GLENWOOD

Student No. | | | | | | | | | |

Student Name: ______________________________

Teacher Name: ______________________________

2019

YEAR 12 / HSC - 2020
ASSESSMENT TASK 1

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 80 minutes
- Write using black pen
- Approved calculators may be used
- A reference sheet is provided
- For Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 45

Section I – 5 marks – Pages 3 - 4

- Attempt Questions 1-5
- Allow about 10 minutes for this section

Section II – 40 marks – Pages 6 - 14

- Attempt Questions 6 - 14
- Allow about 1 hours and 10 minutes for this section

SECTION 1

5 MARKS
Attempt questions 1 – 5
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for questions 1-5

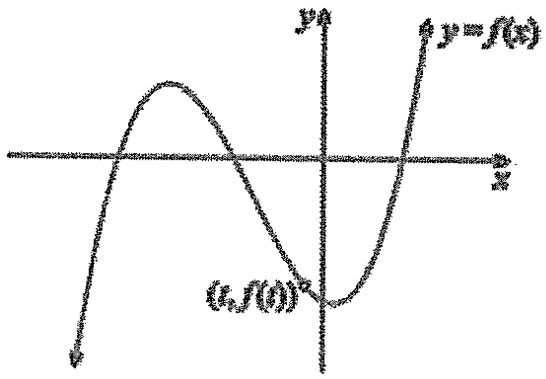
1. If $f(x) = \log_e(2x)$ then find the value of $f'(1)$

(A) $\frac{1}{2}$
(B) 1
(C) $\ln 2$
(D) 2

2. What is the derivative of $\log_2 x$?

(A) $\frac{1}{x}$
(B) $\frac{1}{2x}$
(C) $\ln 2x$
(D) $\frac{1}{x \ln 2}$

3. The diagram below shows the graph of $y = f(x)$. Which of the following statements is true?



(A) $f'(t) > 0$ and $f''(t) < 0$

(B) $f'(t) > 0$ and $f''(t) > 0$

(C) $f'(t) < 0$ and $f''(t) < 0$

(D) $f'(t) < 0$ and $f''(t) > 0$

4. Consider the curve given by $y = \frac{1}{2}x^4 - x^3$. What are the points of inflexion?

(A) only (0, 0)

(B) (0, 0) and $(-1, \frac{5}{8})$

(C) (0, 0) and $(1, -\frac{1}{2})$

(D) (0, 0) and $(\frac{3}{2}, -\frac{27}{32})$

5. The line $y = mx + b$ is a tangent to the curve $y = x^3 - 3x + 2$ at the point $(-2, 0)$. What is value of m and b?

(A) $m = 9$ and $b = -18$

(B) $m = 9$ and $b = 18$

(C) $m = 12$ and $b = -18$

(D) $m = 12$ and $b = 18$

Section II Answer Booklet

Student name

Student Number

Teacher Name

40 marks
Attempt Questions 6 - 14
Allow about 1 hour and 10 minutes for this section

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet. If you use this space, clearly indicate which question you are answering.

Question 6 (2 Marks)

Find the gradient of the tangent to the curve $y = \ln(\tan x)$ at the point on the curve where $x = \frac{\pi}{4}$ 2

Question 7 (9 Marks)

Consider the curve $y = f(x)$ where $f(x) = x^3 - 3x^2 - 9x + 27$

a) Show that $f(x) = (x+3)(x-3)^2$ 2

Question 7 continues on next page

b) Find the stationary points and determine their nature 3

c) Find the point of inflection 2

Question 7 continues on next page

d) Sketch the curve $y = f(x)$, showing all important features 2

Question 8 (3 Marks)

Consider the function $f(x) = x \ln x - x$

a) Find the values of x for which $f(x)$ is decreasing. 2

b) Show that the curve $y = f(x)$ is concave up for all values of x in the domain of $f(x)$ 1

Question 9 (7 Marks)

Differentiate the following

a) $y = \tan(5x - 3)$ 1

b) $y = e^{2x} \sin x$ 2

c) $y = \frac{\tan x}{x}$ 2

Question 9 continues on next page

d) $y = log_e\sqrt{x^2 - 1}$ 2

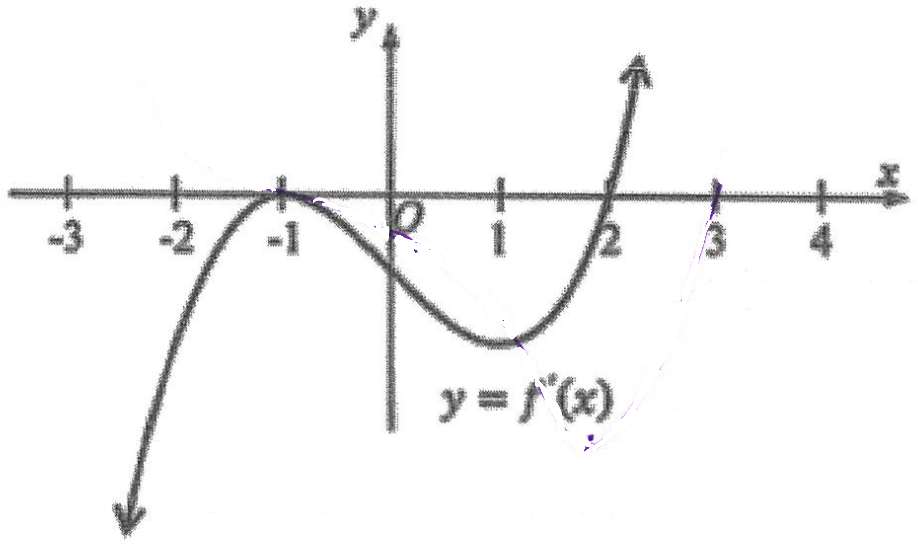
Question 10 (3 Marks)

Find the value of m if $y = 2\sin 3x + 4\cos 2x$ and $\frac{d^2y}{dx^2} = m\sin 3x - 4y$ 3

Questions 6 - 10 are worth 24 marks in total

Question 11 (6 Marks)

The graph below is the curve $y = f'(x)$, the derivative of a function $f(x)$



a) For what values of x does $y = f(x)$ have stationary point? 1

b) For what values of x does $y = f(x)$ have a point of inflection? 1

c) For what values of x does $y = f(x)$ have a horizontal point of inflection? 1

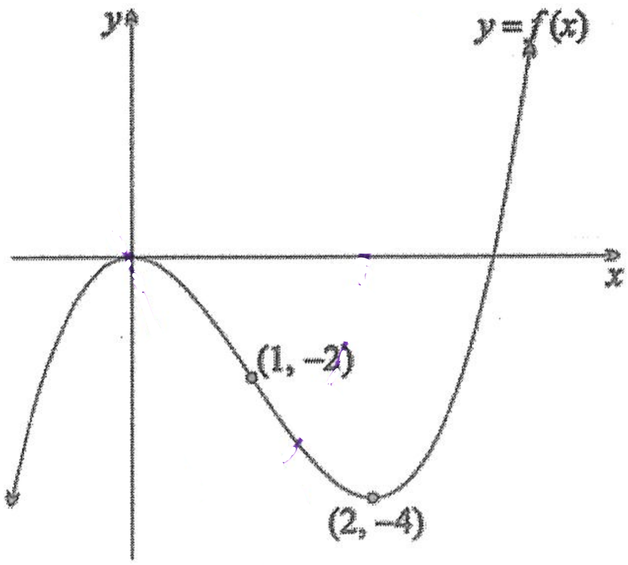
d) Sketch a possible graph of $y = f(x)$ on the same set of axes given above. 3

Question 12 (3 Marks)

Find the equation of the tangent on $y = log_e(2x^2 + 1)$ at the point $(2, log_e 9)$. 3
Express your answer in general form.

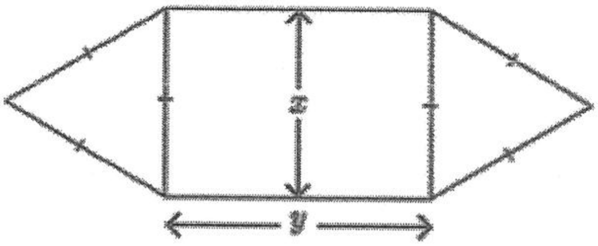
Question 13 (2 Marks) 2

The function $y = f(x)$ is graphed in the diagram below. The points (2,–4) and the origin are stationary points and the point (1, –2) is a point of inflection. The gradient of the tangent at the point of inflection is –3. Sketch the gradient function $y = f'(x)$ on the same set of axes. Mark on your sketch the co-ordinates of all intercepts with the axes and any stationary points.



Question 14 (5 Marks)

The figure shown below consists of a rectangle, x cm by y cm with two equilateral triangles on each side of length x cm. Given that the perimeter of the figure is 26 cm.



a) Show that $A = 13x - \frac{(4-\sqrt{3})x^2}{2}$ where A is the area of the figure 2

b) Hence show that the maximum area of the figure is $\dfrac{13(4+\sqrt{3})}{2}$ 3

END OF EXAMINATION

GLENWOOD

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(A) $\frac{1}{2}$
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2. What is the derivative of $\log_2 x$?

(A) $\frac{1}{x}$
(B) $\frac{1}{2x}$
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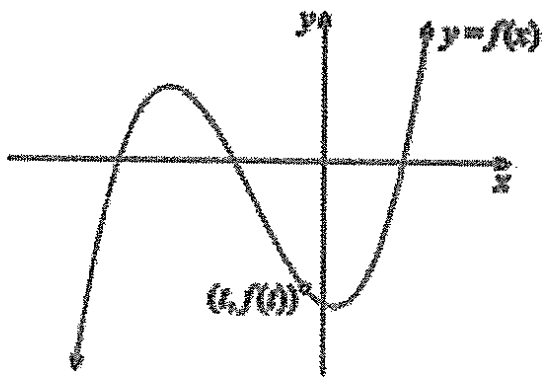
3. The diagram below shows the graph of $y=f(x)$. Which of the following statements is true?



(A) $f'(t)>0$ and $f''(t)<0$

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4. Consider the curve given by $y = \frac{1}{2}x^4 - x^3$. What are the points of inflexion?

(A) only (0, 0)

(B) (0, 0) and $(-1, \frac{5}{8})$

(C) (0, 0) and $(1, -\frac{1}{2})$

(D) (0, 0) and $(\frac{3}{2}, -\frac{27}{32})$

5. The line $y = mx + b$ is a tangent to the curve $y = x^3 - 3x + 2$ at the point $(-2, 0)$. What is value of m and b?

(A) $m = 9$ and $b = -18$

(B) $m = 9$ and $b = 18$

(C) $m = 12$ and $b = -18$

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Section II Answer Booklet

Student name

Student Number

Teacher Name

40 marks
Attempt Questions 6 - 14
Allow about 1 hour and 10 minutes for this section

Instructions

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Question 6 (2 Marks)

Find the gradient of the tangent to the curve $y = \ln(\tan x)$ at the point on the curve where $x = \frac{\pi}{4}$ 2

$$y' = \frac{1}{\tan x} \times \sec^2 x$$

At $x = \frac{\pi}{4}$

$$y' = \frac{1}{\tan\frac{\pi}{4}} \times \sec^2 \frac{\pi}{4}$$

$$= (\sqrt{2})^2 = 2$$

Question 7 (9 Marks)

Consider the curve $y = f(x)$ where $f(x) = x^3 - 3x^2 - 9x + 27$

a) Show that $f(x) = (x+3)(x-3)^2$ 2

$$f(x) = x^3 - 3x^2 - 9x + 27$$
$$= x^2(x-3) - 9(x-3)$$
$$= (x^2 - 9)(x-3)$$
$$= (x-3)(x+3)(x-3)$$
$$= (x+3)(x-3)^2$$

Question 7 continues on next page

b) Find the stationary points and determine their nature 3

$f'(x) = 3x^2 - 6x + 9$

$= 3(x^2 - 2x + 3)$

$= 3(x-3)(x+1)$

for stationary point $f'(x) = 0$

$x = 3, -1$

$f''(x) = 6x - 6$

At $x = 3$

$f''(x) = 12$

Minimum at $x = 3$

At $x = -1$

$f''(x) = -12$

$\therefore$ Maximum at $x = -1$

$f(3) = 0$, $f(-1) = 32$

$\therefore (3,0)$ is the min turning point

$(-1, 32)$ is the max turning point.

c) Find the point of inflection 2

$f''(x) = 6x - 6$

$6x - 6 = 0$

$x = 1$

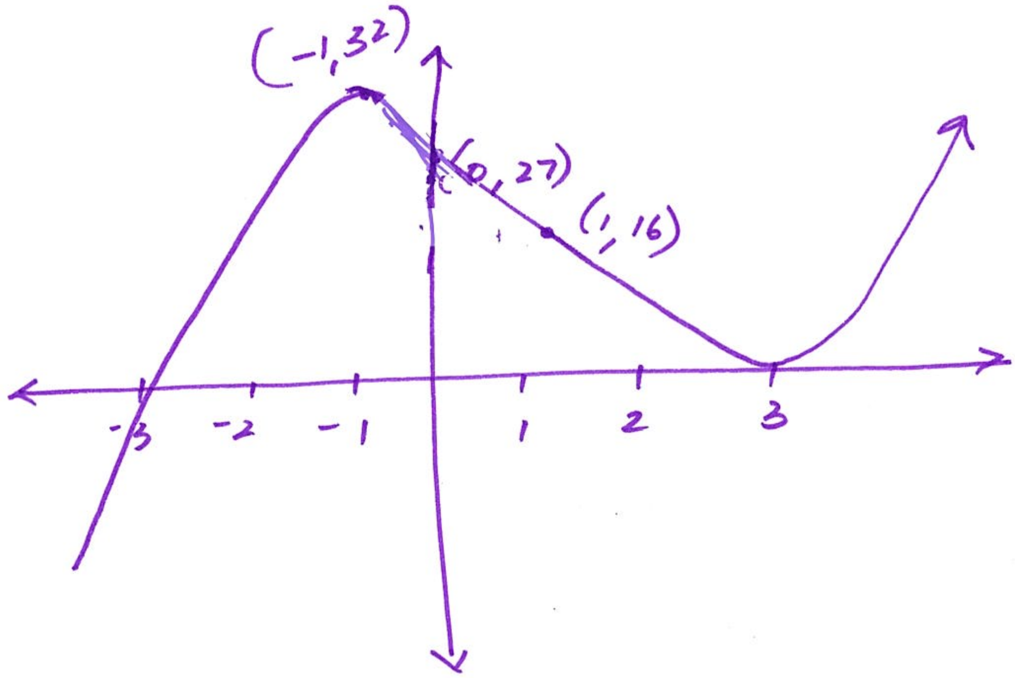
x	0	1	2
$f''(x)$	-6	0	6

$f''(1) = 0$ & concavity changes

$\therefore (1, 16)$ is a point of inflexion

Question 7 continues on next page

d) Sketch the curve $y = f(x)$, showing all important features 2



Question 8 (3 Marks)

Consider the function $f(x) = x\ln x - x$

a) Find the values of x for which $f(x)$ is decreasing. 2

$f'(x) = \left(\ln x + x \cdot \frac{1}{x}\right) - 1$

$= \ln x + 1 - 1$

$= \ln x$

$f''(x) = \frac{1}{x}$

$f'(x) < 0$

$\ln(x) < 0$

$0 < x < 1$

b) Show that the curve $y = f(x)$ is concave up for all values of x in the domain of $f(x)$ 1

Domain is $x > 0$

$f''(x) > 0$ for $x > 0$

curve is concave up through out the domain

Question 9 (7 Marks)

Differentiate the following

a) $y = \tan(5x - 3)$ 1

$y' = 5\sec^2(5x-3)$

b) $y = e^{2x}\sin x$ 2

$y' = e^{2x}\cos x + \sin x \times e^{2x} \times 2$

$= e^{2x}(\cos x + 2\sin x)$

c) $y = \frac{\tan x}{x}$ 2

$y' = \frac{x(\sec^2 x) - 1 \cdot \tan x}{x^2}$

$= \frac{x\sec^2 x - \tan x}{x^2}$

Question 9 continues on next page

d) $y = log_e\sqrt{x^2-1}$ 2

$y' = \frac{1}{\sqrt{x^2-1}} \times \frac{1}{\cancel{2}\sqrt{x^2-1}} \times \cancel{2}x$

$= \frac{x}{x^2-1}$

Question 10 (3 Marks)

Find the value of m if $y = 2\sin 3x + 4\cos 2x$ and $\frac{d^2y}{dx^2} = m\sin 3x - 4y$ 3

$y' = 6\cos 3x - 8\sin 2x$

$y'' = -18\sin 3x - 16\cos 2x$

$-18\sin 3x - 16\cos 2x = m\sin 3x - 4y$

$-18\sin 3x - 16\cos 2x = m\sin 3x - 4(2\sin 3x + 4\cos 2x)$

$-18\sin 3x - \cancel{16\cos 2x} = m\sin 3x - 8\sin 3x - \cancel{16\cos 2x}$

$-18\sin 3x + 8\sin 3x = m\sin 3x$

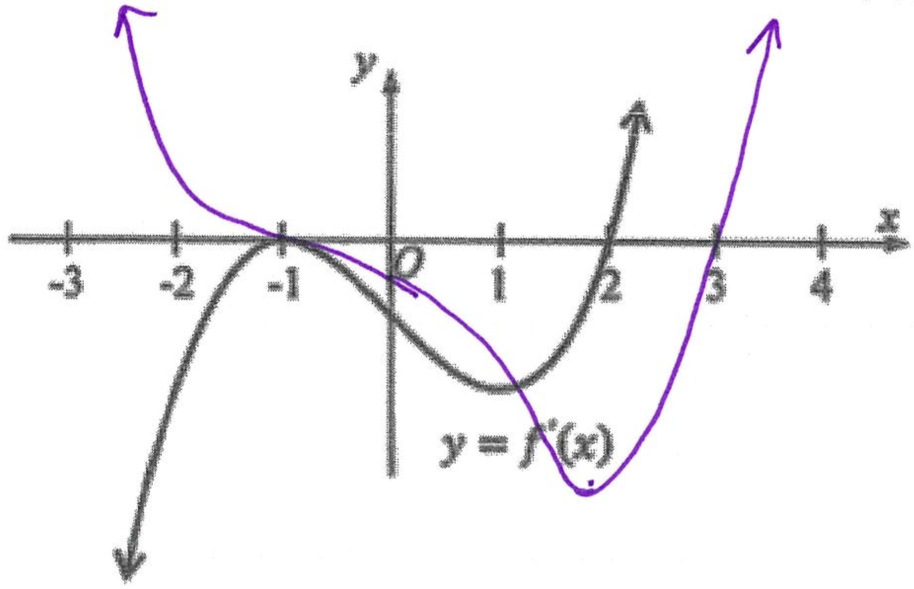
$-10\sin 3x = m\sin 3x$

$m = -10$

Questions 6 - 10 are worth 24 marks in total

Question 11 (6 Marks)

The graph below is the curve $y = f'(x)$, the derivative of a function $f(x)$



a) For what values of x does $y = f(x)$ have stationary point? 1

$x = -1, 2$

b) For what values of x does $y = f(x)$ have a point of inflection? 1

$x = -1$ or 1

c) For what values of x does $y = f(x)$ have a horizontal point of inflection? 1

$x = -1$

d) Sketch a possible graph of $y = f(x)$ on the same set of axes given above. 3

Question 12 (3 Marks)

Find the equation of the tangent on $y = log_e(2x^2 + 1)$ at the point $(2, log_e 9)$. Express your answer in general form. 3

$y = \ln(2x^2+1)$

$y' = \frac{1}{2x^2+1} \times 4x$

At $x=2$, $m = \frac{4(2)}{2(2)^2+1} = \frac{8}{9}$

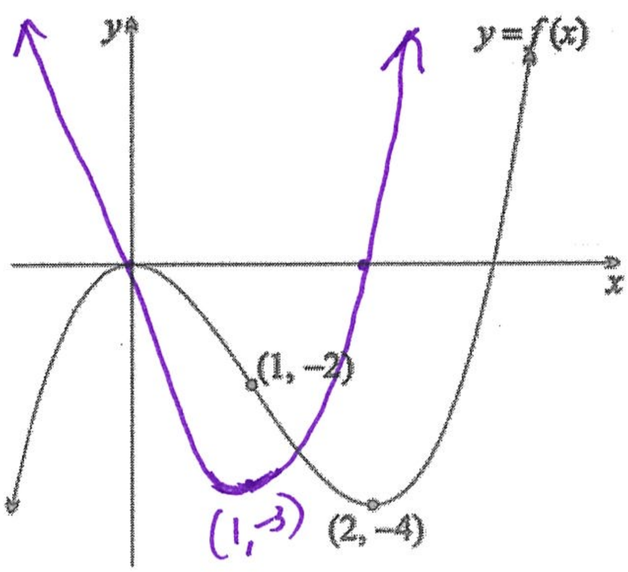
$y - \ln 9 = \frac{8}{9}(x-2)$

$9y - 9\ln 9 = 8x - 16$

$8x - 9y - 16 + 9\ln 9 = 0$

Question 13 (2 Marks)

The function $y = f(x)$ is graphed in the diagram below. The points $(2,-4)$ and the origin are stationary points and the point $(1,-2)$ is a point of inflection. The gradient of the tangent at the point of inflection is -3. Sketch the gradient function $y = f'(x)$ on the same set of axes. Mark on your sketch the co-ordinates of all intercepts with the axes and any stationary points. 2



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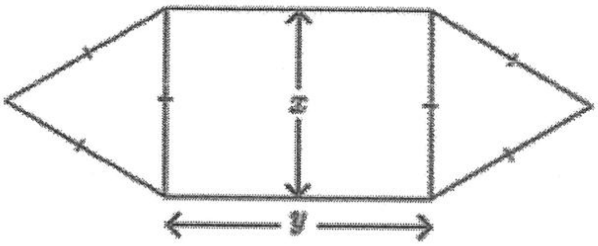

a) Show that $A = 13x - \frac{(4-\sqrt{3})x^2}{2}$ where A is the area of the figure　　2

$2y + 4x = 26$

$y = \frac{26 - 4x}{2}$

$y = 13 - 2x$

$A_{\text{triangle}} = \frac{1}{2} x^2 \times \sin 60$

$= \frac{1}{2} x^2 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} x^2$

$A_{\text{figure}} = \frac{\sqrt{3}}{4} x^2 \times 2 + xy$

$= \frac{\sqrt{3}}{2} x^2 + x(13 - 2x)$

$= \frac{\sqrt{3}}{2} x^2 + 13x - 2x^2$

$= 13x - 2x^2 + \frac{\sqrt{3}}{2} x^2$

$= 13x - \frac{4x^2}{2} + \frac{\sqrt{3}x^2}{2}$

$= 13x - \frac{x^2(4 - \sqrt{3})}{2}$

b) Hence show that the maximum area of the figure is $\frac{13(4+\sqrt{3})}{2}$ 3

$$\frac{dA}{dx} = 13 - \left(\frac{4-\sqrt{3}}{2}\right) \times 2x$$

$$= 13 - (4-\sqrt{3})x$$

For max value

$$\frac{dA}{dx} = 0$$

$$13 - (4-\sqrt{3})x = 0$$

$$(4-\sqrt{3})x = 13$$

$$x = \frac{13}{4-\sqrt{3}}$$

$$\frac{d^2A}{dx^2} = -(4-\sqrt{3}) < 0$$

$\therefore$ Maximum at $x = \frac{13}{4-\sqrt{3}}$

$$\text{Maximum area} = 13\left(\frac{13}{4-\sqrt{3}}\right) - \left(\frac{4-\sqrt{3}}{2}\right)\left(\frac{13}{4-\sqrt{3}}\right)^2$$

$$= \frac{13^2}{4-\sqrt{3}} - \frac{13^2}{2(4-\sqrt{3})}$$

$$= \frac{2\times 13^2 - 13^2(4+\sqrt{3})}{2(4-\sqrt{3})(4+\sqrt{3})}$$

$$= \frac{13^2(4+\sqrt{3})}{2\times 13} = \frac{13(4+\sqrt{3})}{2}$$

END OF EXAMINATION