Gosford High School

Year 11

2008
Preliminary
Higher School Certificate

Mathematics

Assessment Task 3

Time Allowed - 65 minutes

Remember to start each new question on a new page
Students must answer questions using a blue/black pen and/or a sharpened B or HB pencil.
Approved scientific calculators may be used
Students need to be aware that

- 'bald' answers may not gain full marks.
- untidy and/or poorly organised solutions may not gain full marks.

QUESTION 1 (12 marks)

(i) Plot the points A(7,3), B(–2,2) and C(3,–2) on a number plane – use approximately $\frac{1}{4}$ of an A4 page and label your diagram clearly. (2)

(ii) Find the coordinates of the midpoint of AC. (1)

(iii) Find the gradient of the line AB. (1)

(iv) Show that the line l drawn parallel to AB and passing through C has equation

$x - 9y - 21 = 0$. (2)

(v) Find the coordinates of the point D where the line l intersects the x axis. (1)

(vi) Find the perpendicular distance from the line l to the point A. (2)

(vii) Find the area of the triangle ABD. (3)

QUESTION 2 (13 marks)

(a) What is the gradient of a line drawn perpendicular to the line $6x + 2y - 7 = 0$ (2)

(b) Find the equation of the line k which passes through the point $(-\sqrt{3}, 2)$ and makes an angle of $150°$ with the positive direction of the x axis. (2)

(c) Find the point of intersection of the lines $2x - 3y = 5$ and $x + 5y = 4$ (2)

(d) The lines with equations $5x + 2y = 19$ and $4x + ay = b$ are perpendicular and intersect at the point (3, 2). Find the values of 'a' and 'b'. (3)

(e) The equation of any line passing through the point of intersection of the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has the form $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$, where k is a constant.

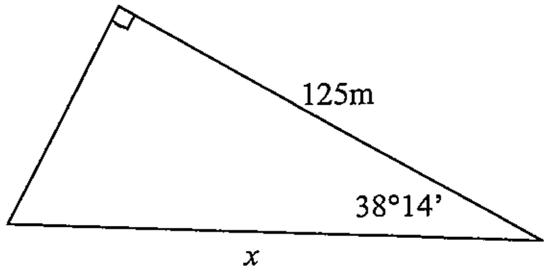
(i) Find the value of the constant k required to find the equation of the line passing through the point (1,–2) and the point of intersection of the lines $3x + 4y - 2 = 0$ and $5x - 2y - 12 = 0$. (2)

(ii) Hence find the equation of the line passing through the point (1,–2) and the point of intersection of the lines $3x + 4y - 2 = 0$ and $5x - 2y - 12 = 0$ (2)

QUESTION 3 (13 marks)

(a)

Find the value of x correct to 3 significant figures



(3)

(b) Solve $\tan x = 2\cdot 36$ for $180° \le x \le 360°$, writing your answer(s) to the nearest degree. (1)

(c) Solve $\sin A = -\frac{\sqrt{3}}{2}$ for $0° \le A \le 360°$ (2)

(d) If $\tan K = -\frac{8}{15}$ and $0° \le K \le 270°$ find the exact value of $\sin K$. (2)

(e) Write the exact value of

(i) $\cot(300°)$ (1)

(ii) $\cos(-45°)$ (1)

(f) A ship leaves port A and sails 30km due North to port B.
The ship then sails 50km due West to port C.

(i) Draw a neat diagram representing the given information. (1)

(ii) Find $\angle BAC$ to the nearest degree. (1)

(iii) Hence find the bearing of port A from port C. (1)

QUESTION 4 (12 marks)

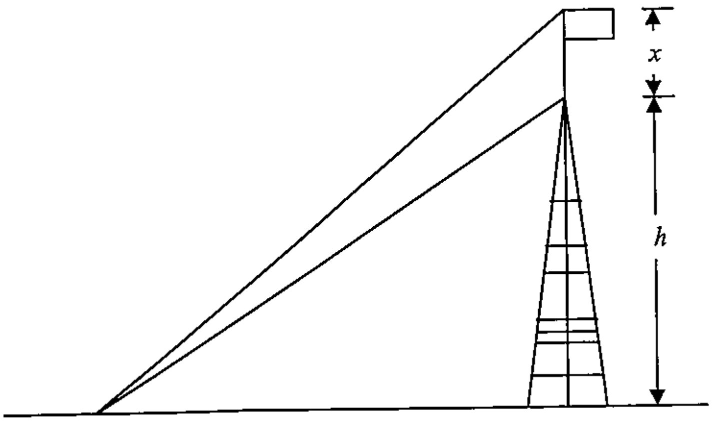
(a) Draw the graph of $y = \sin x$ for $0° \le x \le 360°$, labelling all essential features. (2)

(b) Simplify $\dfrac{\sin(180+\alpha)}{\cos(90-\alpha)}$ (2)

(c) Solve $|\cos\theta| = 1$ for $0° \le \theta \le 360°$ (2)

(d) Simplify $\dfrac{\tan\theta + \cot\theta}{\operatorname{cosec}\theta \cdot \sec\theta}$ (2)

(e)

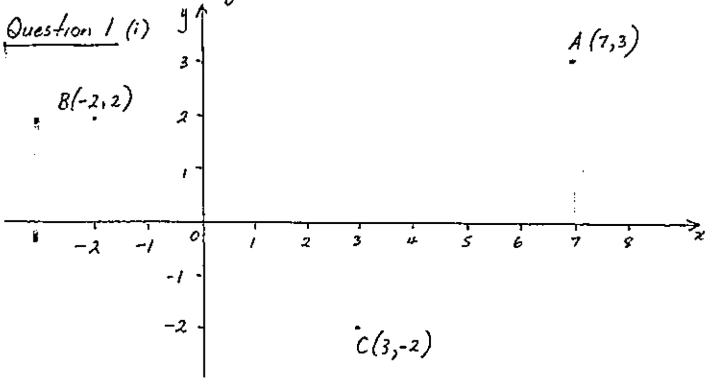


A flagpole x metres high stands on top of a vertical tower. From a point on the ground, the angles of elevation of the top and bottom of the flagpole are $\beta°$ and $\alpha°$ respectively. Show that the height of the tower, h m, is given by

$$h = \frac{x\tan\alpha°}{\tan\beta° - \tan\alpha°}.$$ (4)

Yr 11 Preliminary H.S.C 2unit Task 3 Solutions

Question 1 (i)



(ii) Midpoint of AC $= \left(\frac{7+3}{2}, \frac{3-2}{2}\right)$ using $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= (5, \frac{1}{2})$

iii) Gradient of AB $= \frac{3-2}{7+2}$ using $m = \frac{y_2-y_1}{x_2-x_1}$

$m_{AB} = \frac{1}{9}$

iv) Equation of ℓ is :-

$y+2 = \frac{1}{9}(x-3)$ using $y-y_1 = m(x-x_1)$

$9y+18 = x-3$

$x-9y-21=0$

v) ℓ intersects x axis when $y=0$

$\therefore x-21=0$

$x = 21$.

D has coordinates $(21,0)$

vi) $d = \left|\frac{1(7)+(-9)(3)-21}{\sqrt{1^2+(-9)^2}}\right|$ using $d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

and $a=1$ $x_1 = 7$
$b=-9$ $y_1=3$
$c=-21$

$d = \left|\frac{7-27-21}{\sqrt{82}}\right|$

$d = \left|\frac{-41}{\sqrt{82}}\right|$

$d = \frac{41}{\sqrt{82}}$

vii) $d_{AB} = \sqrt{(7+2)^2+(3-2)^2}$

$= \sqrt{82}$

Area of $\triangle ABD = \frac{1}{2}bh$.

$= \frac{1}{2}\times\sqrt{82}\times\frac{41}{\sqrt{82}}$

$= 20.5$ sq units

Question 2

a) $6x+2y-7=0$

$2y = -6x+7$

$y = -3x+\frac{7}{2}$

$\therefore$ Gradient of perpendicular line is $\frac{1}{3}$

b) Gradient of line $k = \tan 150°$

$= -\frac{1}{\sqrt{3}}$

Equation of line k is

$y-2 = -\frac{1}{\sqrt{3}}(x+\sqrt{3})$

$y\sqrt{3}-2\sqrt{3} = -x-\sqrt{3}$

$x+y\sqrt{3} = \sqrt{3}$

c) $x = 4-5y \longrightarrow 2x-3y=5$

$2(4-5y)-3y=5$

$8-10y-3y=5$

$3 = 13y$

$y = \frac{3}{13}$

$x = 4-\frac{15}{13}$

$x = 2\frac{11}{13}$

Point of Intersection is $(2\frac{11}{13}, \frac{3}{13})$

d) $(3,2)$ satisfies $4x+ay=b$

$12+2a = b \ldots\ldots(1)$

Gradient of $5x+2y=19$ is $-\frac{5}{2} = m_1$

Gradient of $4x+ay=b$ is $-\frac{4}{a} = m_2$

Since $\perp$, $m_1\times m_2 = -1$

$\frac{20}{2a} = -1$

$20 = -2a$

$a = -10$.

$\therefore b = 12-20$ using (1)

$b = -8$

$\therefore a=-10, b=-8$

e) i) Equation of required line is

$(3x+4y-2)+k(5x-2y-12)=0$

$(1,-2)$ satisfies

$\therefore (3-8-2)+k(5+4-12)=0$

$-7-3k=0$

$k = -\frac{7}{3}$

e) ii) Equation required is

$$(3x+4y-2) - \frac{7}{3}(5x-2y-12) = 0$$
$$9x + 12y - 6 - 35x + 14y + 84 = 0$$
$$-26x + 26y + 78 = 0$$
$$x - y - 3 = 0$$

Question 3

a) $\cos 38^\circ 14' = \frac{125}{x}$

$$x = \frac{125}{\cos 38^\circ 14}$$
$$x = 159\,m$$

b) $x = 180^\circ + \tan^{-1}(2\cdot36)$

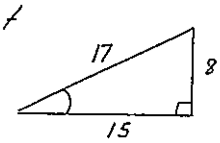
$$\doteqdot 180^\circ + 67\cdot03^\circ$$
$$= 247^\circ \text{ (to the nearest degree)}$$

c) $A = 180^\circ + 60^\circ,\ 360^\circ - 60^\circ$

$$= 240^\circ,\ 300^\circ$$

d) K lies in 2nd Quadrant

$\therefore \sin K = \frac{8}{17}$



b) $\frac{\sin(180+\alpha)}{\cos(90-\alpha)} = \frac{-\sin\alpha}{\sin\alpha}$

$$= -1$$

c) $|\cos\theta| = 1$

$\therefore \cos\theta = \pm 1$

$\therefore \theta = 0^\circ, 180^\circ, 360^\circ$

d) $\frac{\tan\theta + \cot\theta}{\operatorname{cosec}\theta \cdot \sec\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta}}$

(multiply top & bottom by $\sin\theta\cos\theta$)

$$= \frac{\sin^2\theta + \cos^2\theta}{1}$$
$$= \frac{1}{1}$$
$$= 1$$

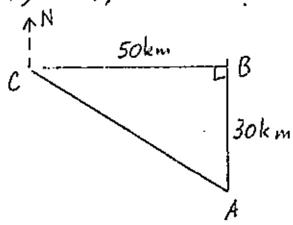
e) i) $\cot 300^\circ = \frac{1}{\tan 300^\circ}$

$$= \frac{1}{-\tan 60^\circ}$$
$$= -\frac{1}{\sqrt{3}}$$

ii) $\cos(-45)^\circ = \cos$

$$= \frac{1}{\sqrt{2}}$$

f) i)



ii) $\tan B\hat{A}C = \frac{50}{30}$

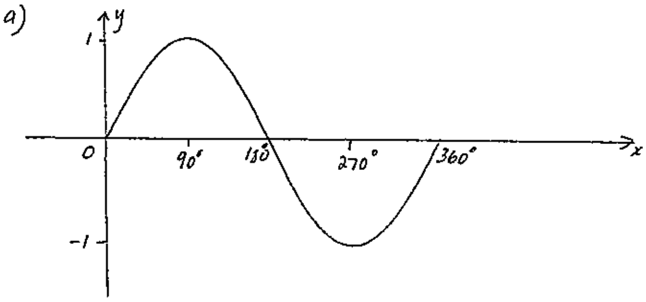
$$B\hat{A}C = \tan^{-1}\left(\tfrac{5}{3}\right)$$
$$= 59^\circ$$

$$\angle NAC = 180^\circ - 59^\circ$$
$$= 121^\circ$$

$\therefore$ Bearing of A from C is $121^\circ N$

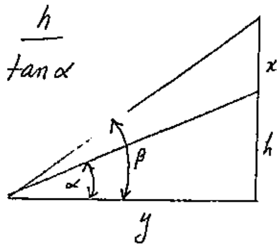
Question 4

a)



e) $\tan\alpha = \frac{h}{y} \Rightarrow y = \frac{h}{\tan\alpha}$

$\tan\beta = \frac{h+x}{y}$



$\therefore \tan\beta = \frac{h+x}{\frac{h}{\tan\alpha}}$

$$\tan\beta = \frac{h\tan\alpha + x\tan\alpha}{h}$$
$$h\tan\beta = h\tan\alpha + x\tan\alpha$$
$$h\tan\beta - h\tan\alpha = x\tan\alpha$$
$$h(\tan\beta - \tan\alpha) = x\tan\alpha$$
$$h = \frac{x\tan\alpha}{\tan\beta - \tan\alpha}$$

e) ii) ∴ Equation required is

$$(3x + 4y - 2) - \frac{7}{3}(5x - 2y - 12) = 0$$
$$9x + 12y - 6 - 35x + 14y + 84 = 0$$
$$-26x + 26y + 78 = 0$$
$$x - y - 3 = 0$$

Question 3

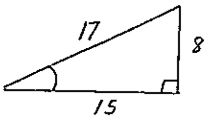
a) $$\cos 38°14' = \frac{125}{x}$$
$$x = \frac{125}{\cos 38°14}$$
$$x = 159m$$

b) $$x = 180° + \tan^{-1}(2.36)$$
$$\doteqdot 180° + 67.03°$$
$$= 247° \text{ (to the nearest degree)}$$

c) $$A = 180° + 60°,\ 360° - 60°$$
$$= 240°,\ 300°$$

d) K lies in 2nd Quadrant

$$\therefore \sin K = \frac{8}{17}$$



b) $$\frac{\sin(180+\alpha)}{\cos(90-\alpha)} = \frac{-\sin\alpha}{\sin\alpha}$$
$$= -1$$

c) $$|\cos\theta| = 1$$
$$\therefore \cos\theta = \pm 1$$
$$\therefore \theta = 0°, 180°, 360°$$

d) $$\frac{\tan\theta + \cot\theta}{\operatorname{cosec}\theta \cdot \sec\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta}}$$

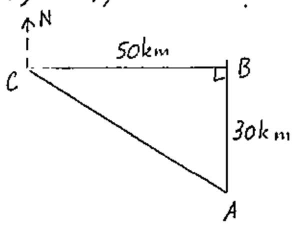
(multiply top & bottom by $\sin\theta\cos\theta$.)

$$= \frac{\sin^2\theta + \cos^2\theta}{1}$$
$$= \frac{1}{1}$$
$$= 1$$

e) i) $$\cot 300° = \frac{1}{\tan 300°}$$
$$= \frac{1}{-\tan 60°}$$
$$= -\frac{1}{\sqrt{3}}$$

ii) $$\cos(-45)° = co$$
$$= \frac{1}{\sqrt{2}}$$

f) i)

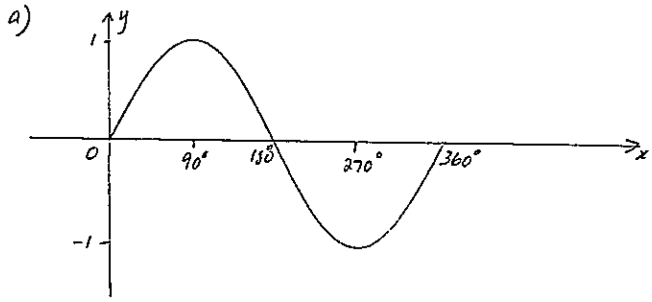


ii) $$\tan B\hat{A}C = \frac{50}{30}$$
$$B\hat{A}C = \tan^{-1}\left(\frac{5}{3}\right)$$
$$= 59°$$
$$\angle NAC = 180° - 59°$$
$$= 121°$$

∴ Bearing of A from C is 121°N

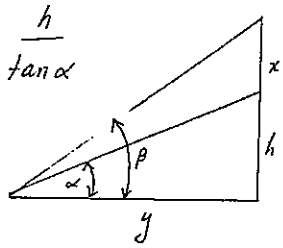
Question 4

a)



e) $$\tan\alpha = \frac{h}{y} \Rightarrow y = \frac{h}{\tan\alpha}$$

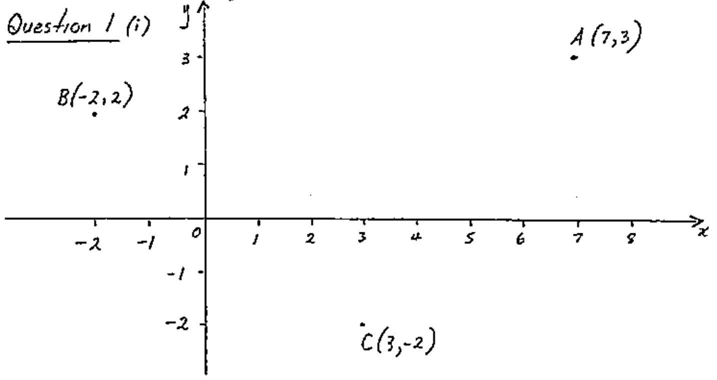
$$\tan\beta = \frac{h+x}{y}$$



$$\therefore \tan\beta = \frac{h+x}{\frac{h}{\tan\alpha}}$$
$$\tan\beta = \frac{h\tan\alpha + x\tan\alpha}{h}$$
$$h\tan\beta = h\tan\alpha + x\tan\alpha$$
$$h\tan\beta - h\tan\alpha = x\tan\alpha$$
$$h(\tan\beta - \tan\alpha) = x\tan\alpha$$
$$h = \frac{x\tan\alpha}{\tan\beta - \tan\alpha}$$

Yr 11 Preliminary H.S.C 2unit Task 3 Solutions

Question 1 (i)



(ii) Midpoint of AC $= \left(\frac{7+3}{2}, \frac{3-2}{2}\right)$ using $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= (5, \frac{1}{2})$

iii) Gradient of AB $= \frac{3-2}{7+2}$ using $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{AB} = \frac{1}{9}$

iv) Equation of ℓ is :-

$y + 2 = \frac{1}{9}(x-3)$ using $y - y_1 = m(x - x_1)$

$9y + 18 = x - 3$

$x - 9y - 21 = 0$

v) ℓ intersects x axis when $y = 0$.

$\therefore x - 21 = 0$

$x = 21$.

D has coordinates $(21, 0)$

vi) $d = \left|\frac{1(7)+(-9)(3)-21}{\sqrt{1^2+(-9)^2}}\right|$ using $d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$

and $a = 1$, $x_1 = 7$

$b = -9$, $y_1 = 3$

$c = -21$

$d = \left|\frac{7-27-21}{\sqrt{82}}\right|$

$d = \left|\frac{-41}{\sqrt{82}}\right|$

$d = \frac{41}{\sqrt{82}}$

vii) $d_{AB} = \sqrt{(7+2)^2 + (3-2)^2}$

$= \sqrt{82}$

Area of $\Delta ABD = \frac{1}{2}bh$.

$= \frac{1}{2} \times \sqrt{82} \times \frac{41}{\sqrt{82}}$.

$= 20.5$ sq units

Question 2

a) $6x + 2y - 7 = 0$

$2y = -6x + 7$

$y = -3x + \frac{7}{2}$

$\therefore$ Gradient of perpendicular line is $\frac{1}{3}$

b) Gradient of line k $= \tan 150^\circ$

$= -\frac{1}{\sqrt{3}}$

Equation of line k is

$y - 2 = -\frac{1}{\sqrt{3}}(x + \sqrt{3})$

$y\sqrt{3} - 2\sqrt{3} = -x - \sqrt{3}$

$x + y\sqrt{3} = \sqrt{3}$

c) $x = 4 - 5y \longrightarrow 2x - 3y = 5$

$2(4 - 5y) - 3y = 5$

$8 - 10y - 3y = 5$

$3 = 13y$

$y = \frac{3}{13}$

$x = 4 - \frac{15}{13}$

$x = 2\frac{11}{13}$

Point of Intersection is $(2\frac{11}{13}, \frac{3}{13})$

d) $(3,2)$ satisfies $4x + ay = b$

$12 + 2a = b \ldots\ldots (1)$

Gradient of $5x + 2y = 19$ is $-\frac{5}{2} = m_1$

Gradient of $4x + ay = b$ is $-\frac{4}{a} = m_2$

Since $\perp$, $m_1 \times m_2 = -1$

$\frac{20}{2a} = -1$

$20 = -2a$

$a = -10$.

$\therefore b = 12 - 20$ using (1)

$b = -8$

$\therefore a = -10, b = -8$

e) i) Equation of required line is

$(3x + 4y - 2) + k(5x - 2y - 12) = 0$

$(1, -2)$ satisfies

$\therefore (3 - 8 - 2) + k(5 + 4 - 12) = 0$

$-7 - 3k = 0$

$k = -\frac{7}{3}$