Girraween High School

Year 11 Mathematics Advanced
2024 Task 1

General Instructions

- Working Time - 90 minutes
- Write using blue or black pen only
- Calculators approved by NESA may be used

For Questions 1-5 (page 1)

- Colour the bubble next to the letter corresponding to the correct response on your paper

For Questions 6-12 (pages 2-26)

- All necessary working should be shown in the space provided
- Marks will be deducted for careless or badly arranged work
- Diagrams are not to scale

Question 1 (1 mark)

What is $\dfrac{2}{2+\sqrt{3}}$ as a fraction with a rational denominator?

A. $4+2\sqrt{3}$

B. $4-2\sqrt{3}$

C. $\dfrac{4+2\sqrt{3}}{7}$

D. $\dfrac{4-2\sqrt{3}}{7}$

Question 2 (1 mark)

If $a+\dfrac{1}{a}=5$, find the value of $a^2+\dfrac{1}{a^2}$

A. 25

B. 27

C. 24

D. 23

Question 3 (1 mark)

Which of the following would not represent a function?

A. $y=\dfrac{-1}{x}$

B. $y=2x^2+\sqrt{2}$

C. $(0,-1),(1,0),(1,1),(2,2)$

D. $y=|x|$

Question 4 (1 mark)

How many real solutions does the equation $x^3+2x^2+3x=0$ have?

A. 0

B. 1

C. 2

D. 3

Question 5 (1 mark)

What is the domain and range of the function $f(x)=\sqrt{1-x^2}$?

A. Domain: $0\leq x\leq 1$, Range: $-1\leq y\leq 1$

B. Domain: $-1\leq x\leq 1$, Range: $-1\leq y\leq 1$

C. Domain: $-1\leq x\leq 1$, Range: $0\leq y\leq 1$

D. Domain: $0\leq x\leq 1$, Range: $0\leq y\leq 1$

Answers to Multiple choice questions

Select the alternative A, B, C or D that best answers the question

1.	A	○	B	○	C	○	D	○
2.	A	○	B	○	C	○	D	○
3.	A	○	B	○	C	○	D	○
4.	A	○	B	○	C	○	D	○
5.	A	○	B	○	C	○	D	○

Question 6 (15 marks)

(a) Evaluate, correct to 2 significant figures $\dfrac{2.1 \times 10^3 \times 6.9 \times 10^2}{\sqrt{3.4 \times 10^8}}$ [2]

(b) Factorise

i. $2x^2 + 3x - 2$ [1]

ii. $x^3 + 5x^2 + x + 5$ [1]

iii. $4a^2(x^3 + 18ab^2) - (32a^5 + 9b^2x^3)$ [2]

(c) Simplify:

i. $\dfrac{x^2 - y^2}{y - x}$ [2]

ii. $\dfrac{x-4}{2} + \dfrac{x+3}{3}$ [2]

iii. $\dfrac{3x-3y}{xy} \div \dfrac{x^2-xy}{x^2}$ [2]

iv. $\dfrac{3}{x^2+2x-8} - \dfrac{2}{x^2+x-6}$ [3]

Question 7 (14 marks)

(a) Express $\dfrac{6+\sqrt{3}}{1-\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b are rational. [2]

(b) Six months before the Beijing Olympics Games, an athlete's best long jump was 0.2 metres less than the qualifying distance. Over the following month he improved his jump by 4.2% to a distance which was 0.1 metres better than the qualifying distance. Find the qualifying distance (in metres correct to 2 decimal places) [2]

(c) If $S = \dfrac{a(1-r^n)}{1-r}$ find the exact value of S when $a = 32$, $n = 8$ and $r = \dfrac{-1}{2}$ [2]

(d) Solve for x by completing the square for $f(x) = x^2 + 10x + 17$ [2]

(e) Find the value of x if $\sqrt{x} = \sqrt{50} - \sqrt{18}$ [2]

(f) For which values of k is the expression $x^2 - (k-4)x + 9$ a perfect square? [2]

(g) ABC is a right angled triangle in which $\angle ACB = 90^o$. [2]
if $AB = \sqrt{1+x^2}$, $BC = \sqrt{1-x^2}$ and $AC = \sqrt{x}$, find the value(s) of x, giving reasons.

Question 8 (14 marks)

(a) i. Factorise fully: $6x^2 + 9x - 6$ [2]

ii. Hence, or otherwise solve $4 - 8x = (3x + 2)(2x - 1)$ [2]

(b) If $x = 3^2$ and $y = 2^4$, express $27^6 \times 4^8$ in terms of x and y. [2]

(c) Simplify fully $\dfrac{2^{-x} \times 4^{2x+1}}{(2^{-x})^3}$ [2]

(d) Solve simultaneously using method of elimination: [3]

$3x + 7y = 27$

$5x + 2y = 16$

(e) Solve simultaneously: [3]

$3x - 2y = 14$

$xy = 12$

Question 9 (14 marks)

(a) State the domain and range of each function.

i. $y = 4^{-x}$ [2]

ii. $y = \sqrt{x-1}$ [2]

iii. $y = \dfrac{-1}{x-2}$ [2]

iv. $y = \dfrac{-2}{x} + 5$ [2]

(b) If $f(x) = x^2 - 9$ and $g(x) = 16x - 73$, for what values of x does $f(x) = g(x)$? [2]

(c) A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} \frac{1}{x} & for\ x > 0 \\ 1 & for\ x = 0 \\ 2^x & for\ x < 0 \end{cases}$$

i. Draw a neat sketch of the graph $y = f(x)$ using the space below [2]

ii. Evaluate $f(2) + f(0) + f(-2)$ [2]

Question 10 (14 marks)

(a) Given $y = -3x^2 + 3x + 2$

i. By completing the square show that $y = -3\left(x - \frac{1}{2}\right)^2 + \frac{11}{4}$. [1]

ii. Find the x-intercepts [2]

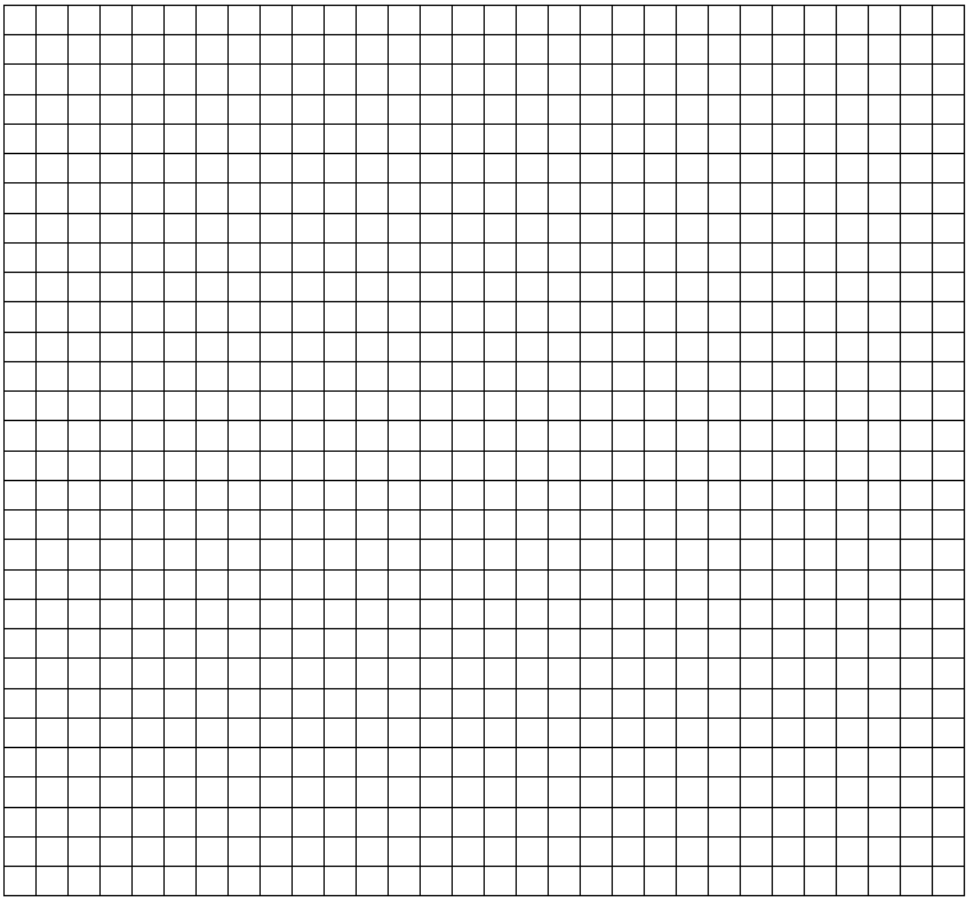
iii. Find the y-intercept(s) [1]

iv. Write the equation of axis of symmetry. [1]

v. Hence sketch the graph of the given equation showing all the above features. [2]

(b) Given $y^2 - 4xy + 1 = 0$. Make y the subject of the formula. [2]

(c) Sketch $y = x(x+1)(x+3)^2$ showing all the necessary features. [2]



(d) Express $\dfrac{2}{6-3\sqrt{3}} - \dfrac{1}{3+2\sqrt{3}}$ as a single fraction with a rational denominator [3]

Question 11 (13 marks)

(a) Factorise by grouping: $x^3 - x^2 + x - 1$ [1]

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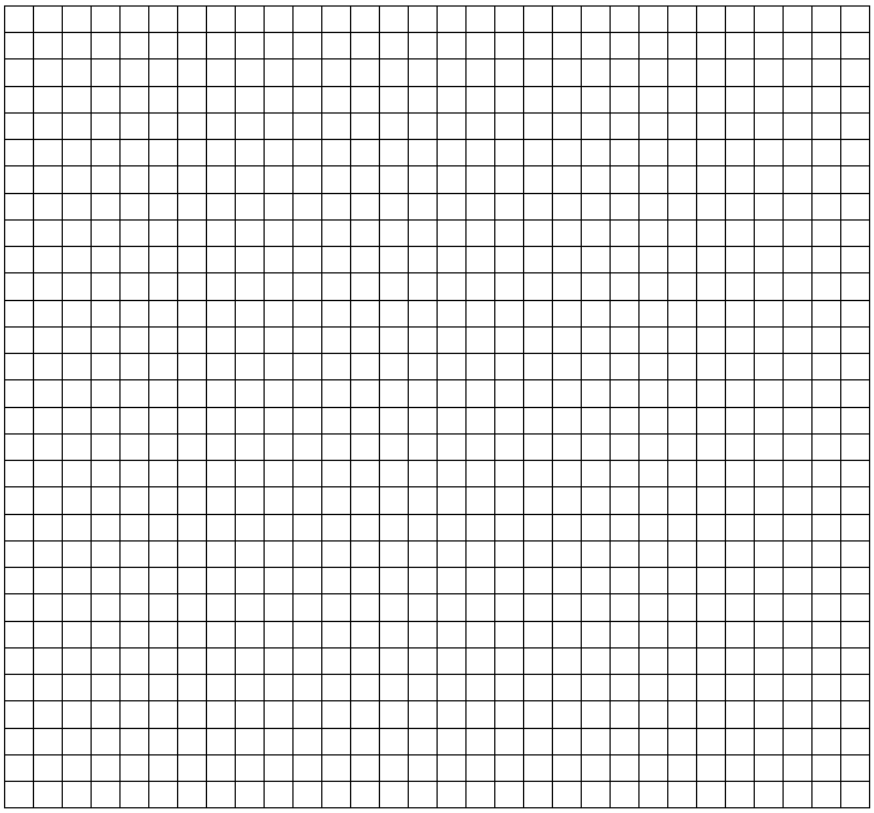
(b) Given $y = \sqrt{16 - x^2}$ [2]

Sketch the function using the grid below. Show all the necessary features



(c) For $f(x) = 3x^2 - 4x$. Find in simplest form $\dfrac{f(x+h) - f(x)}{h}$ [2]

(d) Show that $\frac{1-x^{-1}}{x^{1/2}}$ is same as $\frac{x\sqrt{x}-\sqrt{x}}{x^2}$ in its simplest form [Assume x is an integer] [2]

(e) The speed s of a particle is directly proportional to the square root of its kinetic energy E. When $E = 225$, $s = 40$.

i. Write a formula for s in terms of E. [2]

ii. Find the value of s when $E = 900$. [1]

(f) The shutter speed S of a camera varies inversly as the square of the aperture setting f. When $f = 8$ and $S = 125$

i. Find the formula for S in terms of f. [2]

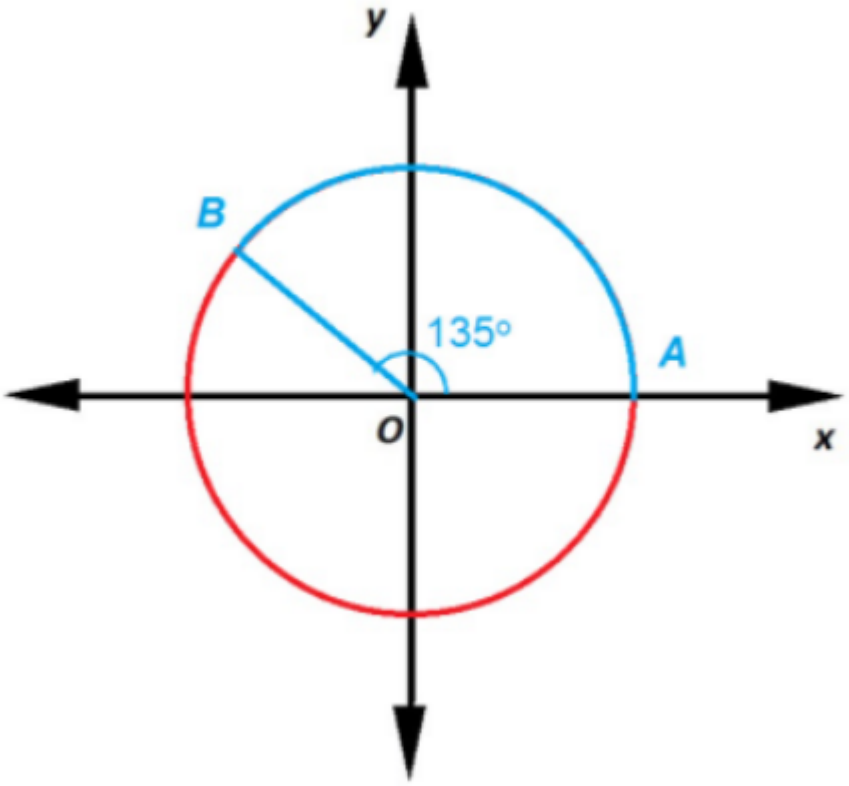
ii. Hence, find the value of S when $f = 4$. [1]

Question 12 (12 marks)

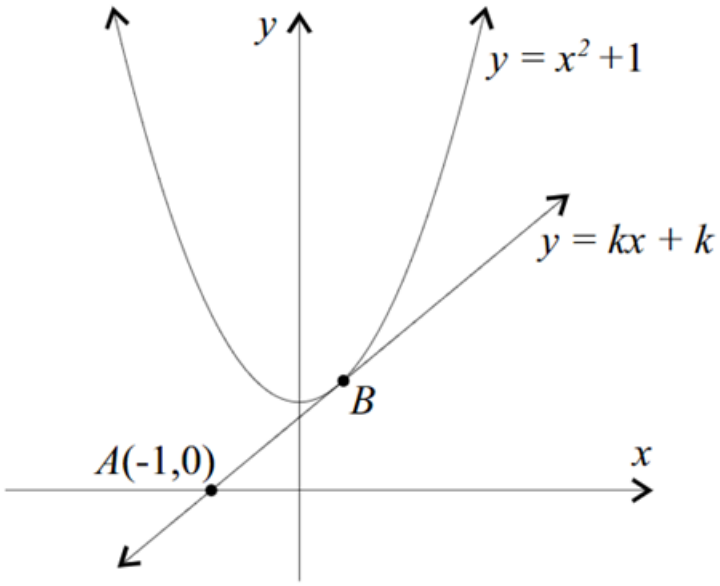
(a) Factorise fully: $80x^4 - 5y^4$ [2]

(b) For every pair of numbers a and b, the function $f(x)$ satisfies $b^2 f(a) = a^2 f(b)$. If $f(x) \neq 0$, find the value of $\dfrac{f(5) - f(1)}{f(2)}$ [2]

(c) The diagram below shows a circle on the coordinate axis with centre at the origin. Minor Arc AB has length 6π. Find the equation of the circle. [2]

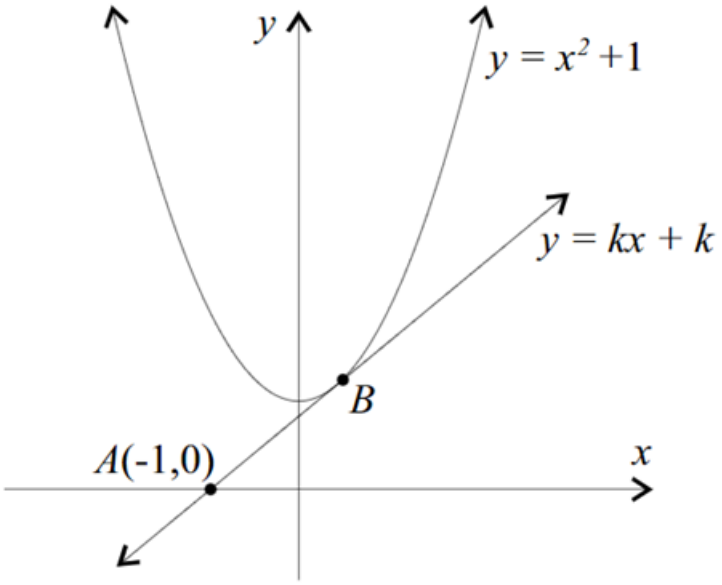


(d) Consider the parabola given by the equation $y = x^2 + 1$ as shown below. The line L through $A(-1, 0)$ has equation $y = kx + k$ for some value k and is tangent to the parabola at some point B in the first quadrant. That is to say, there is only one intersection point between the line and the parabola.



i. Show that k must satisfy the equation $k^2 + 4k - 4 = 0$ [2]

ii. Hence, find the value k, and the coordinates of the point B. [3]



iii. The point $C(-1-\sqrt{2}, 4+2\sqrt{2})$, also lies on the parabola such that the line AC is a tangent to the parabola. By considering the midpoint of BC, or otherwise, find the area of ΔABC [1]

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End of Task 1

Extra working out space.

Mathematics Assessment Solutions
Girraween High School
March 2024

Task: Y11 Adv Assessment Task 1	Year: 2024

Suggested Solutions

Question 1

(1) (B) $\frac{2}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 4-2\sqrt{3}$

(2) (D) $\left(a+\frac{1}{a}\right)^2 = 5^2$

$\left(a^2+2+\frac{1}{a^2}\right) = 25$

$a^2+\frac{1}{a^2} = 23$

(3) (C) Not a function because there exists 'x' values with more that one y-value.

(4) (B) $x(x^2+2x+3) = 0$

$\Delta = \frac{-2 \pm \sqrt{4-4(1)(3)}}{2}$

$\Delta < 0$

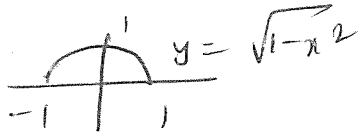
∴ no solutions ; only solution at $x=0$

Marker's Feedback

Suggested Solutions

Question 5

(c)



Question 6

(a) $78.5830\,5$

$\simeq 79$ — (2)

(b) (i) $2x^2+3x-2$

$= (2x-1)(x+2)$

(ii) $x^2(x+5)+1(x+5)$

$(x+5)(x^2+1)$

Marker's Feedback

Q6a Mostly done well however few students are advised to revise significant figures.

6b well done!

6c Some students factorised (x^2+1) to $(x+1)(x-1)$ which is not correct.

Suggested Solutions

Question 6 (iii),

$4a^2(x^3+18ab^2) - 32a^5 + 9b^2x^3$

$x^3(4a^2-9b^2) - 8a^3(4a^2-9b^2)$

$(4a^2-9b^2)(x^3-8a^3)$

$(2a-3b)(2a+3b)(x^3-8a^3)$

or

$(2a-3b)(2a+3b)(x-2a)(x^2+2ax+4a^2)$

(c) i, $\dfrac{(x+y)(x-y)}{-(x-y)}$

$= -(x+y)$

c ii, $\dfrac{3(x-4)+2(x+3)}{6} = \dfrac{3x-12+2x+6}{6}$

$= \dfrac{5x-6}{6}$

Marker's Feedback

6 b iii, Most students had trouble factorising fully. Some students could not recognise that the difference of squares & cubes.

6 c i, Some students had trouble with the difference of squares and negative sign in the denominator.

6 c ii, Mistakes due to calculation errors. Some students forgot the denominator altogether.

Suggested Solutions

Question 6c(iii),

$$\frac{3(x-y)}{xy} \times \frac{x^2}{x(x-y)}$$

$$= \frac{3}{y}$$

6c(iv), $\frac{3}{(x+4)(x-2)} - \frac{2}{(x+3)(x-2)}$

$$\frac{(3x+9) - 2(x+4)}{(x+4)(x-2)(x+3)}$$

$$\frac{3x+9-2x-8}{(x+4)(x-2)(x+3)}$$

$$\frac{(x+1)}{(x+4)(x-2)(x+3)}$$

Marker's Feedback

6c(iii), Some students had trouble identifying and cancelling common terms in the numerator and denominator.

6c(iv), Some students had trouble with lowest common denominator.
Some students made calculation errors in the numerator.

Suggested Solutions

Question 7a

$$\frac{6+\sqrt{3}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{6+6\sqrt{3}+\sqrt{3}+3}{1-3}$$

$$= \frac{9+7\sqrt{3}}{-2} = \frac{-9}{2} - \frac{7\sqrt{3}}{2}$$

(7b) Let 1st Jump = j metres

4.2% of j = 0.3 m

j = 7.14m

Qualifying jump = 7.14 + 0.2

= 7.34m

Marker's Feedback

(a) Students need to write $\frac{-9}{2} - \frac{7\sqrt{3}}{2}$ or equivalent to get full marks. Some are writing $a = \frac{-9}{2}$, $b = \frac{-7}{2}$. They got 1 mark.

(b) Many find this question hard.

Suggested Solutions

Question 7C

$$S = \frac{32\left(1-\left(-\frac{1}{2}\right)^8\right)}{\left(1+\frac{1}{2}\right)}$$

$$S = \frac{2}{3} \times 32 \times \left(1-\frac{1}{256}\right)$$

$$S = 21\frac{1}{4}$$

(7d) $x^2+10x+17=0$

Completing the square on LHS

$$\left(x^2+10x+5^2\right)+17-25=0$$

$$(x+5)^2-8=0$$

$$(x+5)^2=8$$

$$x+5=\pm 2\sqrt{2}$$

$$x=-5\pm 2\sqrt{2}$$

Marker's Feedback

7C. Not putting brackets around $-\frac{1}{2}$ and making mistakes.

7d. $x=-5\pm\sqrt{8}$ is given full mark. Some are factorising the equation $(x+5)^2-8=0$, which is a harder way of getting the solution.

Suggested Solutions

Question 7e

$$\sqrt{x} = \sqrt{50} - \sqrt{18}$$

$$= 5\sqrt{2} - 3\sqrt{2}$$

$$= 2\sqrt{2}$$

$$\sqrt{x} = \sqrt{8}$$

$$x = 8$$

(7f)

$$\Delta = 0$$

$$\Delta = (-(k-4))^2 - 4(1)(9)$$

$$= k^2 - 8k - 20$$

$$= (k-10)(k+2)$$

$$\therefore k = 10, -2$$

Marker's Feedback

7e done well.

7f Getting only one answer.

Suggested Solutions

Question (9)

$$AB^2 = BC^2 + AC^2$$

$$\left(\sqrt{1+x^2}\right)^2 = \left(\sqrt{1-x^2}\right)^2 + \left(\sqrt{x}\right)^2$$

$$1 + x^2 = 1 - x^2 + x$$

$$2x^2 - x = 0$$

$$x(2x-1) = 0$$

$$x = \frac{1}{2} \; ; \quad \because x \neq 0 \text{ as } AC = \sqrt{x}$$

Marker's Feedback

7g $x = 0, \frac{1}{2}$ is given full mark.

$\left(\sqrt{1+x^2}\right)^2 = 1+x^2$ → Many are getting this wrong.

Suggested Solutions

Question 8a,i,

$$6x^2+9x-6 = 3(2x^2+3x-2)$$
$$= 3(2x^2+4x-x-2)$$
$$= 3(2x(x+2)-1(x+2))$$
$$= 3(2x-1)(x+2)$$

8a,ii, $4-8x = (3x+2)(2x-1)$

$$4-8x = 6x^2+x-2$$
$$6x^2+9x-6 = 0$$
$$3(2x-1)(x+2) = 0$$

$2x-1=0$ or $x+2=0$

$x = \frac{1}{2}$; $x = -2$

Marker's Feedback

i) Some Students did not take a factor of 3 out.
Some Students solved it as an equation.

ii) Lot of Students made mistakes, some canceled $(2x-1)$ from both sides.
Some did not solve the equation.

Suggested Solutions

Question 8b

$$27^6 \times 4^8 = (3^3)^6 \times (2^2)^8$$

$$= 3^{18} \times 2^{16}$$

$$= (3^2)^9 \times (2^4)^4$$

$$= x^9 \times y^4$$

8c $$\frac{2^{-x} \times 4^{2x+1}}{(2^{-x})^3} = \frac{2^{-x} \times 2^{2(2x+1)}}{2^{-3x}}$$

$$= 2^{-x+4x+2+3x}$$

$$= 2^{6x+2}$$

Marker's Feedback

b) Lot of students had trouble and poorly done.

c) Poorly done — Need more practice

Suggested Solutions

Question 8d

$3x + 7y = 27$ — (1) $\times 2$ — (3)

$5x + 2y = 16$ — (2) $\times 7$ — (4)

(4) − (3)

$$\begin{array}{r} 35x + 14y = 112 \\ 6x + 14y = 54 \\ (-) \quad\quad\quad\quad\quad\quad \\ \hline 29x \quad\quad = 58 \end{array}$$

$x = 2$

Sub $x = 2$ in eq (1)

$6 + 7y = 27$

$y = 3$ and $x = 2$

Marker's Feedback

Mostly done well.

Suggested Solutions

Question 8e

$3x - 2y = 14$ — (1)

$xy = 12 \Rightarrow y = \frac{12}{x}$ — (2)

Sub (2) into (1)

$3x - 2\left(\frac{12}{x}\right) = 14$

$3x^2 - 24 = 14x$

$3x^2 - 14x - 24 = 0$

$(3x + 4)(x - 6) = 0$

$x = 6$ or $x = -\frac{4}{3}$

$y = 2$ or $y = -9$

Marker's Feedback

Mostly done well but some had trouble with this question.

Suggested Solutions

Question 9a

(i) Domain: All real x or $(-\infty, \infty)$

Range: $y > 0$ or $(0, \infty)$

(ii) Domain: $x \geq 1$ or $[1, \infty)$

Range: $y \geq 0$ or $[0, \infty)$

(iii) Domain: $x \neq 2$ or $(-\infty, 2) \cup (2, \infty)$

Range: $y \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

(iv) Domain: $x \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

Range: $y \neq 5$ or $(-\infty, 5) \cup (5, \infty)$

Marker's Feedback

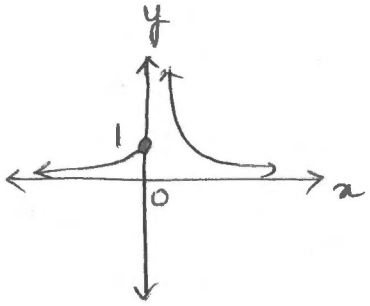
This question was done well. There were some silly mistakes here and there eg. students mixed up domain and range or used > and ≥ interchangeably. Some unconventional notation was marked correct but not encouraged eg. for domain, instead of all real values of x some students wrote [any] and instead of $x \neq 0$ or $y \neq 0$, some students wrote $x < 0, x > 0$ and $y < 0, y > 0$, or $-\infty \leq x \leq \infty$ (cannot reach $-\infty, \infty$) or $0 < x < \infty$ (∞ is redundant)

Suggested Solutions

Question 9b

$$f(x) = g(x)$$
$$x^2 - 9 = 16x - 73$$
$$x^2 - 16x + 64 = 0$$
$$(x-8)^2 = 0$$
$$x = 8$$

9ci,



9cii, $f(2) + f(0) + f(-2)$

$$\frac{1}{2} + 1 + 2^{-2}$$
$$= 1\frac{3}{4}$$

Marker's Feedback

9b - Done very well. Only a few students couldn't attempt or equated f(x) and g(x) and created an incorrect equation then used quadratic formula.

9ci - Done poorly, especially $f(x) = \frac{1}{x}$ for $x > 0$ and $f(x) = 1$ for $x = 0$. Students need to pay attention to the inequalities and draw a closed circle at the y-int.

9cii - Done very well. Only a few students substituted incorrectly $f(x) = \frac{1}{x}$ and $f(x) = 2^x$.

Suggested Solutions

Question 10a

(i), $y = -3(x^2 - x) + 2$

$y = -3\left(x^2 - x + \left(\frac{-1}{2}\right)^2\right) + 2 - 3\left(\frac{-1}{2}\right)^2$

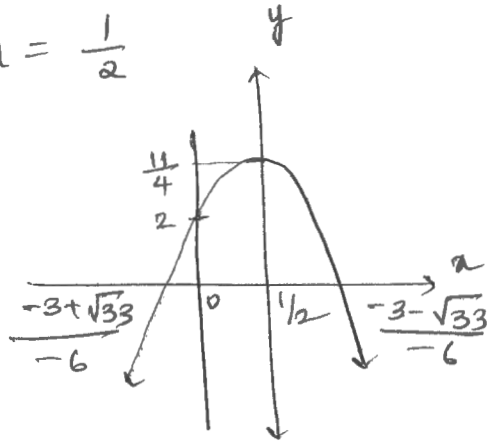
$y = -3\left(x - \frac{1}{2}\right)^2 + \frac{11}{4}$

(ii), $x = \frac{-3 \pm \sqrt{33}}{-6}$

(iii), y-intercept = 2

(iv), $x = \frac{1}{2}$

(v),



Marker's Feedback

(i) Poorly done. Students were thrown by the −3 when completing the square. They often got mixed up with brackets when doing this part as well.

(ii) A lot of students preferred to go back & use the quadratic formula to find x intercepts, rather than solve by completing the square. [still correct & paid].

Suggested Solutions

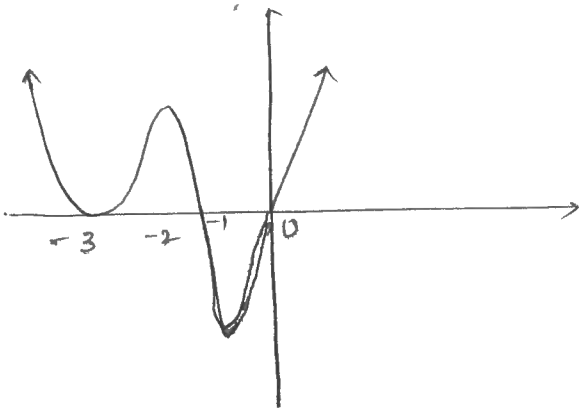
Question 10b

$$y = \frac{4x \pm \sqrt{(4x)^2 - 4(1)(1)}}{2}$$

$$y = \frac{4x \pm \sqrt{16x^2 - 4}}{2}$$

$$y = 2x \pm \sqrt{4x^2 - 1}$$

10c



Marker's Feedback

(10)(b) VERY poorly done. The vast majority of students had no idea to complete the square.

Notes: Some students used the quadratic formula in (10)(b) but the majority who got it out completed the square.

(c) Generally very well done, although some students had the graph upside down!

A lot of students used PENCIL to draw the graph.

Remember: PENCIL DOESN'T SCAN WELL. If you write in pencil in the HSC IT WON'T GET MARKED.

Suggested Solutions

Question 10d

$$\frac{2}{6-3\sqrt{3}} - \frac{1}{3+2\sqrt{3}}$$

$$\frac{2}{(6-3\sqrt{3})} \times \frac{(6+3\sqrt{3})}{(6+3\sqrt{3})} - \frac{1}{(3+2\sqrt{3})} \frac{(3-2\sqrt{3})}{(3-2\sqrt{3})}$$

$$\frac{2(6+3\sqrt{3})}{36-27} - \frac{(3-2\sqrt{3})}{9-12}$$

$$\frac{12+6\sqrt{3}}{9} - \frac{(3-2\sqrt{3})}{-3}$$

$$\frac{12+6\sqrt{3}+9-6\sqrt{3}}{9}$$

$$= \frac{21}{9} \text{ or } 2\frac{1}{3}$$

Marker's Feedback

Most students knew to rationalise denominators, but there were a lot of silly mistakes.

Note: A few students with the new fsc-8200 calculators took some shortcuts. These weren't penalised here, as the question just said to simplify. However, if a question says "show" make sure you SHOW ALL WORKINGS.

Suggested Solutions

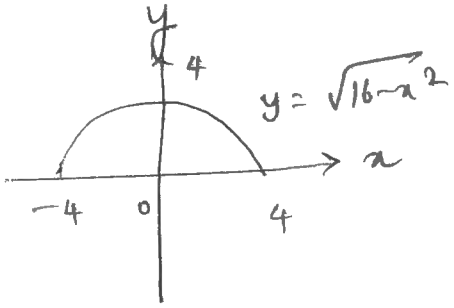
Question 11a

$$x^3 - x^2 + x - 1 = x^2(x-1) + 1(x-1)$$

$$= (x^2+1)(x-1)$$

well done.

11b



- well done.
- scale is important.

11c $f(x) = 3x^2 - 4x$

$$f(x+h) = 3(x+h)^2 - 4(x+h)$$

$$= 3(x^2+2xh+h^2) - 4x - 4h$$

$$= 3x^2 + 6xh + 3h^2 - 4x - 4h$$

multiple algebraic expansion errors

$$\frac{f(x+h)-f(x)}{h} = \frac{(3x^2+6xh+3h^2-4x-4h)-(3x^2-4x)}{h}$$

$$= \frac{(3x^2+6xh+3h^2-4x-4h-3x^2+4x)}{h}$$

$$= \frac{3h^2+6xh-4h}{h} = 3h+6x-4$$

Marker's Feedback

Suggested Solutions

Question 11d

$$\frac{1-x^{-1}}{x^{1/2}}$$

$$\frac{1-\frac{1}{x}}{\sqrt{x}} = \frac{x-1}{x\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{x\sqrt{x}-\sqrt{x}}{x^2}$$

Generally well done.

11e i) $S = k\sqrt{E}$

$40 = k\sqrt{225}$

$k = \frac{40}{15}$

$k = \frac{8}{3} \quad \therefore S = \frac{8}{3}\sqrt{E}$

- poorly done
- a number of students are writing $\sqrt{E} = kS$

Not penalised this time but note 'S' should be the subject.

- many students did not notice 'square root' of E.

ii) $S = \frac{8}{3}\sqrt{900}$

$S = 80$

Marker's Feedback

Suggested Solutions

Question 11 f

(i) $S \propto \frac{1}{f^2}$

$S = \frac{K}{f^2}$

$125 = \frac{K}{f^2}$

$K = 125 \times 8$

$K = 8000$

$\therefore S = \frac{8000}{f^2}$

- Poorly done
- many students did not note 'square' and only used $S = \frac{k}{f}$.

(ii) $S = \frac{8000}{16}$

$S = 500$

well done.

Marker's Feedback

Suggested Solutions

Question 12

(a) $80x^4 - 5y^4$

$5(16x^4 - y^4)$

$5(4x^2+y^2)(4x^2-y^2)$

$5(4x^2+y^2)(2x+y)(2x-y)$

(b) $b^2 f(a) = a^2 f(b)$

$\frac{f(a)}{f(b)} = \frac{a^2}{b^2}$

$\frac{f(5)-f(1)}{f(2)} = \frac{f(5)}{f(2)} - \frac{f(1)}{f(2)}$

$= \frac{25}{4} - \frac{1}{4}$

$= 6$

Marker's Feedback

12 (a) Some students didn't factorise fully.

12 (b) Poorly done.

Suggested Solutions

Question 12

(c) Arc $AB = 6\pi$

$$\frac{135}{360} = 2\pi r = 6\pi$$

$$\therefore r = 8$$

$\therefore$ The equation of the circle is

$$x^2 + y^2 = 8^2$$

$$x^2 + y^2 = 64$$

(d) At the intersection point

$$y = x^2 + 1 \quad \& \quad y = kx + k$$

$$x^2 + 1 = kx + k$$

$$x^2 - kx + (1 - k) = 0$$

If there is only one solution then

$\Delta = 0$, so

$$\Delta = k^2 - 4(1 - k) = 0$$

$$\Rightarrow k^2 - 4k - 4 = 0$$

Marker's Feedback

12(c) Generally well done.

12(d)(i) Some students had trouble with this question.

Suggested Solutions

Question 12d(ii),

$$K = \frac{-4 \pm \sqrt{16+16}}{2}$$

$$K = \frac{-4 \pm 4\sqrt{2}}{2}$$

$$= -2 \pm 2\sqrt{2} \quad (K>0)$$

So $K = -2+2\sqrt{2}$

Now $x^2 - Kx + (1-K) = 0$ gives

$$x = \frac{K \pm \sqrt{\Delta}}{2}$$

So if $K = -2+2\sqrt{2}$ and $\Delta = 0$ gives

$$x = -1+\sqrt{2}$$

and $y = x^2+1$

$$y = (-1+\sqrt{2})^2 + 1$$

$$= 4-2\sqrt{2}$$

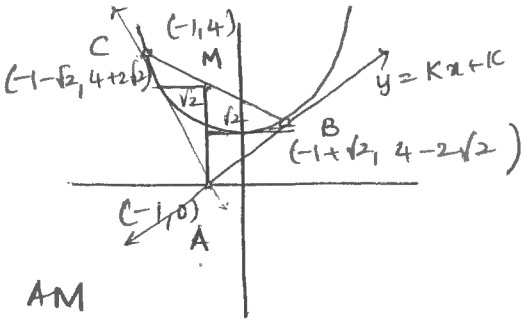
$\therefore B(-1+\sqrt{2}, 4-2\sqrt{2})$

Marker's Feedback

12 d (ii) Some students found the value of k, but couldn't find coordinates of B.
Some got wrong value for k (common error k=2) and then found coordinates of B.

Suggested Solutions

Question 12 d (iii)



Midpoint of BC is $(-1, 4)$ and is also vertically above $(-1, 0)$

ΔAMB and ΔAMC have base AM and heights of $\sqrt{2}$ in the horizontal direction.

$$\text{So Area}_{\Delta AMB} = \text{Area}_{\Delta AMC} = \frac{1}{2} \times 4 \times \sqrt{2}$$

$$= 2\sqrt{2}$$

$$\therefore \text{The Area}_{\Delta ABC} = 4\sqrt{2} \text{ square units.}$$

Marker's Feedback

12 d (iii) Poorly done.