GIRRAWEEN HIGH SCHOOL

YEAR 12 - TASK 1

2020

MATHEMATICS ADVANCED

Time allowed: 90 minutes

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- Board-approved calculators may be used.
- The BOSTES Reference Sheet may be used (see separate laminated sheet).
- **DIAGRAMS ARE NOT DRAWN TO SCALE**.
- **TOTAL MARKS: 70**

Use the space provided to show your answers for each question including all relevant working out.

Extra working pages are attached at the end of this paper. Clearly write the question numbers.

- **Section I (5 marks): Questions 1 – 5**
  - Select A, B, C or D that best answers the question.
- **Section II (65 marks): Questions 6 – 10**
  - All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.

SECTION I: MULTIPLE CHOICE QUESTIONS 1 – 5

Select A, B, C or D that best answers the question on your ANSWER BOOKLET.

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1. What is the first term of the sequence, $T_n = 3n - 7$ greater than 130?

(A) T_{41}
(B) T_{45}
(C) T_{46}
(D) T_{50}

2. If $f(x) = 3^x$ and $g(x) = -2x$, what is the value of $g \circ f(2)$?

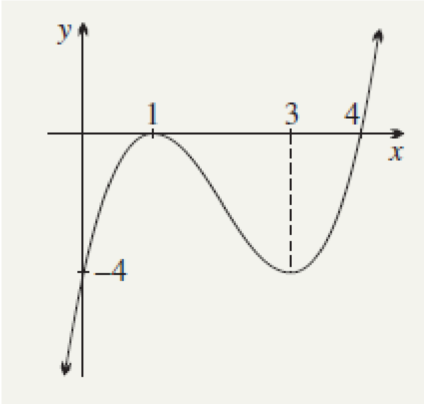
(A) -18
(B) -12
(C) $1/81$
(D) 81

3. What is $P(X \text{ is odd})$ for a discrete probability distribution tabulated below?

$\boldsymbol{x}$	0	1	2	3	4	5
$\boldsymbol{P(X = x)}$	0.07	0.15	0.24	0.26	0.17	0.11

(A) 0.41
(B) 0.48
(C) 0.52
(D) 0.59

4. A sketch of $y = x^3 - 6x^2 + 9x - 4$ is shown on the right.
What is the solution of $x^3 + 9x > 6x^2 + 4$?



(A) $x < 1$
(B) $1 < x < 4$
(C) $x > 4$
(D) $0 < x < 4$

5. The n^{th} term of the sequence $1, -2, 3, -4, 5, -6, 7, -8, \ldots$ is:

(A) $(-1)^{n-1}(2n-1)$
(B) $(-1)^{n+1}n$
(C) $(-1)^{n+1}(2n-1)$
(D) $(-1)^{n+1}(n+1)$

Answers to Multiple Choice Questions

Select the alternative A, B, C or D that best answers the question.

1.	A	○	B	○	C	○	D	○
2.	A	○	B	○	C	○	D	○
3.	A	○	B	○	C	○	D	○
4.	A	○	B	○	C	○	D	○
5.	A	○	B	○	C	○	D	○

SECTION II: QUESTIONS 6 – 10

Show all necessary working for each question.

Marks may be deducted for careless or badly arranged work.

QUESTION 6 (11 Marks)

(a) Find x and write out the three numbers if $(x-2), (x+2), (5x-2)$ form a geometric sequence.

(3 marks)

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(b) The sum of the second and the fifth term of an arithmetic sequence is 32 whilst the sum of the third and the eighth term is 48.

(i) Find the first term and the common difference. (2 marks)

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(ii) Find the sum of the first 30 terms. (1 mark)

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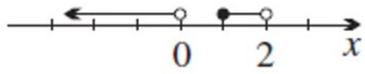
~~(c)~~ Consider the intervals shown on the number line below.



(i) Write the grouped compound interval using inequality interval notation. (1 mark)

(ii) Write the graphed compound interval using bracket interval notation. (1 mark)

(d) The sum of a series is given as $S_n = n^2 - 3^n$.

(i) Find S_{14}. (1 mark)

(ii) Find the 13th term of the series. (2 marks)

QUESTION 7 (15 Marks)

(a) Consider a pair of equations: $y = |2x| + 1$ and $y = -x + 2$.

(i) Sketch the graph of $y = |2x| + 1$. (2 marks)

(ii) Write down the equations of the right-hand and left-hand branches of the first equation. (1 mark)

(iii) On the same graph in (i), draw the graph of $y = -x + 2$. (1 mark)

(iv) Find the points of intersection. (2 marks)

(v) Hence solve $|2x| + 1 \leq -x + 2$. (1 mark)

(b) Use the table below to find $Var(X)$. (3 marks)

$\boldsymbol{x}$	0	2	4	6	8	Sum
$\boldsymbol{p(x)}$	0.18	0.22	0.26	0.19	0.15	

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(c) Jessica decides to save money by putting some coins in a jar every day. On the first day she will put in $10c$, on the second, $15c$, and so on, with the amount she puts into the jar increasing by $5c$ each day.

(i) How much will she put in on the 20^{th} day? (2 marks)

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(ii) For how many days must she save before the jar holds \$75.00? (3 marks)

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QUESTION 8 (14 Marks)

(a) Evaluate:

(i) (2 marks)

$$\sum_{n=3}^{7}(11-3n)$$

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(ii) (2 marks)

$$\sum_{k=1}^{39}(-1)^{k-1}$$

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(b) The Spring Hill Zoo has 20 wallabies. Six are from Snake Gully, five are from Dingo Ridge, and nine are from Acacia Flat. On a specific day, the zookeeper selected three wallabies at random and took them to breakfast.

(i) Draw up a probability distribution table for the number of wallabies from Acacia Flat taken to breakfast. (2 marks)

(ii) Find $E(X)$. (1 mark)

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(iii) Find $P(X \geq E(X))$ (1 mark)

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(c) Use the formula for the limiting sum of a geometric sequence to express $2.2\dot{7}$ as a fraction in lowest terms. (2 marks)

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(d) When a plastic matter is recycled, typically one fourth of it is wasted every time it goes through one recycling process. A plastic recycling plant receives 150 tonnes of matter made of plastic.

(i) How much of this will be available for use after it goes through the recycling process 10 times? (2 marks)

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(ii) After how many recycling processes would more than half the starting mass be wasted? (2 marks)

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QUESTION 9 (13 Marks)

(a) Consider the sum of n terms of the arithmetic sequence: $101 + 96 + 91 + \cdots$.

(i) Find the expression for the sum of n terms, S_n. (2 marks)

(ii) Find how many terms must be taken to make the sum negative. (2 marks)

(iii) Find the maximum sum of the sequence and the value of n where it reaches this. (2 marks)

(b) Consider $y = x^3 + 2x^2 - 15x$.

(i) Determine whether the function is even or odd (or neither). (2 marks)

(ii) Find all intercepts. (2 marks)

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(iii) Draw up a table of signs to analyse any change of sign of the function. (1 mark)

(iv) Sketch the curve showing important features. (2 marks)

QUESTION 10 (12 Marks)

(a) Consider the geometric series:

$$1 + \left(\sqrt{\alpha} - 2\right) + (\sqrt{\alpha} - 2)^2 + (\sqrt{\alpha} - 2)^3 + \cdots$$

(i) Find the largest positive integral value of α for the series to have a limiting sum. (2 marks)

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(ii) Find the limiting sum for this series when $\alpha = 8$.

Express your answer as a surd with a rational denominator. (2 marks)

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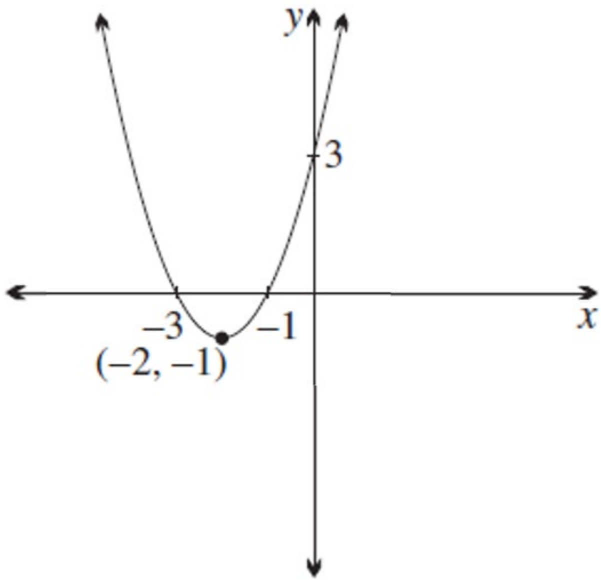
~~(b)~~ Given below is the graph of $y = f(x)$.



~~(i)~~ State the transformations done on the standard curve, $y = x^2$, to obtain the graph given above.

(2 marks)

~~(ii)~~ Hence write down its equation. (1 mark)

~~(iii)~~ On the same diagram given above, sketch $y = -e^x + 1$. (1 mark)

~~(iv)~~ Using the sketch, find the number of solutions of the two functions.

(1 mark)

(c) The probability distribution of a discrete random variable, X, is given by the table below.

$\boldsymbol{x}$	-1	0	1	2	3
$\boldsymbol{p(x)}$	$\frac{3}{4}p$	$\frac{2}{10}$	$(1-2p)$	$\frac{1}{2}p^2$	$\frac{3}{10}$

Show that $p = \frac{1}{2}$ is the only possible value for this probability distribution table. (3 marks)

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End of examination

Extra working pages – Clearly write the question numbers

FINAL MARK

GIRRAWEEN HIGH SCHOOL
YEAR 12
MATHEMATICS (ADVANCED)
2020 TASK 1

Student number: Solutions

QUESTION	MARK	MA11-7	MA12-1	MA12-4	MA12-10
1-5	/5	✓		✓	
6	/11			✓	
7	/15	✓	✓	✓	
8	/14	✓	✓	✓	✓
9	/13		✓	✓	✓
10	/12	✓	✓	✓	✓
TOTAL					
	/70	/46	/54	/70	/39

Year 11 Mathematics Advanced outcomes

A student:

MA11-7 uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions

Year 12 Mathematics Advanced outcomes

A student:

MA12-1 uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts

MA12-2 models and solves problems and makes informed decisions about financial situations using mathematical reasoning and techniques

MA12-3 applies calculus techniques to model and solve problems

MA12-4 applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems

MA12-5 applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs

MA12-6 applies appropriate differentiation methods to solve problems

MA12-7 applies the concepts and techniques of indefinite and definite integrals in the solution of problems

MA12-8 solves problems using appropriate statistical processes

MA12-9 chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use

MA12-10 constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context

GIRRAWEEN HIGH SCHOOL

YEAR 12 - TASK 1

2020

MATHEMATICS ADVANCED

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  - ➢ Select A, B, C or D that best answers the question.
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(C) T_{46}
(D) T_{50}

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(A) -18
(B) -12
(C) $1/81$
(D) 81

3. What is $P(X \text{ is odd})$ for a discrete probability distribution tabulated below?

x	0	1	2	3	4	5
$P(X = x)$	0.07	0.15	0.24	0.26	0.17	0.11

(A) 0.41
(B) 0.48
(C) 0.52
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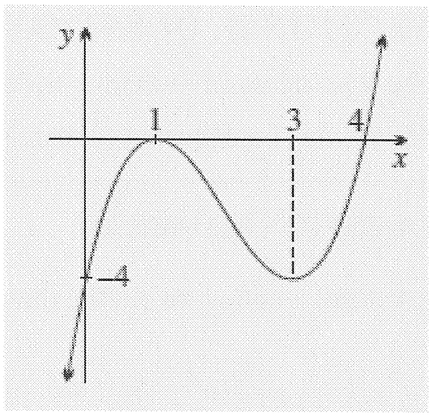
4. A sketch of $y = x^3 - 6x^2 + 9x - 4$ is shown on the right.

What is the solution of $x^3 + 9x > 6x^2 + 4$?



(A) $x < 1$

(B) $1 < x < 4$

(C) $x > 4$

(D) $0 < x < 4$

5. The n^{th} term of the sequence $1, -2, 3, -4, 5, -6, 7, -8, \ldots$ is:

(A) $(-1)^{n-1}(2n-1)$

(B) $(-1)^{n+1}n$

(C) $(-1)^{n+1}(2n-1)$

(D) $(-1)^{n+1}(n+1)$

Answers to Multiple Choice Questions

Select the alternative A, B, C or D that best answers the question.

1.	A	○	B	○	C	●	D	○
2.	A	●	B	○	C	○	D	○
3.	A	○	B	○	C	●	D	○
4.	A	○	B	○	C	●	D	○
5.	A	○	B	●	C	○	D	○

SECTION II: QUESTIONS 6 – 10

Show all necessary working for each question.

Marks may be deducted for careless or badly arranged work.

QUESTION 6 (11 Marks)

(a) Find x and write out the three numbers if $(x-2), (x+2), (5x-2)$ form a geometric sequence.

(3 marks)

$$\frac{x+2}{x-2} = \frac{5x-2}{x+2}$$

$x^2+4x+4 = 5x^2-12x+4$

$4x^2-16x = 0$

$x^2-4x = 0$

$x(x-4) = 0$

$x=0$ or $x=4$

Thus the sequence could be:

$-2, 2, -2, \ldots$

or $2, 6, 18, \ldots$

(b) The sum of the second and the fifth term of an arithmetic sequence is 32 whilst the sum of the third and the eighth term is 48.

(i) Find the first term and the common difference. (2 marks)

$T_2 + T_5 = 32$

$T_2 = a + d$

$T_5 = a + 4d$

$2a + 5d = 32 \ldots (1)$

$T_3 + T_8 = 48$

$T_3 = a + 2d$

$T_8 = a + 7d$

$2a + 9d = 48 \ldots (2)$

$(2) - (1)$: $4d = 16$

$\therefore d = 4$

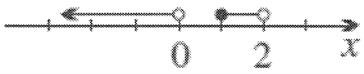
Sub into (1) : $2a + 20 = 32$ $\therefore a = 6$

(ii) Find the sum of the first 30 terms. (1 mark)

$S_{30} = \frac{30}{2}[12 + (29 \times 4)]$

$= 1920$

(c) Consider the intervals shown on the number line below.



(i) Write the grouped compound interval using inequality interval notation. (1 mark)

$x<0$ or $1\le x<2$

(ii) Write the graphed compound interval using bracket interval notation. (1 mark)

$(-\infty,0)\cup[1,2)$

(d) The sum of a series is given as $S_n = n^2 - 3^n$.

(i) Find S_{14}. (1 mark)

$S_{14} = 14^2 - 3^{14}$

$= -4782773$

(ii) Find the 13th term of the series. (2 marks)

$S_{13} = 13^2 - 3^{13}$

$= -1594154$

$S_{12} = 12^2 - 3^{12}$

$= -531297$

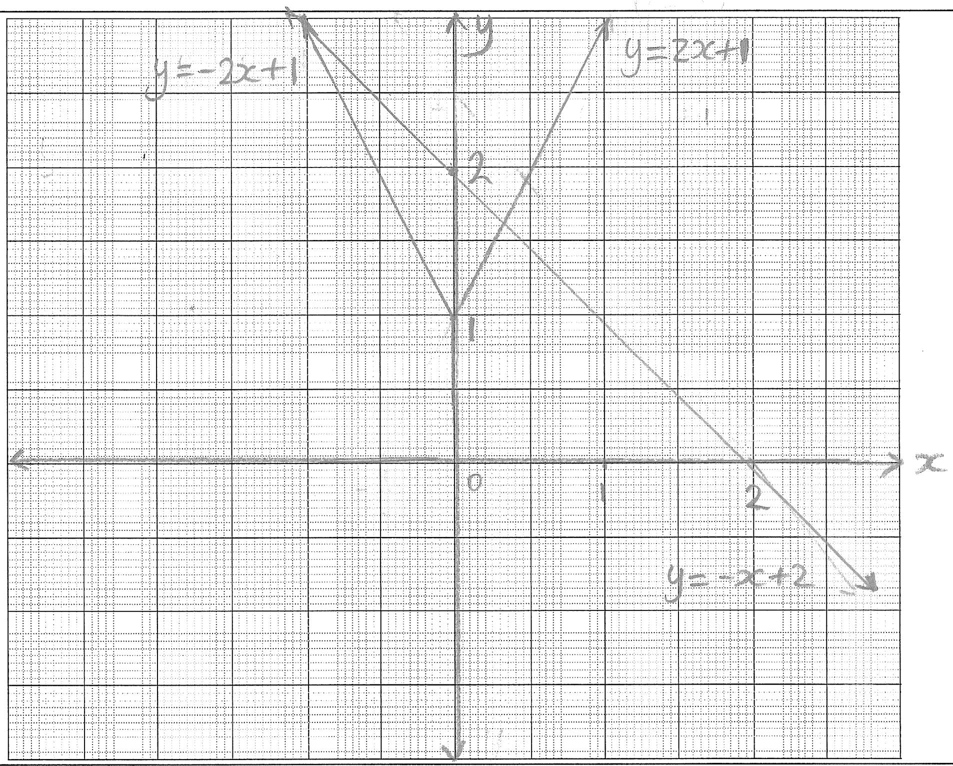
$T_{13} = S_{13} - S_{12}$

$= -1062857$

QUESTION 7 (15 Marks)

(a) Consider a pair of equations: $y = |2x| + 1$ and $y = -x + 2$.

(i) Sketch the graph of $y = |2x| + 1$. (2 marks)



(ii) Write down the equations of the right-hand and left-hand branches of the first equation. (1 mark)

(iii) On the same graph in (i), draw the graph of $y = -x + 2$. (1 mark)

(iv) Find the points of intersection. (2 marks)

$2x + 1 = -x + 2$

$x = \frac{1}{3}$

Sub into $y = -x + 2$,

$y = \frac{5}{3}$

$-2x + 1 = -x + 2$

$x = -1$

Sub into $y = -x + 2$,

$y = 3$

$\therefore$ Points of intersections are $(\frac{1}{3}, \frac{5}{3})$ & $(-1, 3)$

(v) Hence solve $|2x| + 1 \leq -x + 2$. (1 mark)

$-1 \leq x \leq \frac{1}{3}$

(b) Use the table below to find $Var(X)$. (3 marks)

x	0	2	4	6	8	Sum	
$p(x)$	0.18	0.22	0.26	0.19	0.15	1	
$xp(x)$	0	0.44	1.04	1.14	1.2	3.82	$= \mu$
$x^2p(x)$	0	0.88	4.16	6.84	9.6	21.48	$= E(X^2)$

$Var(X) = E(X^2) - \mu^2 = 6.8876$

(c) Jessica decides to save money by putting some coins in a jar every day. On the first day she will put in $10c$, on the second, $15c$, and so on, with the amount she puts into the jar increasing by $5c$ each day.

(i) How much will she put in on the 20th day? (2 marks)

$0.1,\ 0.15,\ 0.20, \ldots$

$a = 0.1$, $d = 0.05$

$T_{20} = a + (n-1)d$

$= 0.1 + (19 \times 0.05)$

$= 1.05 \quad \therefore \$1.05$

(ii) For how many days must she save before the jar holds \$75.00? (3 marks)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$75 = \frac{n}{2}(0.2 + 0.05n - 0.05)$

$150 = 0.05n^2 + 0.15n$

$n^2 + 3n - 3000 = 0$

$n = \dfrac{-3 \pm \sqrt{12009}}{2}$

$\therefore n = 53.3$

$\therefore$ 54 days

QUESTION 8 (14 Marks)

(a) Evaluate:

(i) (2 marks)

$$\sum_{n=3}^{7}(11-3n)$$

$= (11-9) + (11-12) + (11-15) + \ldots + (11-21)$

$= 2 - 1 - 4 + \ldots - 10$

AP with $a=2$, $d=-3$, $n=5$ $\quad S_5 = \frac{n}{2}(a+l)$

$= \frac{5}{2}(2-10)$

$= -20$

(ii) (2 marks)

$$\sum_{k=1}^{39}(-1)^{k-1}$$

$= (-1)^0 + (-1)^1 + (-1)^2 + \ldots + (-1)^{37} + (-1)^{38}$

$= 1 - 1 + 1 - 1 + \ldots - 1 + 1$

$= 1$

(b) The Spring Hill Zoo has 20 wallabies. Six are from Snake Gully, five are from Dingo Ridge, and nine are from Acacia Flat. On a specific day, the zookeeper selected three wallabies at random and took them to breakfast.

(i) Draw up a probability distribution table for the number of wallabies from Acacia Flat taken to breakfast. (2 marks)

x	0	1	2	3
$P(X=x)$	$\frac{11}{76}$	$\frac{33}{76}$	$\frac{33}{95}$	$\frac{7}{95}$

(ii) Find $E(X)$. (1 mark)

$E(X) = \left(0 \times \frac{11}{76}\right) + \left(1 \times \frac{33}{76}\right) + \left(2 \times \frac{33}{95}\right) + \left(3 \times \frac{7}{95}\right)$

$= \frac{27}{20} = 1.35$

(iii) Find $P(X \geq E(X))$ (1 mark)

$P(X \geq E(X)) = P(X=2) + P(X=3) = \frac{8}{19}$

(c) Use the formula for the limiting sum of a geometric sequence to express $2.2\dot{7}$ as a fraction in lowest terms. (2 marks)

$2.2\dot{7} = 2.2 + (0.07 + 0.007 + 0.0007 + \ldots)$

$= 2.2 + $ (GP with $a = 0.07$, $r = 0.1$ $\therefore S_\infty$ exists)

$= 2.2 + \left(\frac{0.07}{1-0.1}\right)$

$= 2.2 + \frac{7}{90}$

$= 2\frac{5}{18}$

(d) When a plastic matter is recycled, typically one fourth of it is wasted every time it goes through one recycling process. A plastic recycling plant receives 150 tonnes of matter made of plastic.

(i) How much of this will be available for use after it goes through the recycling process 10 times? (2 marks)

$150,\ 112.5,\ 84.375, \ldots$

GP with $a = 150$ $\qquad T_n = ar^{n-1}$

$r = \frac{3}{4}$ $\qquad T_{11} = 150 \times \left(\frac{3}{4}\right)^{10}$

$T_{11} = ?$ $\qquad \doteqdot 8.45$ tonnes

(ii) After how many recycling processes would more than half the starting mass be wasted? (2 marks)

Find n when $T_n = \frac{a}{2}$: $\frac{a}{2} = a\left(\frac{3}{4}\right)^{n-1}$

$\frac{1}{2} = \left(\frac{3}{4}\right)^{n-1}$

$\ln(0.5) = \ln(0.75)^{n-1}$

$n - 1 = \frac{\ln(0.5)}{\ln(0.75)}$

$n \doteqdot 3.41$ (4th term)

$\therefore$ After 3rd process less than $\frac{1}{2}$ of 150t will remain

QUESTION 9 (13 Marks)

(a) Consider the sum of n terms of the arithmetic sequence: $101 + 96 + 91 + \cdots$.

(i) Find the expression for the sum of n terms, S_n. (2 marks)

$a = 101$, $d = -5$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[202 - 5n + 5]$

$\therefore S_n = \frac{n}{2}(207 - 5n)$

(ii) Find how many terms must be taken to make the sum negative. (2 marks)

Find n when $S_n < 0$

$\frac{n}{2}(207 - 5n) < 0$

Sub. $S_n = 0$,

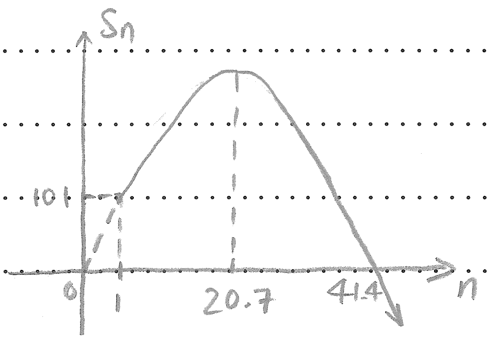
$\frac{n}{2}(207 - 5n) = 0$

$\therefore n = 41.4$

Since $n \geq 1$

$n > 41.4$

$\therefore$ 42 terms



(iii) Find the maximum sum of the sequence and the value of n where it reaches this. (2 marks)

When $n = 21$, max. S_n occurs

$S_{21} = \frac{21}{2}(207 - 105)$

$= 1071$

(b) Consider $y = x^3 + 2x^2 - 15x$.

(i) Determine whether the function is even or odd (or neither). (2 marks)

$f(x) = x^3 + 2x^2 - 15x$

$f(-x) = (-x)^3 + 2(-x)^2 - 15(-x)$

$= -x^3 + 2x^2 + 15x$

$-f(x) = -x^3 - 2x^2 + 15x$

$\therefore f(x)$ is neither.

(ii) Find all intercepts. (2 marks)

x-int : $x^3+2x^2-15x=0$ $\qquad$ y-int : at $y=0$

$x(x^2+2x-15)=0$

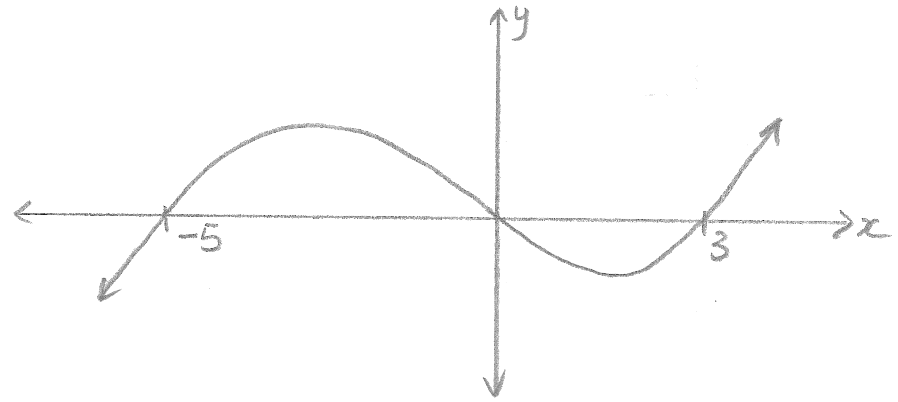
$x(x+5)(x-3)=0$

$x=0$ or $x=-5$ or $x=3$

(iii) Draw up a table of signs to analyse any change of sign of the function. (1 mark)

x	-6	-5	-1	0	1	3	4
y	-54	0	16	0	-16	0	36
sign	$-$	0	$+$	0	$-$	0	$+$

(iv) Sketch the curve showing important features. (2 marks)



QUESTION 10 (12 Marks)

(a) Consider the geometric series:

$$1 + (\sqrt{\alpha} - 2) + (\sqrt{\alpha} - 2)^2 + (\sqrt{\alpha} - 2)^3 + \cdots$$

(i) Find the largest positive integral value of α for the series to have a limiting sum. (2 marks)

For a S_∞ to exist,

$-1 < r < 1$

$-1 < (\sqrt{\alpha} - 2) < 1$

$1 < \sqrt{\alpha} < 3$

$1 < \alpha < 9$

$\therefore$ largest positive integral value of α is 8

(ii) Find the limiting sum for this series when $\alpha = 8$.

Express your answer as a surd with a rational denominator. (2 marks)

$a = 1$

$r = \sqrt{8} - 2$

$= 2\sqrt{2} - 2$

$S_\infty = \frac{a}{1-r}$

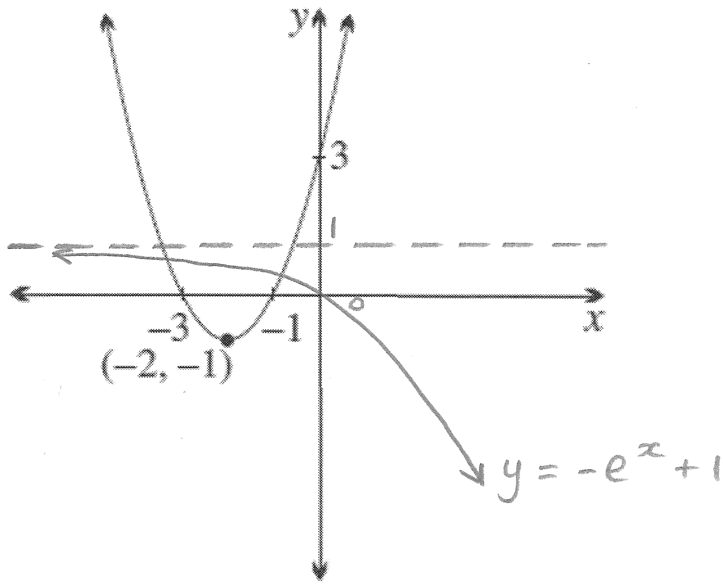
$= \frac{1}{1-(2\sqrt{2}-2)}$

$= \frac{1}{3-2\sqrt{2}} \cdot \frac{(3+2\sqrt{2})}{(3+2\sqrt{2})}$

$= \frac{3+2\sqrt{2}}{9-(2\sqrt{2})^2}$

$= 3+2\sqrt{2}$

(b) Given below is the graph of $y = f(x)$.



(i) State the transformations done on the standard curve, $y = x^2$, to obtain the graph given above. (2 marks)

$y=x^2$ is shifted to the left by 2 units.
Then it is moved down by 1 unit.
or $y=x^2$ is shifted to the right by 2 units
Then it is moved down by 1 unit
Then it is reflected about the y-axis

(ii) Hence write down its equation. (1 mark)

$y = (x+2)^2 - 1$

(iii) On the same diagram given above, sketch $y = -e^x + 1$. (1 mark)

(iv) Using the sketch, find the number of solutions of the two functions. (1 mark)

2 Solutions

(c) The probability distribution of a discrete random variable, X, is given by the table below.

x	-1	0	1	2	3
$p(x)$	$\frac{3}{4}p$	$\frac{2}{10}$	$(1-2p)$	$\frac{1}{2}p^2$	$\frac{3}{10}$

Show that $p = \frac{1}{2}$ is the only possible value for this probability distribution table. (3 marks)

$$\frac{3}{4}p + \frac{2}{10} + (1-2p) + \frac{1}{2}p^2 + \frac{3}{10} = 1$$

$$\frac{1}{2}p^2 - \frac{5}{4}p + \frac{3}{2} = 1$$

$$2p^2 - 5p + 6 = 4$$

$$2p^2 - 5p + 2 = 0$$

$$2p^2 - 4p - p + 2 = 0$$

$$2p(p-2) - (p-2) = 0$$

$$(2p-1)(p-2) = 0$$

$$\therefore p = \frac{1}{2} \text{ or } p = 2$$

however when $p=2$, $P(X=1)$ becomes -3 which is not a possible value for probability.

hence $p = \frac{1}{2}$ is the only solution

End of examination