
Student Number

2020 Glenwood High School

YEAR 12

Assessment Task 1

MATHEMATICS ADVANCED

General Instructions

* Reading time - 5 minutes
* Working time – 80 minutes
* Write using black pen
* Approved calculators may be used
* A refence sheet is provided

Total marks: **45**

Section I - 5 marks (pages 3-4)

* Attempt Questions 1-5
* Allow about 10 minutes for this section

Section II - 40 marks (pages 6-15)

* Attempt Questions 6-14
* Allow about 1 hour and 10 minutes for this section

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SECTION 1

5 MARKS
Attempt questions 1 – 5
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for questions 1-5

1 What is the derivative of $f(x) = x^3 + \frac{3}{x^2}$?

(A) $3x^2 - \frac{3}{x}$

(B) $3x^2 - \frac{6}{x^3}$

(C) $3x^2 + \frac{6}{x}$

(D) $3x^2 + \frac{3}{x^3}$

2 If $f(x)$ has a local maximum at $x = 2$, which of the following is true?

(A) $f'(2) = 0$ and $f''(2) < 0$

(B) $f'(2) = 0$ and $f''(2) > 0$

(C) $f'(2) > 0$ and $f''(2) < 0$

(D) $f'(2) = 0$ and $f''(2) = 0$

3 If $f(x) = e^{x^2}$, then $f'(x)$ is:

(A) $x^2 e^{x^2-1}$

(B) $2xe^{2x}$

(C) $2xe^{x^2}$

(D) $2e^{x^2}$

4 The gradient of the tangent to the curve $y = \sqrt{x}$ is $\frac{1}{4}$ at point Q.
What is the coordinate of point Q ?

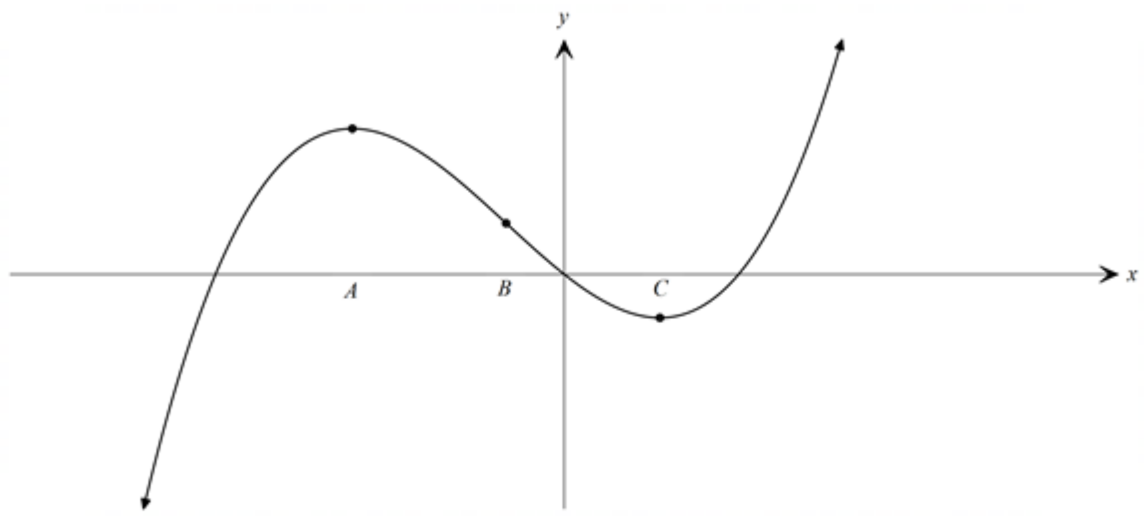
(A) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

(B) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(C) $(4, -2)$

(D) $(4,2)$

5 The graph of $y = f(x)$ is shown below.
$x = A$ and $x = C$ are stationary points, and $x = B$ is a point of inflection.



Over what domain is $f'(x) < 0$ and $f''(x) > 0$?

(A) $(-\infty, A)$

(B) (A, B)

(C) (B, C)

(D) (C, ∞)

2020 Year 12 Assessment Task 1

Student Number

Section II Answer Booklet

Mathematics Advanced

40 marks
Attempt Questions 6 – 14
Allow about 1 hour and 10 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet.
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Question 6 (6 marks)

Find the first derivative of the following:

i) $y = e^{\cos(x+1)}$ **1**

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ii) $y = 4x\ 10^{x}$ **2**

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iii) $y = \ln\left(\frac{\sqrt{(x^2-4)}}{3x+1}\right)$ **3**

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Question 7 (9 marks)

A function is given by $f(x) = 3x - x^3 - 1$

i) Find the coordinates of the stationary points and determine their nature. **3**

ii) Find the coordinate of any point of inflection **2**

Question 7 continues next page

iii) Sketch the curve $y = f(x)$ in the domain $[-3,2]$, labelling all important features **except** the x-intercepts. **3**

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iv) What is the global maximum value of this function in the domain $[-3,2]$? **1**

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Question 8 (2 marks)

Consider the function $f(x) = x^3 - 3x^2 + kx + 8$. For what values of k is $f(x)$ an increasing function ? **2**

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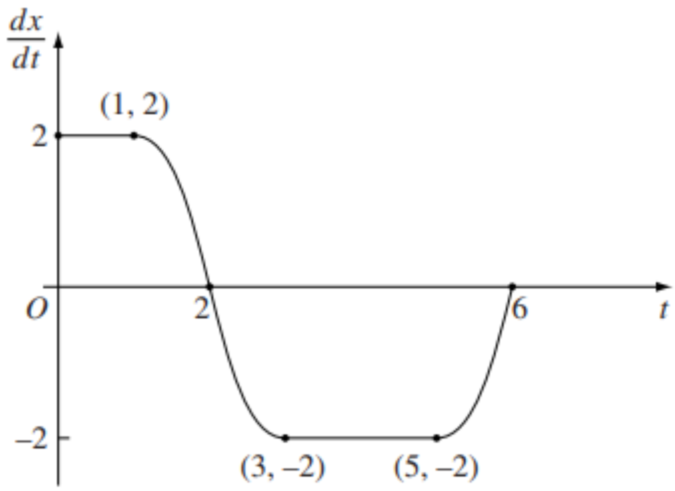
Question 9 (2 marks)

The graph below shows the derivative function, $\frac{dx}{dt}$. **2**

Draw a sketch of the second derivative, $\frac{d^2x}{dt^2}$, on the same axes, in the domain [0,6].



Question 10 (5 marks)

For the curve $y = \frac{\sin x}{1+\cos x}$

i) Show that the point $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right)$ lies on the curve **1**

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ii) Show that $\frac{dy}{dx} = \frac{1}{1+\cos x}$ **2**

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Question 10 continues next page

iii) Find the equation of the tangent to the curve at $x = \frac{\pi}{3}$ **2**

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Question 11 (2 marks)

Show that the curve $f(x) = \ln(x + 2)$ is concave down for all values of x in its domain. **2**

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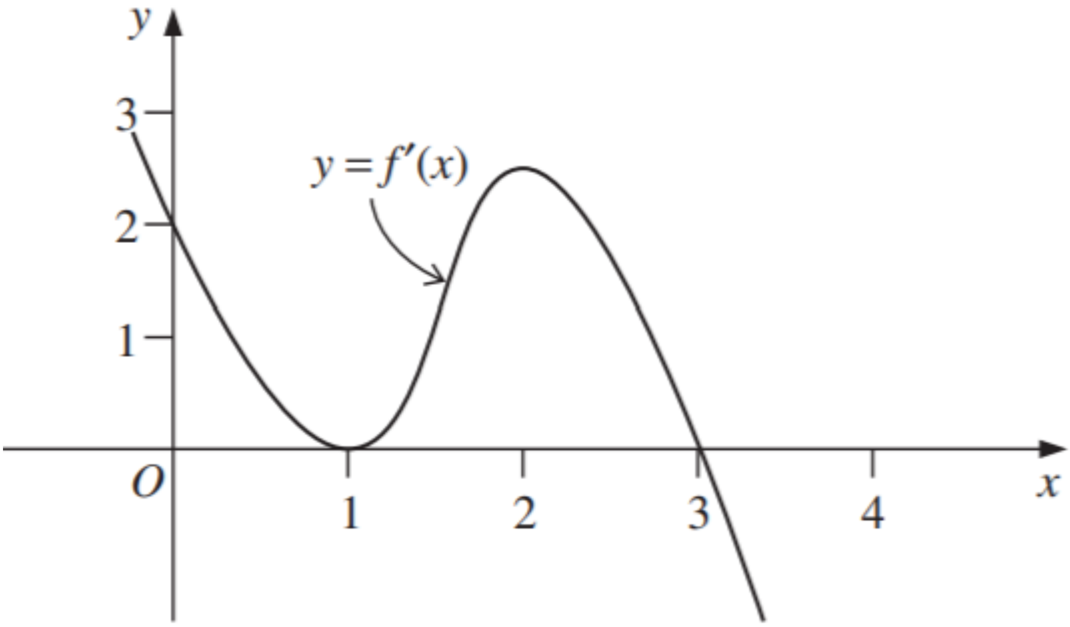
Question 12 (3 marks)

The diagram below shows a sketch of the gradient function, $f'(x)$. **3**

Sketch the graph of the function $y = f(x)$, given that $f(0) = 0$, on the same set of axes.
Clearly label stationary points and points of inflections.



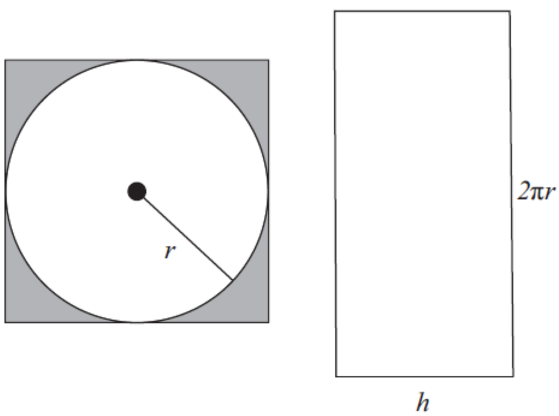
Question 13 (6 marks)

A company is manufacturing barrels with a **constant volume** specified by the customer.

The barrels are open cylinders with a circular base but no top. Each barrel has height h metres, radius r metres and volume V cubic metres.

In the manufacturing process, the circular base and the material for the curved side are cut from flat sheets of metal, which are then folded to form the open cylinder.

The metal used to construct a barrel is shown by the total area in the diagram below. The shaded metal remaining after the circular base is removed is wasted.



Question 13 continues next page

i) Show that the area of the sheet metal required to construct a barrel is given by **2**

$$A = 4r^2 + \frac{2V}{r}$$

ii) Hence, show that the minimum area of metal, A, assuming V is a constant, occurs when **2**

$$r^3 = \frac{V}{4}$$

Question 13 continues next page

iii) Hence, show that the minimum area of metal, A, required to make a barrel is **2**

$$A = 3 \times \sqrt[3]{V^2} \times \sqrt[3]{4}$$

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Question 14 (5 marks)

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Question 14 continues next page

ii) Using the result from i), show that the derivative of $y = \frac{\sec x}{\sin x + \cos x}$ can be expressed as **3**

$$\frac{dy}{dx} = \frac{\tan^2 x + 2\tan x - 1}{1 + 2\sin x \cos x}$$

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END OF EXAMINATION

Section II Extra writing space

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Solution

Student Number

2020 Glenwood High School
YEAR 12
Assessment Task 1

MATHEMATICS ADVANCED

B
A
C
D
C

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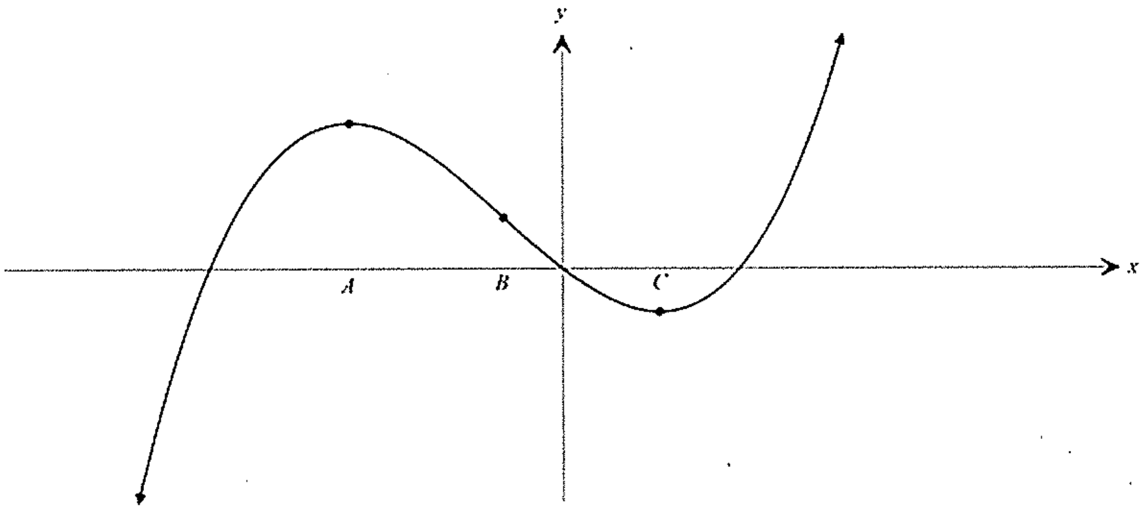

Over what domain is $f'(x) < 0$ and $f''(x) > 0$?

(A) $(-\infty, A)$

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Question 6 (6 marks)

Find the first derivative of the following:

i) $y = e^{\cos(x+1)}$ **1**

$y' = -\sin(x+1)\, e^{\cos(x+1)}$

ii) $y = 4x\ 10^x$ **2**

$y' = 4.10^x + 4x.10^x \ln 10$ $u = 4x$

$= 10^x(4 + 4x\ln 10)$ $u' = 4$

or : $4.10^x(1 + x\ln 10)$ $v = 10^x$

$v' = 10^x.\ln 10$

iii) $y = \ln\left(\frac{\sqrt{(x^2-4)}}{3x+1}\right)$ **3**

$y = \ln(x^2-4)^{1/2} - \ln(3x+1)$ ① SPLIT INTO 2

$y' = \dfrac{\frac{1}{2}(x^2-4)^{-1/2}.2x}{(x^2-4)^{1/2}} - \dfrac{3}{(3x+1)}$

$= \dfrac{x}{(x^2-4)} - \dfrac{3}{3x+1}$

$= \dfrac{x+12}{(x^2-4)(3x+1)}$

Question 7 (9 marks)

A function is given by $f(x) = 3x - x^3 - 1$

i) Find the coordinates of the stationary points and determine their nature. **3**

$f'(x) = 3 - 3x^2$

$f''(x) = -6x$

For stationary points, $f'(x) = 0$

$3 - 3x^2 = 0$

$x^2 = 1$

$x = \pm 1$

at $x = 1$, $f(1) = 1$ $\therefore$ Maximum turning point

$f''(1) = -6 < 0$ at $(1, 1)$

at $x = -1$, $f(-1) = -3$

$f''(-1) = 6 > 0$ $\therefore$ Minimum turning point at $(-1, -3)$

ii) Find the coordinate of any point of inflection **2**

$f''(x) = -6x$

$-6x = 0$

$x = 0$, $f(0) = -1$

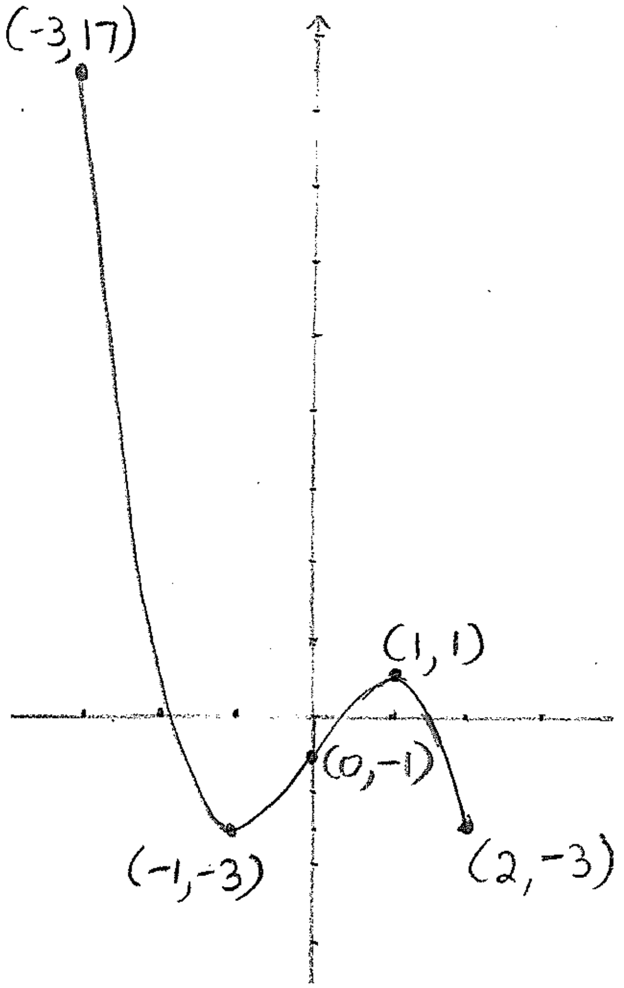
Check concavity change

x	-1	0	1
$f''(x)$	6	0	-6
	$\smile$		$\frown$

Since there is a change in concavity at $x = 0$, there is a point of inflection at $(0, -1)$.

Question 7 continues next page

iii) Sketch the curve $y = f(x)$ in the domain $[-3,2]$, labelling all important features **except** the x-intercepts. **3**



at $x = -3$, $f(-3) = 17$

at $x = 2$, $f(2) = -3$

y-intercept, $x = 0$, $f(0) = -1$

iv) What is the global maximum value of this function in the domain $[-3,2]$? **1**

Global maximum value is 17

Question 8 (2 marks)

Consider the function $f(x) = x^3 - 3x^2 + kx + 8$. For what values of k is $f(x)$ an increasing function ? **2**

$f'(x) = 3x^2 - 6x + k$

If $f(x)$ is an increasing function, then $f'(x) > 0$

$f'(x)$ is concave up, $a > 0$

thus $\Delta < 0$, for $f'(x) > 0$

$\Delta = (-6)^2 - 4(3)(k)$

$= 36 - 12k$

$36 - 12k < 0$

$12k > 36$

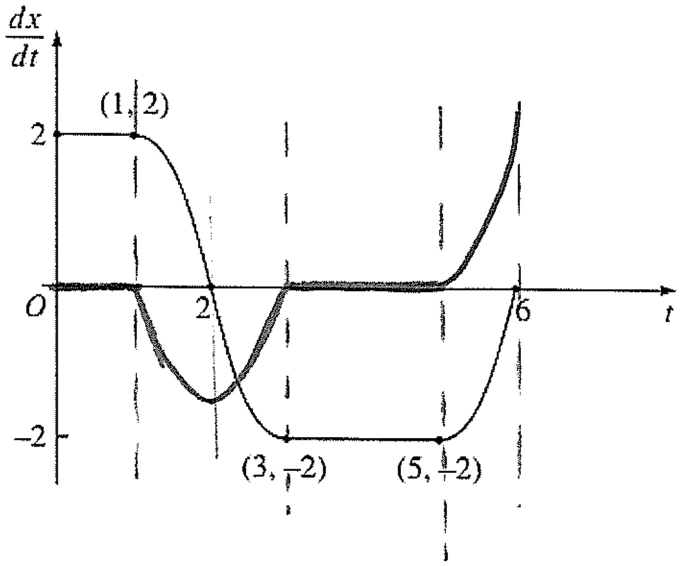
$k > 3$

$\therefore$ when $k > 3$, $f(x)$ is an increasing function

Question 9 (2 marks)

The graph below shows the derivative function, $\frac{dx}{dt}$. **2**

Draw a sketch of the second derivative, $\frac{d^2x}{dt^2}$, on the same axes, in the domain [0,6].



Question 10 (5 marks)

For the curve $y = \frac{\sin x}{1+\cos x}$

i) Show that the point $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right)$ lies on the curve **1**

at $x = \frac{\pi}{3}$, $y = \frac{\sin\frac{\pi}{3}}{1+\cos\pi/3}$

$= \frac{\sqrt{3}/2}{1+\frac{1}{2}}$

$= \frac{\sqrt{3}}{3}$

Hence, $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right)$ lies on the curve.

ii) Show that $\frac{dy}{dx} = \frac{1}{1+\cos x}$ **2**

$y = \frac{\sin x}{1+\cos x}$

$u = \sin x$

$u' = \cos x$

$v = 1+\cos x$

$v' = -\sin x$

$\frac{dy}{dx} = \frac{\cos x(1+\cos x) + \sin^2 x}{(1+\cos x)^2}$ ✓

$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

$= \frac{1+\cos x}{(1+\cos x)^2}$ ✓

$\frac{dy}{dx} = \frac{1}{1+\cos x}$

Question 10 continues next page

iii) Find the equation of the tangent to the curve at $x = \frac{\pi}{3}$ **2**

At $x = \frac{\pi}{3}$, $\frac{dy}{dx} = \frac{1}{1+\cos\pi/3}$

$= \frac{1}{1+\frac{1}{2}}$

$= \frac{2}{3}$

Equation of tangent with gradient $\frac{2}{3}$ through $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right)$

$y - \frac{\sqrt{3}}{3} = \frac{2}{3}\left(x - \frac{\pi}{3}\right)$

$y = \frac{2x}{3} - \frac{2\pi}{9} + \frac{\sqrt{3}}{3}$

In general form $6x - 9y - 2\pi + 3\sqrt{3} = 0$

Question 11 (2 marks)

Show that the curve $f(x) = \ln(x + 2)$ is concave down for all values of x in its domain. **2**

concave down, $f''(x) < 0$

$f'(x) = \frac{1}{x+2}$

$= (x+2)^{-1}$

$f''(x) = -\frac{1}{(x+2)^2}$ ✓

Domain of $f(x)$ is $x > -2$

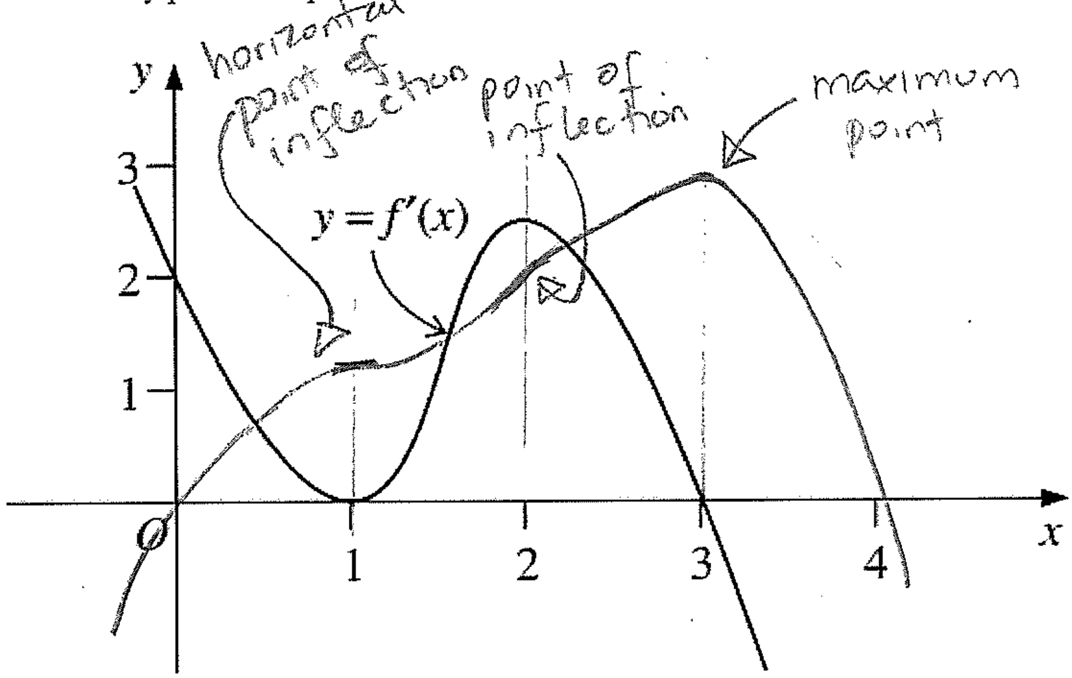
$f''(x) < 0$ for all values of x. ✓

Hence, curve is concave down for all values in its domain

Question 12 (3 marks)

The diagram below shows a sketch of the gradient function, $f'(x)$. **3**

Sketch the graph of the function $y = f(x)$, given that $f(0) = 0$, on the same set of axes. Clearly label stationary points and points of inflections.



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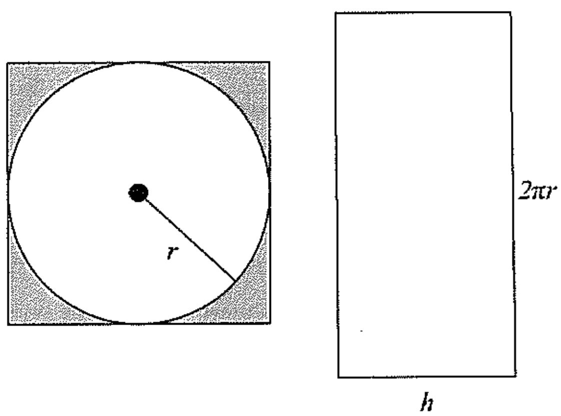

Question 13 continues next page

i) Show that the area of the sheet metal required to construct a barrel is given by **2**

$$A = 4r^2 + \frac{2V}{r}$$

area of metal required

$A = (2r)^2 + 2\pi r h$

but $V = \pi r^2 h$

$h = \frac{V}{\pi r^2}$

$\therefore A = 4r^2 + 2\pi r\left(\frac{V}{\pi r^2}\right)$

$A = 4r^2 + \frac{2V}{r}$

ii) Hence, show that the minimum area of metal, A, assuming V is a constant, occurs when **2**

$$r^3 = \frac{V}{4}$$

$A = 4r^2 + 2Vr^{-1}$

$\frac{dA}{dr} = 8r - \frac{2V}{r^2}$ since V is a constant

$\frac{d^2A}{dr^2} = 8 + \frac{4V}{r^3}$ Test $r^3 = V/4$ is a minimum

Minimum area, $\frac{dA}{dr} = 0$ at $r^3 = V/4$, $\frac{d^2A}{dr^2} = 8 + \frac{4V}{V/4}$

$8r - \frac{2V}{r^2} = 0$ $= 24 < 0$

$8r^3 - 2V = 0$

$8r^3 = 2V$ hence, minimum area occurs

$r^3 = \frac{2V}{8}$ when $r^3 = V/4$

$r^3 = \frac{V}{4}$

Marks

Question 13 continues next page

iii) Hence, show that the minimum area of metal, A, required to make a barrel is **2**

$$A = 3 \times \sqrt[3]{V^2} \times \sqrt[3]{4}$$

From (i), $A = 4r^2 + 2V/r$

when $r^3 = V/4$, $A = \frac{4r^3 + 2V}{r}$

$r = \frac{\sqrt[3]{V}}{\sqrt[3]{4}}$,

$A = \frac{4\left(\frac{V}{4}\right) + 2V}{\frac{\sqrt[3]{V}}{\sqrt[3]{4}}}$

$A = 3V \times \frac{\sqrt[3]{4}}{\sqrt[3]{V}}$

$A = 3V \times \sqrt[3]{4} \times V^{-1/3}$

$= 3 \times V^{2/3} \times \sqrt[3]{4}$

$\therefore A = 3 \times \sqrt[3]{V^2} \times \sqrt[3]{4}$

Question 14 (5 marks)

i) Show that $\frac{d}{dx}(\sec x) = \frac{\sin x}{\cos^2 x}$ **2**

$\frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{d}{dx}(\cos x)^{-1}$

$= -(\cos x)^{-2}(-\sin x)$

$= \frac{\sin x}{\cos^2 x}$

$\frac{\cos x \times 0 - 1 \times -\sin x}{\cos^2 x}$

Question 14 continues next page

ii) Using the result from i), show that the derivative of $y = \frac{\sec x}{\sin x + \cos x}$ can be expressed as **3**

$$\frac{dy}{dx} = \frac{\tan^2 x + 2\tan x - 1}{1 + 2\sin x \cos x}$$

$u = \sec x$ From (i), $\frac{d}{dx}(\sec) = \frac{\sin x}{\cos^2 x}$

$u' = \frac{\sin x}{\cos^2 x}$

$v = \sin x + \cos x$

$v' = \cos x - \sin x$

$$\frac{dy}{dx} = \frac{\frac{\sin x}{\cos^2 x}(\sin x + \cos x) - \left(\frac{1}{\cos x}\right)(\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$= \frac{\frac{\sin^2 x}{\cos^2 x} + \frac{\sin x}{\cos x} - 1 + \frac{\sin x}{\cos x}}{\sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$= \frac{\tan^2 x + \tan x - 1 + \tan x}{1 + 2\sin x \cos x}$$

$$\frac{dy}{dx} = \frac{\tan^2 x + 2\tan x - 1}{1 + 2\sin x \cos x}$$

END OF EXAMINATION