GIRRAWEEN HIGH SCHOOL

Year 11 Mathematics **Task 3** **June 2009**

Part A: 45 Minutes

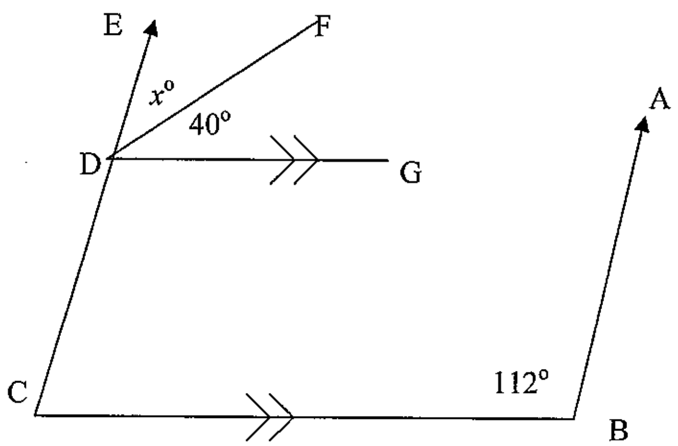
Instructions:

- Start each question on a new page
- Show all necessary working
- Marks will be deducted for careless or badly arranged work
- Board approve calculators may be used
- Diagrams are NOT to scale

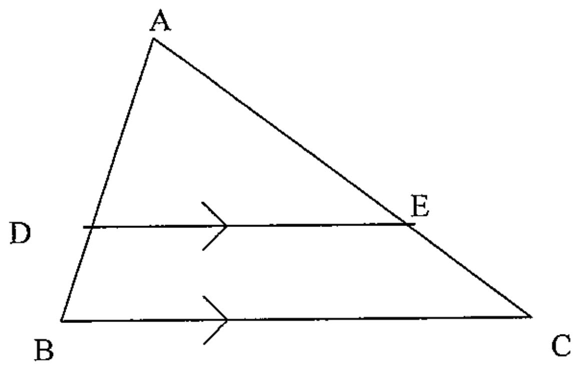
QUESTION 1 (13 marks)

(a) Find the interior and exterior angle size of a regular 15 sided polygon. (3)

(b) Find the value of x, giving reasons. (5)

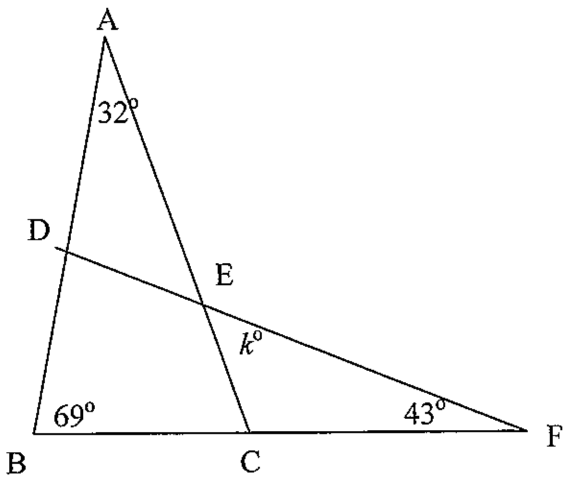


(c) In ΔABC, DE is drawn parallel to BC. Show that ΔADE is similar to ΔABC. (5)

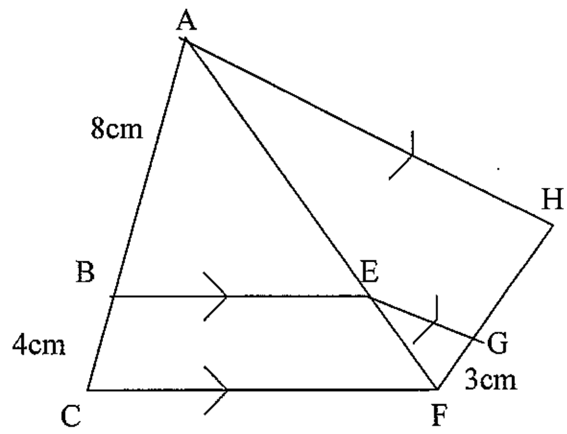


QUESTION 2 (8 marks)

(a) In the diagram find k, giving reasons. (4)

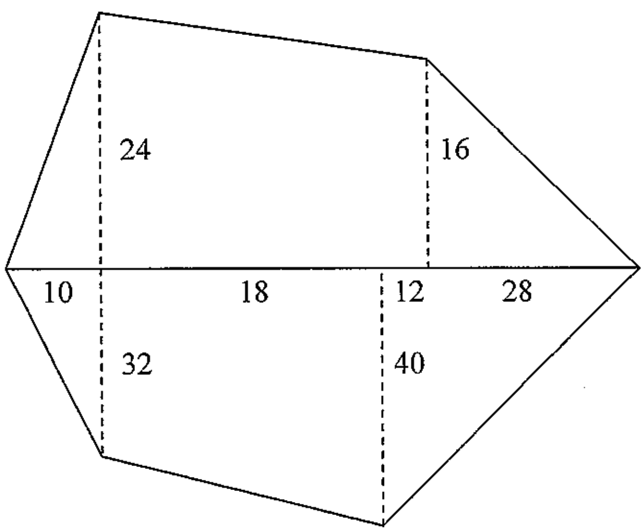


(b) Find the value of GH, giving reasons. (4)



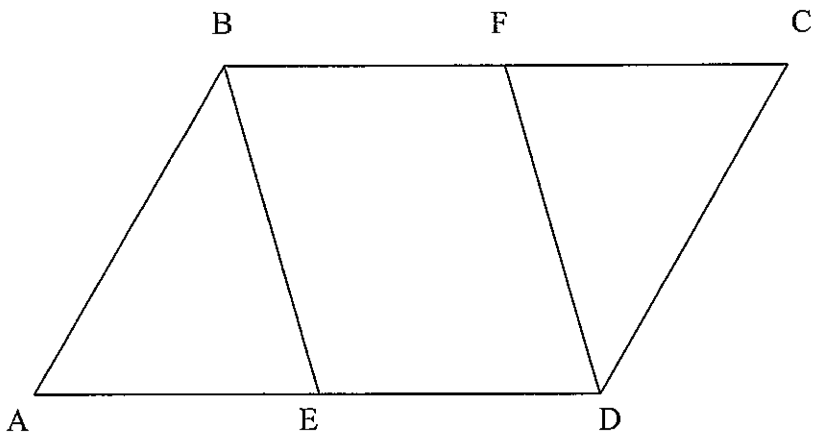
QUESTION 3 (6 marks)

Find the area of the following field.



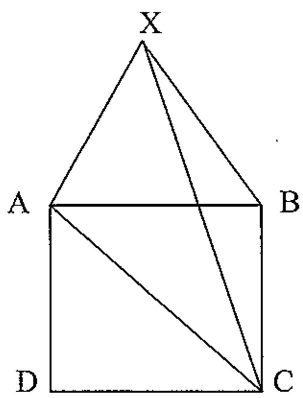
QUESTION 4 (6 marks)

ABCD and *DEBF* are parallelograms. Prove that $AE = CF$.



QUESTION 5 (6 marks)

ABCD is a square and *ABX* is an equilateral triangle standing on the side *AB* as shown. *CX* and *AC* are joined. By finding the actual size of the angles or otherwise, prove that $\angle ACX = 2 \times \angle BCX$ and $\angle AXC = 3 \times \angle BXC$.



GIRRAWEEN HIGH SCHOOL

Year 11 Mathematics | **Task 3** | **June 2009**

Part B: 45 Minutes

Instructions:

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QUESTION 1 (14 marks)

(a) The points A (-2, 1), B (6, 5) and C (10, 3) form a triangle

(i) Draw the triangle on a set of axes. (1)

(ii) Find the equation of the line AB, in general form. (3)

(iii) Find the length of AB. (2)

(iv) Find the perpendicular distance from C to AB. (3)

(v) Find the area of ΔABC. (2)

(b) Show that the three points (3, 5), (1, 1) and (-2, -5) are collinear. (3)

QUESTION 2 (10 marks)

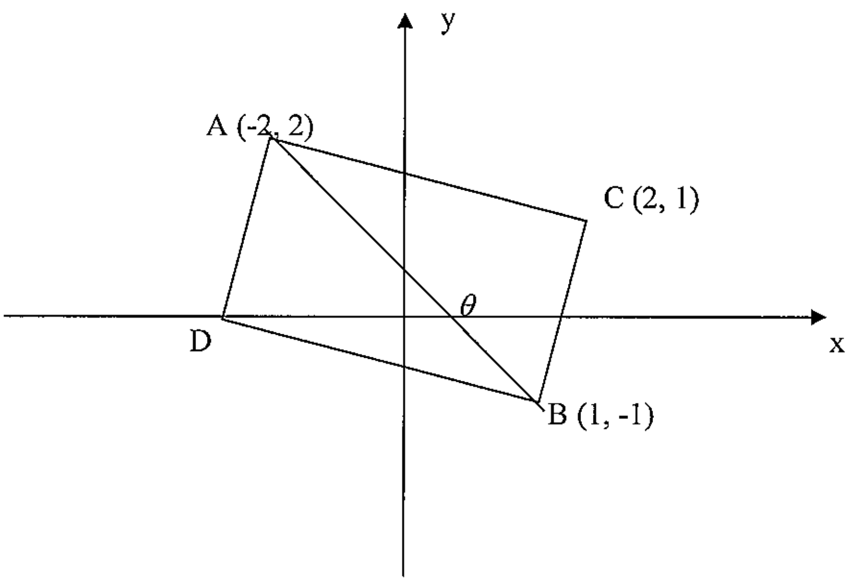
(a) Show that the triangle with vertices A (0, 0), B (3, 4) and C (-1, 1) is isosceles. (3)

(b) What is the value of p if (4, p) lies on the line $3x + 2y - 1 = 5$. (2)

(c) The line $5x + ky = 12$ passes through the point (2, -1). Find the value of k. (2)

(d) Find the equation of the line through the point (0, 4) and parallel to part (c). (3)

QUESTION 3 (8 marks)



ABCD is a parallelogram. The diagonals meet at *M*.

(a) Find θ, to the nearest degree. (3)

(b) Find the midpoint *M* of *AB*. (1)

(c) Find the coordinates of the vertex *D*. (4)

QUESTION 4 (6 marks)

Find the equation of the line which passes through the point of intersection of the lines $x+3y-4=0$ and $3x-4y+1=0$ and also passes through the point (3, -1).

Yr 11 TASK 3 PART A.

Q1a) Interior $= \frac{180°(15-2)}{15}$

$= \frac{180° \times 13}{15}$

$= 156°$ 2

Exterior $= 360° \div 15$

$= 24°$ 1

b) $\angle DCB = 180° - 112°$

$= 68°$ 1

(Co-interior $\angle$'s, $AB \parallel DC$) 1

$\angle EDG = 68°$ 1

(Corresponding $\angle$'s, $DG \parallel CB$) 1

$\therefore x° + 40° = 68°$

$x = 28°$ 1

c) In $\triangle ABC$ and $\triangle ADE$

$\angle BAC$ is common 1

$\angle ABC = \angle ADE$ 1

(corresponding $\angle$'s, $DE \parallel BC$) 1

$\angle ACB = \angle AED$ 1

(Corresponding $\angle$'s, $DE \parallel BC$) 1

$\therefore \triangle ABC \parallel\!| \triangle ADE$.

(equiangular)

Q2a) $\angle ECF = 69° + 32°$

$= 101°$ 1

(exterior angle sum of $\triangle ABC$)

$\angle CEF = 180° - (101° + 43°)$

$= 36°$ 1

($\angle$ sum of $\triangle CEF$) 1

$\therefore k = 36°$

b) $\frac{AB}{BC} = \frac{8}{4}$

$\frac{AB}{BC} = \frac{AE}{EF}$ (ratio of intercepts, $BE \parallel CF$) 1

$\frac{AE}{EF} = \frac{HG}{GF}$ (ratio of intercepts $AH \parallel EG$) 1

$\frac{8}{4} = \frac{HG}{3}$ 1

$\therefore HG = 6$ cm. 1

Q3)

Area I $= \frac{1}{2} \times 56 \times 10$

$= 280$ units^2 1

Area II $= \frac{1}{2} \times 16 \times 28$

$= 224$ units^2 1

Area III $= \frac{1}{2} \times 40 \times 40$.

$= 800$ units^2 1

Area IV $= \frac{30}{2}(16+24)$

$= 600$ units^2 1

Area V $= \frac{18}{2}(32+40)$

$= 648$ units^2 1

$\therefore$ Total Area

$= 280 + 224 + 800 + 600 + 648$

$= 2552$ 1

Q4)

$AB = CD$ (opposite sides of parallelogram are equal) (S) 1

$\angle BAE = \angle DCF$ (opposite angles of parallelogram are equal) (A) 1

$\angle BED = \angle BFD$ (opposite angles of parallelogram are equal) 1

$\therefore \angle AEB = \angle CFD$ (straight angle) (A) 1

$\therefore \triangle ABE \equiv \triangle CDF$ (AAS)

$\therefore AE = CF$ (matching sides of congruent $\triangle$'s)

Q5) $\angle XBC = 90° + 60°$

$= 150°$ 1

(angle of square and equilateral $\triangle$) 1

$BX = BC$ (side length of square and equilateral are equal) 1

$\therefore \triangle BCX$ is isosceles $\triangle$.

$\angle BXC = \angle BCX = \frac{180 - 150}{2}$ 1

$= 15$

(angles opposite equal sides of isosceles $\triangle$) 1

$\angle AXB = 60°$ (angle of equil. $\triangle$)

$\angle AXC = \angle AXB - \angle BXC$

$= 60 - 15$

$= 45°$

$\angle BCA = 45°$

(diagonal bisects angle of vertex in a square)

$\angle ACX = \angle BCA - \angle BCX$

$= 45 - 15$

$= 30°$

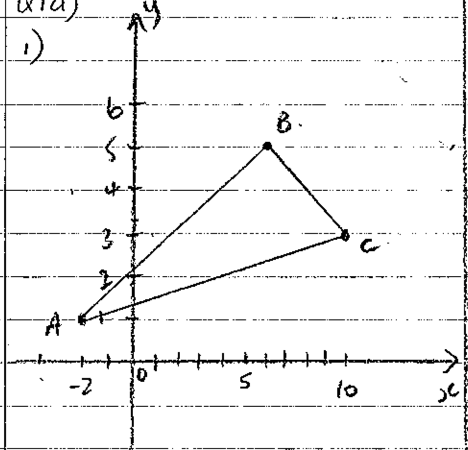
$\therefore \angle ACX = 2 \times \angle BCX$

and $\angle AXC = 3 \times \angle BXC$

Yr 11 Task 3 Part B

Q1a)

i)



ii) $A(-2,1)$ $B(6,5)$

$m = \frac{5-1}{6+2}$

$= \frac{4}{8} = \frac{1}{2}$ 1

$y - 1 = \frac{1}{2}(x+2)$ 1

$y - 1 = \frac{x}{2} + 1$

$2y - 2 = x + 2$

$\therefore x - 2y + 4 = 0$ 1

iii)

$d_{AB} = \sqrt{(6+2)^2 + (5-1)^2}$ 1

$= \sqrt{8^2 + 4^2}$

$= \sqrt{64 + 16}$

$= \sqrt{80} = 4\sqrt{5}$. 1

iv)

$p = \frac{|(1)(10) + (-2)(3) + (4)|}{\sqrt{1^2 + (-2)^2}}$ 1

$= \frac{|10 - 6 + 4|}{\sqrt{5}}$ 1

$= \frac{8}{\sqrt{5}} = \frac{8\sqrt{5}}{5}$. 1

v) Area ΔABC

$= \frac{1}{2} \times \frac{8\sqrt{5}}{5} \times 4\sqrt{5}$. 1

$= \frac{16 \times 5}{5}$

$= 16$ units^2.

b) $m_1 = \frac{1-5}{1-3}$ $m_2 = \frac{-5-1}{-2-1}$

$= \frac{-4}{-2}$ $= \frac{-6}{-3}$

$= 2$ $= 2$.

$\therefore$ $(3,5)$, $(1,1)$ and $(-2,-5)$ are collinear.

Q2a)

$d_{AB} = \sqrt{(3-0)^2 + (4-0)^2}$

$= \sqrt{3^2 + 4^2}$

$= \sqrt{25}$

$= 5$. 1

$d_{BC} = \sqrt{(1-3)^2 + (1-4)^2}$

$= \sqrt{(-4)^2 + (-3)^2}$

$= \sqrt{25}$

$= 5$. 1

$\therefore \Delta ABC$ is isosceles. 1

b) $(4,p)$ $3x + 2y - 1 = 5$

$3(4) + 2(p) - 1 = 5$. 1

$12 + 2p - 1 = 5$.

$2p = -6$

$p = -3$. 1

c) $5x + ky = 12$ $(2,-1)$

$5(2) + k(-1) = 12$

$10 - k = 12$

$k = -2$.

d) $(0,4)$ $m = \frac{5}{2}$. 1

$5x - 2y = 12$

$2y = 5x - 12$

$y = \frac{5}{2}x - 6$ 1

$y - 4 = \frac{5}{2}(x - 0)$ 1

$y - 4 = \frac{5x}{2}$

$y = \frac{5}{2}x + 4$. 1

Q3) a) $\tan\theta$ = gradient AB

$m_{AB} = \frac{2+1}{-2-1}$

$= \frac{3}{-3} = -1$ 1

$\tan\theta = -1$ $\tan\theta = 1$.

$\therefore \theta = 180 - 45°$ $\theta = 45°$

$= 135°$

b) $M_{AB} = \left(\frac{-2+1}{2}, \frac{2-1}{2}\right)$

$= \left(-\frac{1}{2}, \frac{1}{2}\right)$

c) $M_{CD} = \left(\frac{2+x}{2}, \frac{1+y}{2}\right)$,

$\frac{2+x}{2} = \frac{-1}{2}$ $\frac{1+y}{2} = \frac{1}{2}$

$2 + x = -1$ $1 + y = 1$

$x = -3$ $y = 0$

$\therefore D = (-3, 0)$

(Q4) $(3,-1)$

$x+3y-4=0$

$3x-4y+1=0$

$(x+3y-4)+k(3x-4y+1)=0$ 1

$(3+(3)(-1)-4)+k((3)(3)-4(-1)+1)=0$

$(3-3-4)+k(9+4+1)=0$

$-4+14k=0$ 1

$14k=4$

$k=\frac{4}{14}$

$=\frac{2}{7}$ 1

$(x+3y-4)+\frac{2}{7}(3x-4y+1)=0$ 1

$7(x+3y-4)+2(3x-4y+1)=0$ 1

$7x+21y-28+6x-8y+2=0$

$13x+13y-26=0$

$x+y-2=0$ 1

or $x+3y-4=0$ (1) $(\times 3)$

$3x-4y+1=0$ (2)

sub 2 from 3.

$3x+9y-12=0$ (3)

$3x-4y+1=0$

$13y-13=0$

$13y=13$

$y=1$

$\therefore x+3-4=0$

$x-1=0$

$x=1$

$m=\frac{-1-1}{3-1}$

$=\frac{-2}{2}=-1$

$y-1=-1(x-1)$

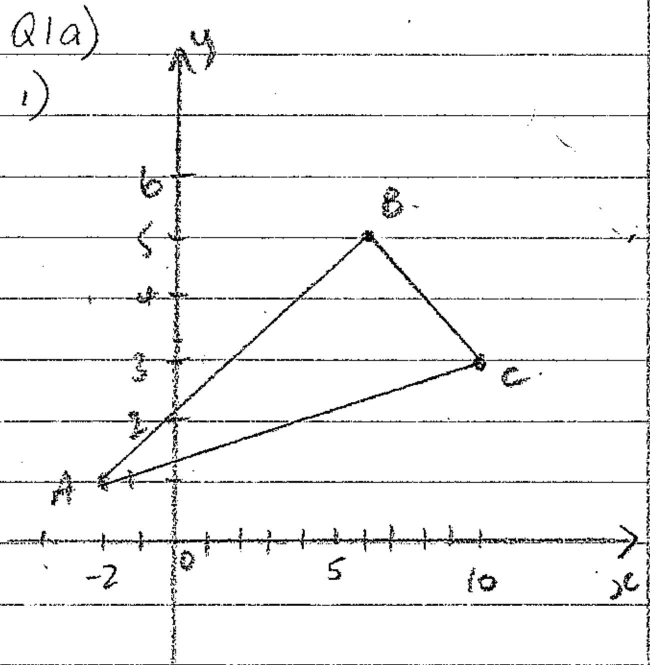
$y-1=-x+1$

$x+y-2=0$

Yr 11 Task 3 Part B

Q1a)

i)



ii) $A(-2,1)$ $B(6,5)$

$m = \frac{5-1}{6+2}$

$= \frac{4}{8} = \frac{1}{2}$ 1

$y-1 = \frac{1}{2}(x+2)$ 1

$y-1 = \frac{x}{2}+1$

$2y-2 = x+2$

$\therefore x-2y+4=0$ 1

iii)

$d_{AB} = \sqrt{(6+2)^2+(5-1)^2}$ 1

$= \sqrt{8^2+4^2}$

$= \sqrt{64+16}$

$= \sqrt{80} = 4\sqrt{5}$ 1

iv)

$p = \frac{|(1)(10)+(-2)(3)+(4)|}{\sqrt{1^2+(-2)^2}}$ 1

$= \frac{|10-6+4|}{\sqrt{5}}$ 1

$= \frac{8}{\sqrt{5}} = \frac{8\sqrt{5}}{5}$ 1

v) Area $\triangle ABC$

$= \frac{1}{2} \times \frac{8\sqrt{5}}{5} \times 4\sqrt{5}$ 1

$= \frac{16\times 5}{5}$

$= 16$ units^2

b) $m_1 = \frac{1-5}{1-3}$ $m_2 = \frac{-5-1}{-2-1}$

$= \frac{-4}{-2}$ $= \frac{-6}{-3}$

$= 2$ $= 2$

$\therefore$ $(3,5)$, $(1,1)$ and $(-2,-5)$ are collinear.

Q2a)

$d_{AB} = \sqrt{(3-0)^2+(4-0)^2}$

$= \sqrt{3^2+4^2}$

$= \sqrt{25}$

$= 5$. 1

$d_{BC} = \sqrt{(-1-3)^2+(1-4)^2}$

$= \sqrt{(-4)^2+(-3)^2}$

$= \sqrt{25}$

$= 5$. 1

$\therefore \triangle ABC$ is isosceles. 1

b) $(4,p)$ $3x+2y-1=5$

$3(4)+2(p)-1=5$ 1

$12+2p-1=5$

$2p=-6$

$p=-3$ 1

c) $5x+ky=12$ $(2,-1)$

$5(2)+k(-1)=12$

$10-k=12$

$k=-2$

d) $(0,4)$ $m=\frac{5}{2}$ 1

$5x-2y=12$

$2y=5x-12$

$y=\frac{5}{2}x-6$ 1

$y-4=\frac{5}{2}(x-0)$ 1

$y-4=\frac{5x}{2}$

$y=\frac{5}{2}x+4$ 1

Q3a) $\tan\theta =$ gradient AB

$m_{AB}=\frac{2+1}{-2-1}$

$=\frac{3}{-3}=-1$ 1

$\tan\theta=-1$ $\tan\theta=1$

$\therefore \theta = 180-45°$ $\theta=45°$

$=135°$

b) $M_{AB}=\left(\frac{-2+1}{2}, \frac{2-1}{2}\right)$

$=\left(-\frac{1}{2}, \frac{1}{2}\right)$

c) $M_{CD}=\left(\frac{2+x}{2}, \frac{1+y}{2}\right)$

$\frac{2+x}{2}=\frac{-1}{2}$ $\frac{1+y}{2}=\frac{1}{2}$

$2+x=-1$ $1+y=1$

$x=-3$ $y=0$

$\therefore D=(-3,0)$

(Q4) $(3,-1)$

$x+3y-4=0$

$3x-4y+1=0$

$(x+3y-4)+k(3x-4y+1)=0$ 1

$(3+(3)(-1)-4)+k((3)(3)-4(-1)+1)=0$

$(3-3-4)+k(9+4+1)=0$

$-4+14k=0$ 1

$14k=4$

$k=\frac{4}{14}$

$=\frac{2}{7}$ 1

$(x+3y-4)+\frac{2}{7}(3x-4y+1)=0$ 1

$7(x+3y-4)+2(3x-4y+1)=0$ 1

$7x+21y-28+6x-8y+2=0$

$13x+13y-26=0$

$x+y-2=0$ 1

or $x+3y-4=0$ (1) $(\times 3)$

$3x-4y+1=0$ (2)

sub 2 from 3.

$3x+9y-12=0$ (3)

$3x-4y+1=0$

$13y-13=0$

$13y=13$

$y=1$

$\therefore x+3-4=0$

$x-1=0$

$x=1$

$m=\frac{-1-1}{3-1}$

$=\frac{-2}{2}=-1$

$y-1=-1(x-1)$

$y-1=-x+1$

$x+y-2=0$