parallelogram ABCD. (3 marks).

(b) What is the value of q if (−3, q) lies on the line $4x+3y+6=0$? (2 marks)

(c) Find the value of p if the line $px+2y-7=0$ is parallel to the line $4x+y-5=0$. (3 marks)

(d) Find the equation of the line that is perpendicular to $3x+y+2=0$ and passes through the point (−2, −1). (4 marks)

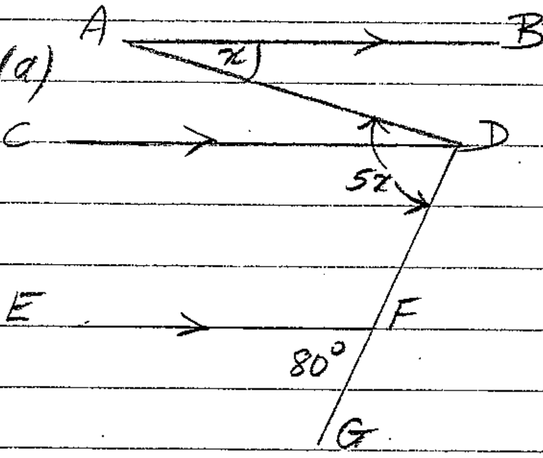
(e) Find the value of g if the lines $2x-y+3=0$, $3x+y+2=0$, and $x+4y-g=0$ are concurrent. (3 marks)

(f) Find the equation of the line through the point of intersection of the lines $3x+4y=2$, and $5x-2y=12$ and the point (1, −2). (4 marks)

END OF TASK

Task 3, Mathematics, June 2008, Solutions

1 (a)



$\angle ADC = x$ (alternate $\angle$s, $AB \parallel CD$)

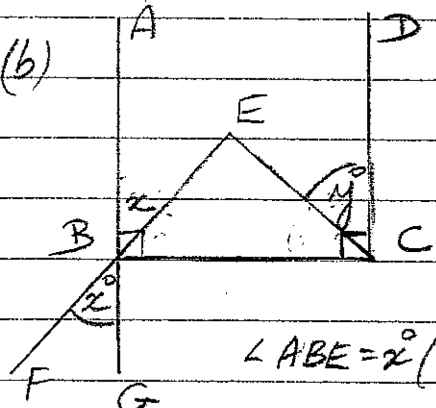
$\therefore \angle CDF = 4x$

$\angle CDF = \angle EFG$ (corresponding $\angle$s, $CD \parallel EF$)

$\therefore 4x = 80$

$\therefore x = 20$ (4)

(b)



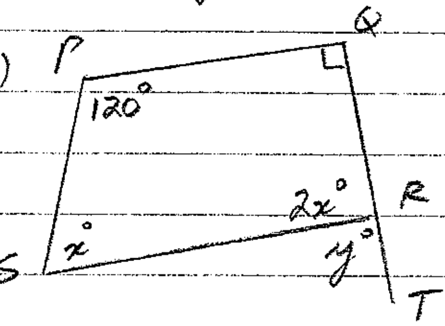
$\angle ABE = x°$ (vertically opposite $\angle$s)

$\therefore \angle EBC = 90 - x = \angle ECB$

$\therefore (90 - x) + y = 90$ (adjacent complementary $\angle$s)

$\therefore y = x$ (4)

(c)



$2x + x + 120 + 90 = 360$ ($\angle$ sum of a quadrilateral)

$3x + 210 = 360$

$\therefore 3x = 150$

$\therefore x = 50$.

$2x + y = 180$ (QRT a straight $\angle$)

$\therefore 100 + y = 180$

$\therefore y = 80$. (4)

(d) Exterior $\angle$ is $15°$

$\therefore$ no. of sides $= \dfrac{360}{15}$

$= 24$ (2)

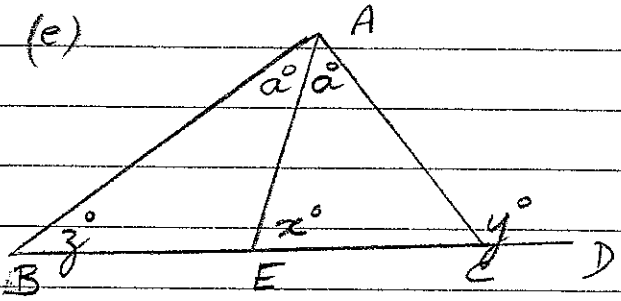
OR $165n = (n-2) \times 180$

$165n = 180n - 360$

$15n = 360$

$\therefore n = 24$.

(e)



$y = a + x$ (exterior $\angle$ = sum of interior opp. $\angle$s)

$x = a + z$ (exterior $\angle$ = sum of interior opp. $\angle$s)

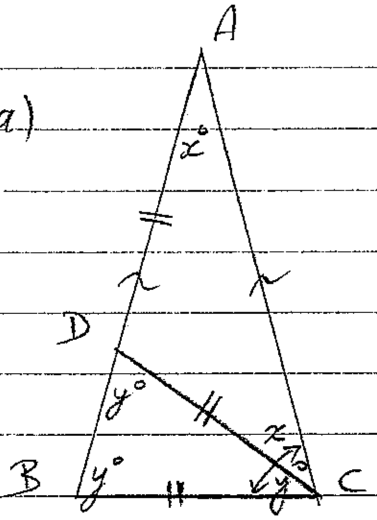
$\therefore a = x - z$

$\therefore y = (x - z) + x$

$\therefore y + z = 2x$ (5)

2\. (a)

(i)



①

Let $\angle ABC = \angle ACB = y^\circ$

$\angle ACD = x^\circ$ ($\angle$s opposite = sides)

$\angle BDC = y^\circ$ ($\angle$s opposite = sides)

$\therefore x + 2y = 180$ ($\angle$ sum of $\triangle ABC$)

But $\angle BDC = x + x$ (exterior $\angle$ = sum of int. opp. $\angle$s)

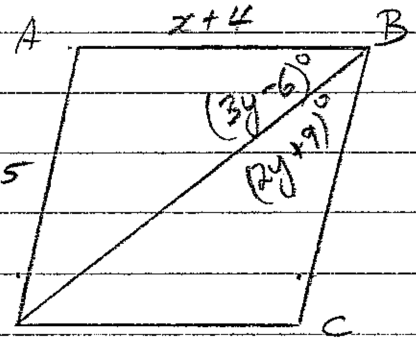
$\therefore y = 2x$

$\therefore x + 2(2x) = 180$

$5x = 180$ ⑥

$\therefore x = 36$.

(b)



$x+4 = 4x-5$ (sides of rhombus are equal)

$4+5 = 4x - x$

$3x = 9$

$\therefore x = 3$.

$3y-6 = 2y+9$ (diagonals bisect $\angle$s through which they pass)

$3y-2y = 6+9$

$y = 15$ ⑥

(c) $8{\cdot}5^2 = 72{\cdot}25$

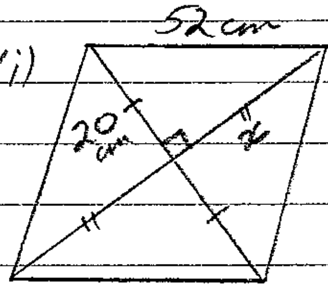
$4^2 + 7{\cdot}5^2 = 16 + 56{\cdot}25$

$= 72{\cdot}25$ ②

Since $8{\cdot}5^2 = 4^2 + 7{\cdot}5^2$

triangle is right-angled

(d) (i)



$52^2 = x^2 + 20^2$

$2704 = x^2 + 400$

$x^2 = 2304$

$\therefore x = 48$ ③

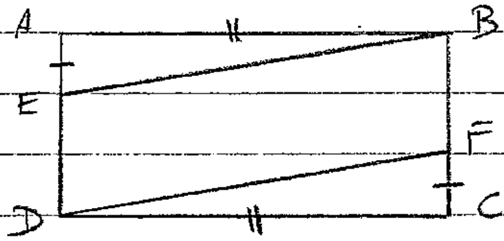
$\therefore$ length of longer diagonal is 96 cm.

(ii)

$A = \frac{1}{2} \times 40 \times 96$

$= 1920\ cm^2$. ②

3\.



(i) In $\triangle$s AEB and CFD

AB = CD (opposite sides of a rectangle)

AE = CF (given)

$\angle EAB = \angle FCD = 90^\circ$ ($\angle$s of a rectangle)

$\therefore \triangle AEB \equiv \triangle CFD$ (SAS)

④

(ii) $EB = FB$ (corresponding sides of congruent Δs)

$AD = BC$ (opposite sides of a rectangle)

$\therefore AE + ED = CF + FB$

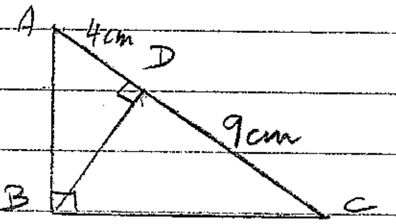
But $AE = CF$ (given)

$\therefore ED = FB$

Since both pairs of opposite sides are equal,
EBFD is a parallelogram (4)

(b)

(i)



(1)

(ii) Let $\angle DBC = \alpha^\circ$

$\therefore \angle DBA = (90 - \alpha)^\circ$ (adj. compl. $\angle$s)

$\angle BDC = 90^\circ$ (adj. compl. $\angle$s)

$\angle BCD = (90 - \alpha)^\circ$ ($\angle$ sum of ΔBDC)

$\therefore$ In Δs ABD and BCD

$\angle ADB = \angle BDC$ (proved above)

& $\angle ABD = \angle BCD$ (proved above)

$\therefore \Delta ABD \mid\mid\mid \Delta BCD$ (equiangular)

(5)

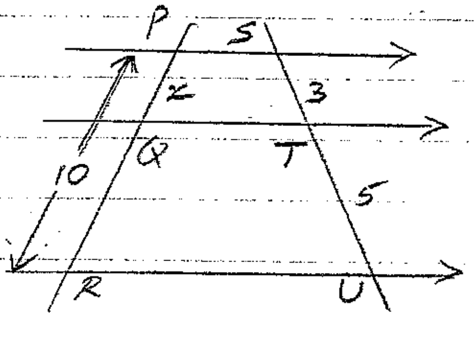
(iii) $\frac{BD}{CD} = \frac{AD}{BD}$ (matching sides of sim. Δs in same ratio)

$\therefore \frac{BD}{9} = \frac{4}{BD}$

$BD^2 = 36, \therefore BD = 6$ cm.

(3)

(c)



$\frac{x}{10 - x} = \frac{3}{5}$ (intercepts cut by // lines in same ratio)

$\therefore 5x = 3(10 - x)$

$5x = 30 - 3x$

$\therefore 8x = 30$

$\therefore x = 3{\cdot}75$ units (3)

4. (a)

(i) $m_{AD} = \frac{-3-5}{0+5} = -\frac{8}{5}$

$\tan\theta = \frac{-8}{5}$

$\therefore \theta = 180^\circ - 58^\circ$

$= 122^\circ$ (2)

(ii) $M\left(\frac{-5+3}{2}, \frac{5+1}{2}\right)$

$\therefore M(-1, 3)$ (2)

(iii) $C(x, y)$; $A(0, -3)$

$\therefore \frac{x+0}{2} = -1$; $\frac{y-3}{2} = 3$

$\therefore x = -2$, $y - 3 = 6$

$y = 9$

$\therefore C(-2, 9)$ (2)

(iv) $m_{AB} = \frac{1+3}{3-0} = \frac{4}{3}$; $A(0, -3)$

equation of AB is

$y + 3 = \frac{4}{3}(x - 0)$

$3y + 9 = 4x$

$4x - 3y - 9 = 0$ (3)

(v) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \frac{|(4 \times 5) + (-3 \times 5) - 9|}{\sqrt{4^2 + (-3)^2}}$

$\therefore d = \frac{|-20-15-9|}{5}$

$= 8.8$ units (3)

(vi) $AB = \sqrt{(3-0)^2 + (1+3)^2}$

$= \sqrt{9+16}$

$= 5$

Area $= AB \times d$

$= 5 \times 8.8$

$= 44$ unit^2. (3)

(b) $4x-3+3q+6 \quad 0$

$3q-6=0$

$3q=6$

$\therefore q = 2$ (2)

(c) $2y = -px+7$

$\therefore y = -\frac{p}{2}x + \frac{7}{2}$

$\therefore m_1 = \frac{-p}{2}$

& $y = -4x+5$

$\therefore m_2 = -4$

Now $m_1 = m_2$ if lines are $\parallel$

$\therefore -\frac{p}{2} = -4$ (3)

$\therefore p = 8$

(d) $y = -3x+2$

$\therefore m_1 = -3$

$\therefore$ gradient of line $\perp$ to $y=-3x+2$ has gradient $\frac{1}{3}$

$\therefore$ equation is given by

$y+1 = \frac{1}{3}(x+2)$

$3y+3 = x+2$

$3y = x-1$

$\therefore x-3y-1=0$ (4)

(e) $2x-y+3=0$ --- (1)

$3x+y+2=0$ --- (2)

(1) + (2) gives

$5x+5=0$

$\therefore 5x=-5$

$\therefore x=-1$

Sub. $x=-1$ into (1)

$\therefore -2-y+3=0$

$\therefore y=1$

Sub. $(-1,1)$ into $x+4y-g=0$

$\therefore -1+4-g=0$

$\therefore g=3$ (3)

(f) Method I: $3x+4y=2$ — (1)

$5x-2y=12$ — (2)

$2\times$(2) $\Rightarrow$ $10x-4y=24$ — (3)

(1) + (3) $\Rightarrow$ $13x=26$

$\therefore x=2$

Sub. $x=2$ into (1),

$\therefore 6+4y=2$

$4y=-4$

$\therefore y=-1$

$\therefore$ point of intersection is $(2,-1)$

& point $(1,-2)$

$m = \frac{-2+1}{1-2} = \frac{-1}{-1} = 1$ & $(2,-1)$

$\therefore$ equation is

$y+1 = 1(x-2)$

$y+1 = x-2$ (4)

$\therefore y = x-3$

Method II (k method)

$$(3x+4y-2)+k(5x-2y-12)=0.$$

at $(1,-2)$, $\quad 3-8-2+k(5+4-12)=0.$

$$-7-3k=0$$

$$\therefore 3k=-7$$

$$\therefore k=-\frac{7}{3}.$$

$$\therefore (3x+4y-2)-\frac{7}{3}(5x-2y-12)=0.$$

$$9x+12y-6-35x+14y+84=0.$$

$$-26x+26y+78=0$$

$$\therefore -x+y+3=0$$

$$\therefore y=x-3.$$