HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 11 Assessment 2 Term 2 2021

STUDENT NUMBER: _____

General Instructions

- Reading Time – 5 minutes
- Working Time – 65 minutes
- Write using black pen
- Calculators approved by NESA may be used
- In Questions 6 – 11, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination
- Students are permitted to refer to one double-sided A4 page of handwritten topic summaries which they prepared prior this assessment

Total marks – 52

Section I Pages 2 – 4

5 marks

Attempt Questions 1 – 5

Allow about 8 minutes for this section

Answer on the Objective Response Answer Sheet provided

Section II Pages 5 – 14

47 marks

Attempt Questions 6 -11

Allow about 57 minutes for this section

Write your answers in the space provided.

Question	***1-5***	***6***	***7***	***8***	***9***	***10***	***11***	***Total***
Total	*/5*	*/9*	*/7*	*/8*	*/7*	*/8*	*/8*	*/52*

This assessment task constitutes 30% of the Preliminary Higher School Certificate Course School Assessment

Section I

5 marks
Attempt Questions 1 – 5
Allow about 8 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 5

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1 Given $f(x) = x + 5$ and $g(x) = x - 2$, what is the domain of $y = \dfrac{f(x)}{g(x)}$?

(A) $(2, \infty)$

(B) $(-\infty, -5) \cup (-5, \infty)$

(C) $(-\infty, 2] \cup [2, \infty)$

(D) $(-\infty, 2) \cup (2, \infty)$

2 Which equation is a many to one relation?

(A) $y = \pm\sqrt{9 - x^2}$

(B) $y = \sqrt{9 - x^2}$

(C) $x = \sqrt{9 - y^2}$

(D) $x = \pm\sqrt{9 - y^2}$

3 Which of the following is the vertex of the parabola $y = 2x^2 + 8x - 11$?

(A) $(-2, -19)$

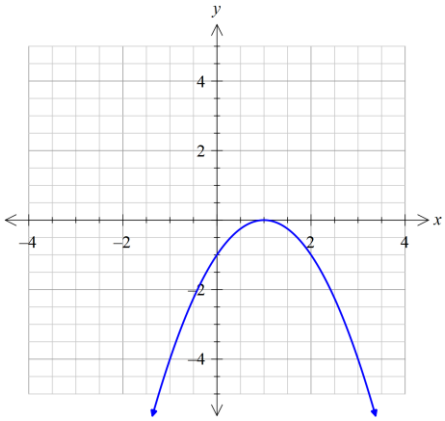
(B) $(2,13)$

(C) $(-2,13)$

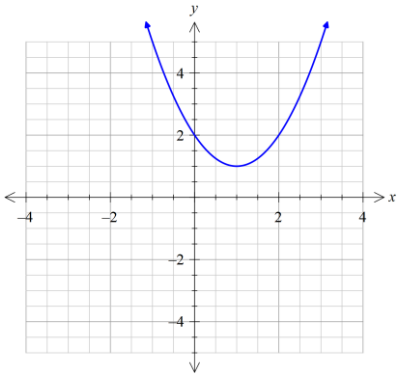
(D) $(2,19)$

4 Which graph below represents a quadratic function in the form $y = ax^2 + bx + c$, where $\Delta > 0$ and $a > 0$?

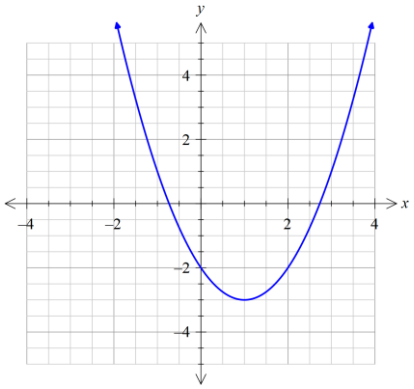
(A)



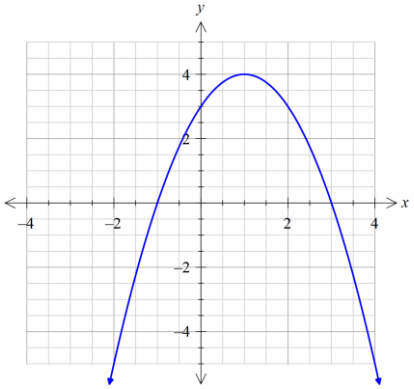
(B)



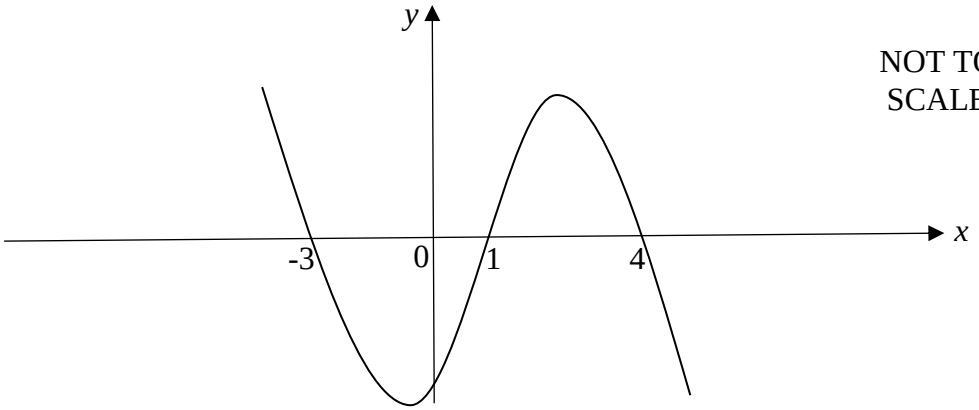
(C)



(D)



5 Which of the following is the equation for the function below?



(A) $y = (x-1)(x+3)(x-4)$

(B) $y = (1-x)(x+3)(x-4)$

(C) $y = (1-x)(x+3)(4-x)$

(D) $y = (x+1)(x-3)(x+4)$

End of Section I

Section II
47 marks
Attempt Questions 6-11

Question 6 (9 marks)

(a) Convert 24° to radians, expressing your answer in terms of π in simplified form. **1**

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(b) What is the exact value of $\sin 240°$? **2**

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(c) Is the function $f(x) = \dfrac{2x^2}{x^3+1}$ odd, even or neither? Show all working. **2**

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Question 6 continues on next page

Question 6 (continued)

(d) A function is defined by

$$g(x) = \begin{cases} -x^3 & for \quad x < -1 \\ x & for \quad -1 \le x \le 1 \\ x^2 & for \quad x > 1 \end{cases}$$

(i) Evaluate $g(-2) + g(0) + g(2)$. **1**

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(ii) Sketch $g(x)$, showing all important features. **3**

End of Question 6

Question 7 (7 marks)

(a) Find the point(s) of intersection of the line $x - y - 4 = 0$ and hyperbola $y = \frac{5}{x}$. **3**

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(b) Find the angle $3x + 4y + 9 = 0$ makes with the x-axis in the positive direction. Express your answer correct to the nearest minute. **2**

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(c) The frequency, in chirps/minute, of a cricket's chirp is directly proportional to the temperature, in ºC, of its surroundings. On a day where the temperature was 30ºC, the frequency of chirps was 280 chirps/minute.

(i) Find a formula for *C* in terms of *T*, where *C* is the frequency of chirps and *T* is the surrounding temperature. **1**

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(ii) Use the formula found in part (i) to calculate the temperature of the surrounding environment when the chirping reaches 350 chirps/minute. Answer correct to the nearest degree. **1**

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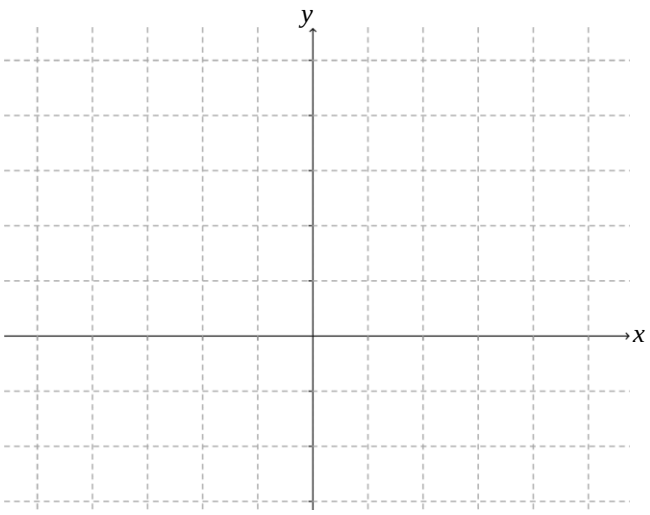
Question 8 (8 marks)

(a) (i) Sketch $y = |x| - 1$ and $y = 4$ on the number plane below, showing the x and y-intercepts. **2**



(ii) Using your graph, or otherwise, solve $|x| - 1 = 4$. **2**

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(b) A sheet of cardboard, measuring 30cm by 24cm, has equal squares of side length x cm cut out of its corners. The cut-out sides are then folded up to form an open box.

(i) Write down an expression for the area of the <u>base</u> of the box in terms of x. **1**

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(ii) Find the dimensions of the base of the box given that its area is 216cm^2. **3**

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Question 9 (7 marks)

(a) If $f(x) = \frac{2}{x}$, show that $f(x+2) - f(x) = -\frac{4}{x^2 + 2x}$. **2**

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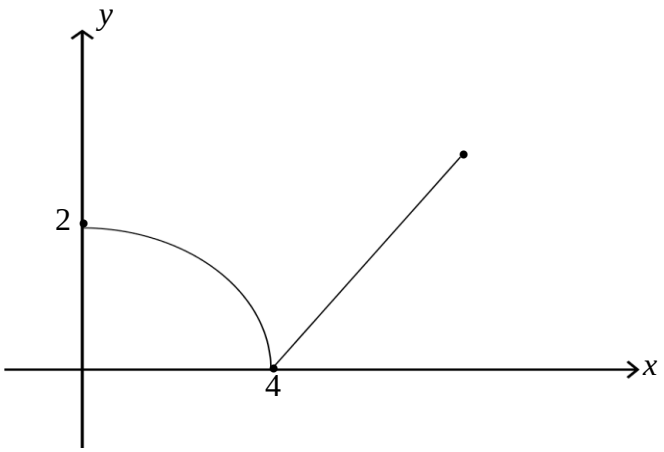
(b) The diagram below shows the graph of $y = f(x)$ with endpoints (0,2) and (7,3).



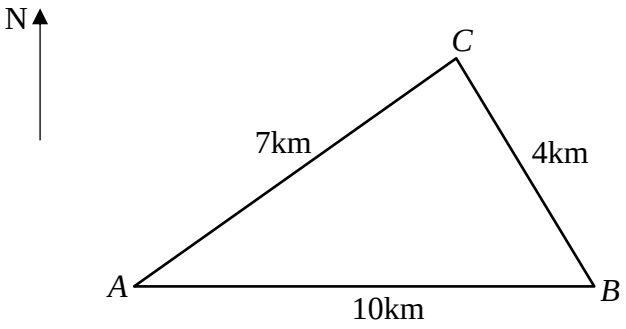
NOT TO SCALE

Sketch $y = f(x+1) + 3$ in the space below, showing the new endpoints. **2**

Question 9 continues on next page

Question 9 (continued)

(c) In the diagram below, building *B* is 10km due east of building *A*. Building *C* is 7km from building *A* and 4km from building *B*.



NOT TO SCALE

(i) Find the size of $\angle ABC$, correct to the nearest degree. **2**

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(ii) Find the bearing of building *B* from building *C*, correct to the nearest degree. **1**

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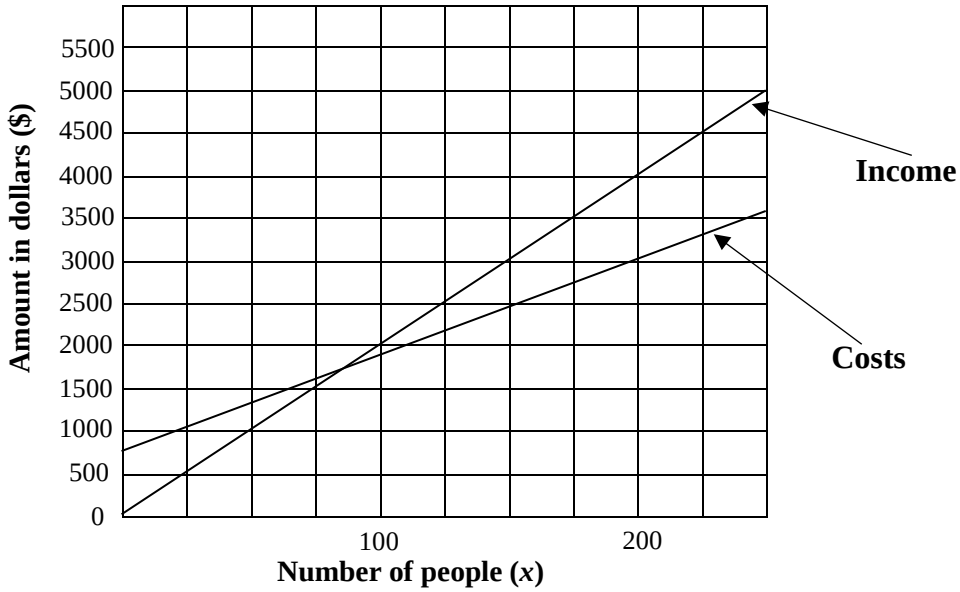
Question 10 (8 marks)

(a) Susan and Nick are planning a fund-raising dance. They hire a hall for $400 and a band for $300. Refreshments will cost $12 per person. They set the entry ticket at $20 per person. The graph below shows planned income and costs for the dance.



(i) Write a formula for the cost, $\$C$, of running the dance for x people. **1**

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(ii) Estimate the minimum number of people needed at the dance to cover the costs. **1**

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(iii) How much profit will be made if 150 people attend the dance? **1**

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(iv) Susan and Nick plan to sell 200 tickets. They want to make a profit of $1 500. What should be the price of a ticket assuming all 200 tickets will be sold? **2**

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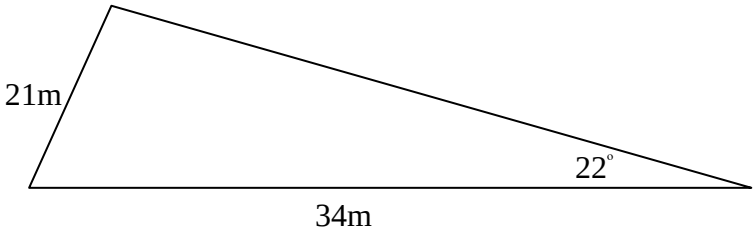
Question 10 continues on next page

(b) Calculate the value of θ, correct to the nearest minute. **3**



NOT TO SCALE

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Question 11 (8 marks)

(a) Solve for x: $\cos x = -\frac{\sqrt{2}}{2}$ for $0° \leq x \leq 360°$. **2**

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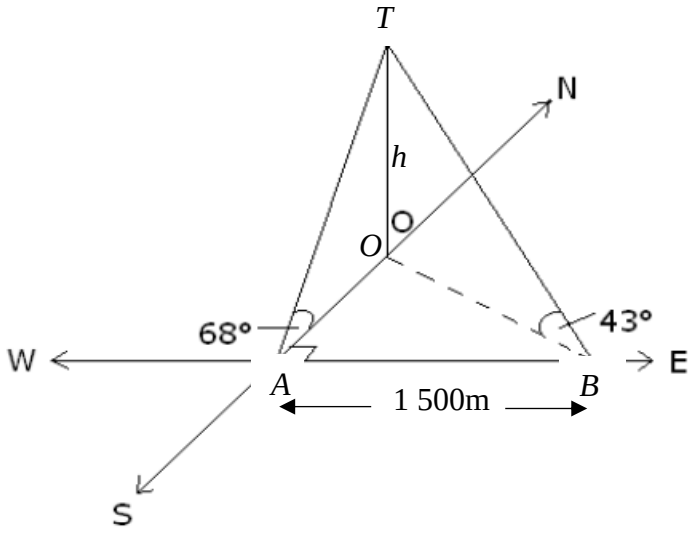
(b) A lighthouse of height h metres, is observed from two points, A and B, in the same horizontal plane as the base of the lighthouse. From point A, due south of the lighthouse, the angle of elevation to the top of the lighthouse is 68°. From point B, due east of point A, the angle of elevation to the top of the lighthouse is 43°. Points A and B are 1 500 metres apart.



NOT TO SCALE

(i) Show that $OA = h \tan 22°$ **2**

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Question 11 continues on next page

Question 11 (continued)

(ii) Show that $h = \dfrac{1\,500}{\sqrt{\tan^2 47° - \tan^2 22°}}$ **3**

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(iii) Find the height of the tower correct to the nearest metre. **1**

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END OF PAPER

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Mathematics Advanced

Year 11 Assessment 2
Term 2 2021

STUDENT NUMBER: _____

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

1. A○ B○ C○ D○
2. A○ B○ C○ D○
3. A○ B○ C○ D○
4. A○ B○ C○ D○
5. A○ B○ C○ D○

HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 11 Assessment 2 Term 2 2021

STUDENT NUMBER: SOLUTIONS and markers feedback

General Instructions

- Reading Time – 5 minutes
- Working Time – 65 minutes
- Write using black pen
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Total	/5	/9	/7	/8	/7	/8	/8	/52

This assessment task constitutes 30% of the Preliminary Higher School Certificate Course School Assessment

Section I

5 marks
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1 If $f(x) = x+5$ and $g(x) = x-2$, what is the domain of $y = \dfrac{f(x)}{g(x)}$?

$y = \dfrac{x+5}{x-2}$

(A) $(2, \infty)$

(B) $(-\infty, -5) \cup (-5, \infty)$

(C) $(-\infty, 2] \cup [2, \infty)$

(D) $(-\infty, 2) \cup (2, \infty)$

D
B
A
C
B

2 Which equation is a many to one relation?

(A) $y = \pm\sqrt{9-x^2}$

(B) $y = \sqrt{9-x^2}$

(C) $x = \sqrt{9-y^2}$

(D) $x = \pm\sqrt{9-y^2}$

3 Which of the following is the vertex of the parabola $y = 2x^2 + 8x - 11$?

$y = 2(x^2+4x+4) - 11 - 8$

$y = 2(x+2)^2 - 19$

(A) $(-2, -19)$

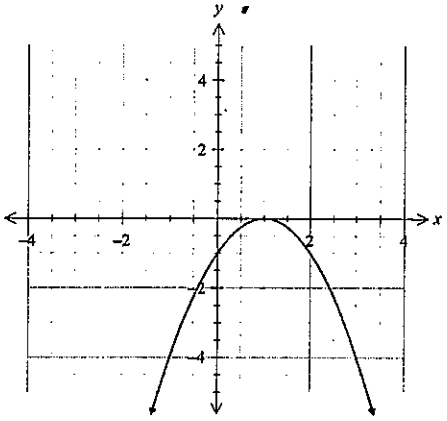
(B) $(2, 13)$

(C) $(-2, 13)$

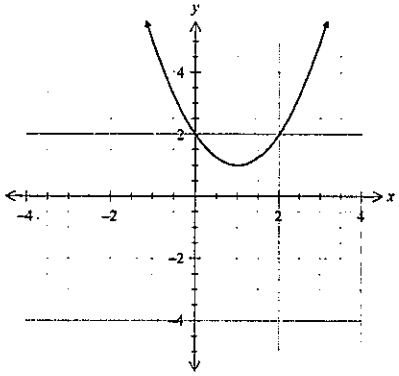
(D) $(2, 19)$

4 Which graph below represents a quadratic function in the form $y = ax^2 + bx + c$, where $\Delta > 0$ and $a > 0$?

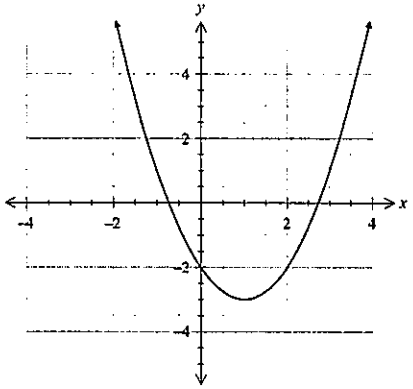
(A)



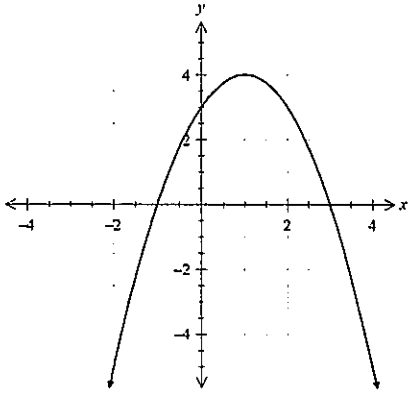
(B)



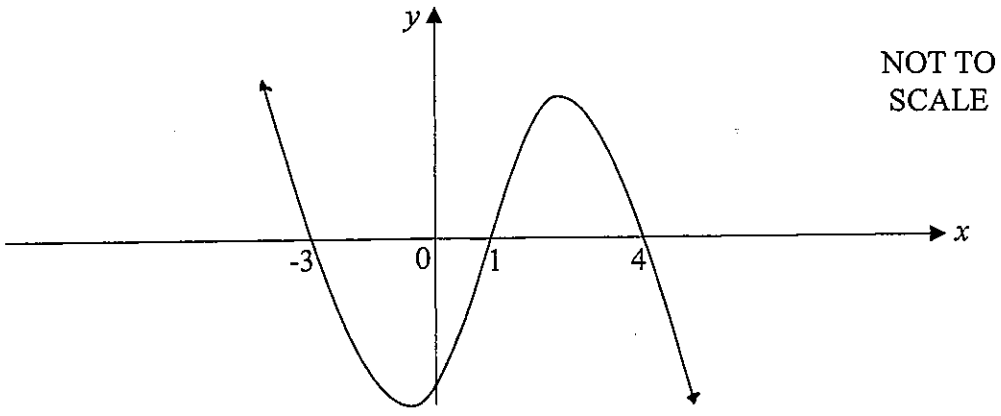
(C)



(D)



5 Which of the following is the equation for the function below?



NOT TO SCALE

(A) $y = (x-1)(x+3)(x-4)$

(B) $y = (1-x)(x+3)(x-4)$

(C) $y = (1-x)(x+3)(4-x)$

(D) $y = (x+1)(x-3)(x+4)$

End of Section I

Section II

47 marks
Attempt Questions 6-11

Question 6 (9 marks)

(a) Convert 24° to radians, expressing your answer in terms of π in simplified form. **1**

$\frac{24\pi}{180}$

$= \frac{2\pi}{15}$

Done well by most students

(b) What is the exact value of $\sin 240°$? **2**

$\sin 240^\circ$ — Done well by most students.

$= \sin(180+60)$ — A few missed the negative.

$= -\sin 60^\circ$

$= -\frac{\sqrt{3}}{2}$

(c) Is the function $f(x) = \frac{2x^2}{x^3+1}$ odd, even or neither? Show all working. **2**

$f(x) = \frac{2x^2}{x^3+1}$

$f(-x) = \frac{2(-x)^2}{(-x)^3+1}$ — MUST show the $(-x)$ being substituted

$= \frac{2x^2}{-x^3+1}$ not even as $f(-x) \neq f(x)$ — MUST justify why it is neither

$= \frac{-2x^2}{x^3-1}$ not odd as $f(-x) \neq -f(x)$

$\therefore$ neither odd nor even.

A common error was stating $\frac{2x^2}{-x^3+1} = -\frac{2x}{x^3+1}$

$\frac{2x^2}{-x^3+1} = \frac{2x^2}{-(x^3-1)}$

$= -\frac{2x}{x^3-1}$

Question 6 continues on next page

Question 6 (continued)

(c) A function is defined by

$$g(x) = \begin{cases} -x^3 & for \quad x < -1 \\ x & for \quad -1 \le x \le 1 \\ x^2 & for \quad x > 1 \end{cases}$$

(i) Evaluate $g(-2) + g(0) + g(2)$. **1**

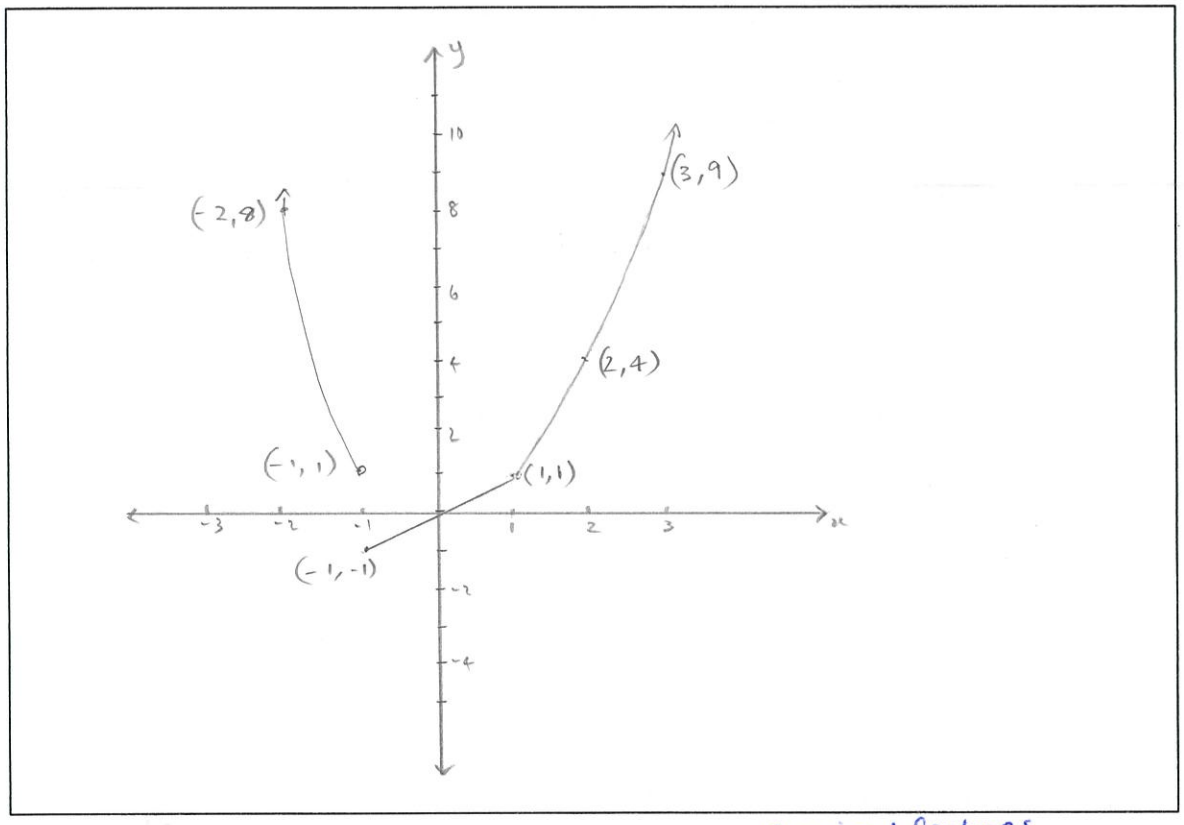
$g(-2) + g(0) + g(2)$

$= -(-2)^3 + 0 + 2^2$

$= 8 + 4$

$= 12$

(ii) Sketch $g(x)$, showing all important features. **3**



Required features
- cubic $y = -x^3$ for $x < -1$
- line $y = x$ for $-1 \le x \le 1$
- parabola $y = x^2$ for $x > 1$
- appropriate scale

End of Question 6

Question 7 (7 marks)

(a) Find the point(s) of intersection of the line $x-y-4=0$ and hyperbola $y=\frac{5}{x}$. **3**

$x-y-4=0$ —(1) $y=\frac{5}{x}$ —(2)

sub. (2) into (1) $x-\frac{5}{x}-4=0$

$x^2-4x-5=0$

$(x-5)(x+1)=0$

$\therefore x=5,\ x=-1$

when $x=5$, $y=\frac{5}{5}$ $y=1$ when $x=-1$, $y=\frac{5}{-1}$ $y=-5$

$\therefore (5,1)$ & $(-1,-5)$ are pts of intersection

Mostly well done
some did not make the quad. = 0
∴ struggled to find correct sol.

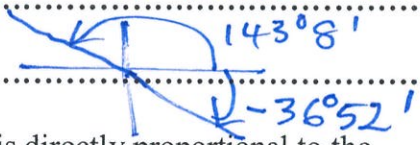
(b) Find the angle $3x+4y+9=0$ makes with the x-axis in the positive direction. Express your answer correct to the nearest minute. **2**

$4y=-3x-9$

$m=-\frac{3}{4}$ $\therefore \theta = 143°8'$

$\tan\theta=-\frac{3}{4}$ ← a lot do not know what this means

$\therefore \theta = 143.1301024$



(c) The frequency, in chirps/minute, of a cricket's chirp is directly proportional to the temperature, in °C, of its surroundings. On a day where the temperature was 30°C, the frequency of chirps was 280 chirps/minute.

(i) Find a formula for C in terms of T, where C is the frequency of chirps and T is the surrounding temperature. **1**

$C=kT$ $\therefore k=\frac{28}{3}$

$280=k(30)$ $\therefore C=\frac{28}{3}T$

A lot got this as $C=\frac{3}{28}T$

(ii) Use the formula found in part (i) to calculate the temperature of the surrounding environment when the chirping reaches 350 chirps/minute. Answer correct to the nearest degree. **1**

$350=\frac{28}{3}T$

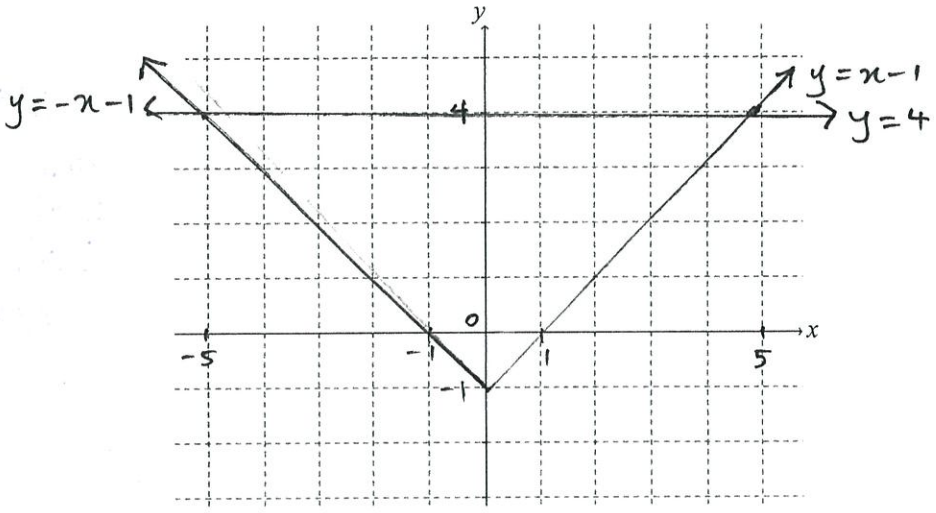
$T=37.5°$

$T=38°$

Mostly well done even when (i) was incorrect.

Question 8 (8 marks)

(a) (i) Sketch $y = |x| - 1$ and $y = 4$ on the number plane below, showing the x and y-intercepts. **2**



Many did well.
Some sketched $y = |x-1|$ instead.
A few did not label the graphs drawn.

(ii) Using your graph, or otherwise, solve $|x| - 1 = 4$. **2**

$x = \pm 5$

Many did well
A few did not solve for x but gave the points of intersection which is not required.

(b) A sheet of cardboard, measuring 30cm by 24cm, has equal squares of side length x cm cut out of its corners. The cut-out sides are then folded up to form an open box.

(i) Write down an expression for the area of the base of the box in terms of x. **1**

$A = (30-2x)(24-2x)$

Many did well.
Algebraic technical problem: common factor of 4, NOT 2.

(ii) Find the dimensions of the base of the box given that its area is 216cm^2. **3**

$216 = (30-2x)(24-2x)$

$216 = 720 - 60x - 48x + 4x^2$

$4x^2 - 108x + 504 = 0$ $\quad 126 = 21 \times 6$

$x^2 - 27x + 126 = 0$

$(x-21)(x-6) = 0$

$\therefore x = 21 \quad x = 6 \quad \therefore x = 6$ is only soln

~~XX~~ not possible because $x < 12$ or other explanations.

dimensions of base : $(30-2(6)) \times (24-2(6))$

$= 18\text{cm} \times 12\text{cm}$

Question 9 (7 marks)

(a) If $f(x) = \frac{2}{x}$, show that $f(x+2) - f(x) = -\frac{4}{x^2+2x}$. **2**

$f(x+2) - f(x)$

$= \frac{2}{x+2} - \frac{2}{x}$

$= \frac{2x - 2(x+2)}{x(x+2)}$

$= \frac{2x - 2x - 4}{x^2+2x}$

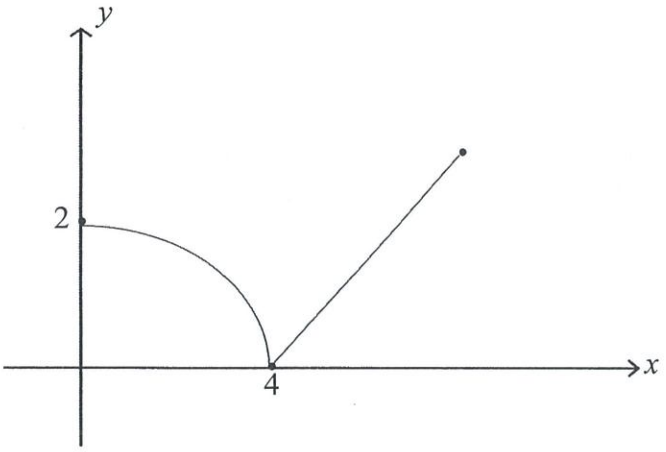
$= \frac{-4}{x^2+2x}$

$= RHS$

It is a Show question, need to show both of these lines.

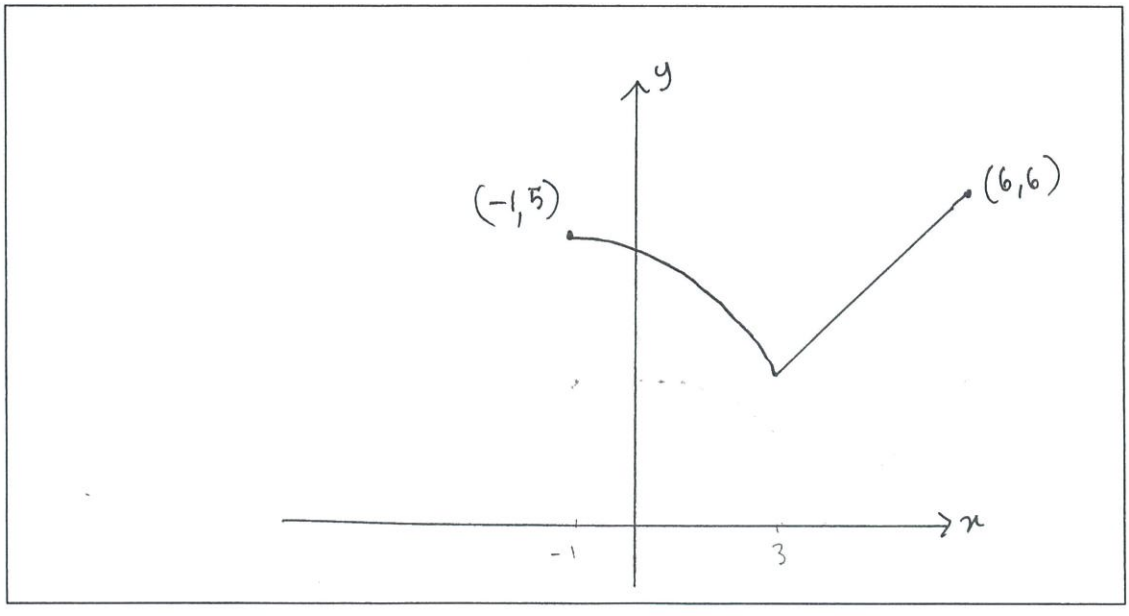
many still cannot expand $-2(x+2) \neq -2x+4$.

(b) The diagram below shows the graph of $y = f(x)$ with endpoints (0,2) and (7,3).



NOT TO SCALE

Sketch $y = f(x+1) + 3$ in the space below, showing the new endpoints. **2**

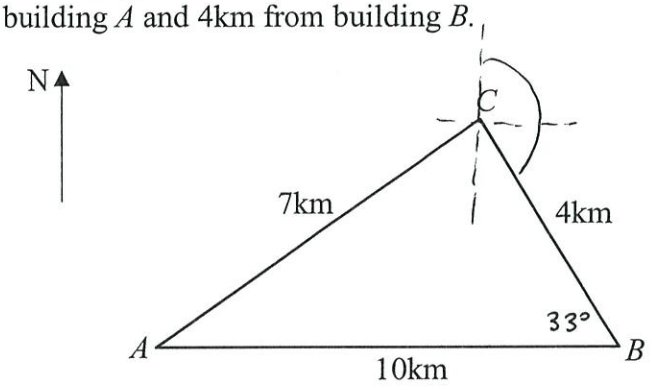


Need to show endpoints.

Question 9 continues on next page

Question 9 (continued)

(c) In the diagram below, building B is 10km due east of building A. Building C is 7km from building A and 4km from building B.



NOT TO SCALE

(i) Find the size of $\angle ABC$, correct to the nearest degree. **2**

let $\angle ABC = \theta$

$\cos\theta = \dfrac{10^2+4^2-7^2}{2(10)(4)}$

$\theta = 33.1229$

$\theta = 33°$

Done well.

Some did not round to nearest degree. This error was ignored.

(ii) Find the bearing of building B from building C, correct to the nearest degree. **1**

bearing $= 90+33$

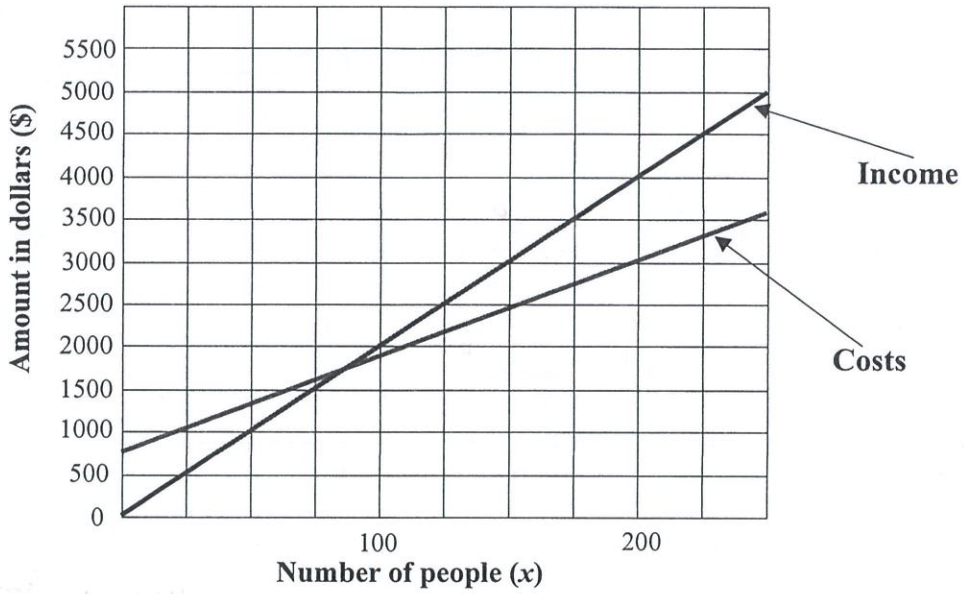
$= 123°T.$

Done Well

Question 10 (8 marks)

(a) Susan and Nick are planning a fund-raising dance. They hire a hall for \$400 and a band for \$300. Refreshments will cost \$12 per person. They set the entry ticket at \$20 per person. The graph below shows planned income and costs for the dance.

NOT TO SCALE



(i) Write a formula for the cost, \$$C$, of running the dance for x people. **1**

$C = 12x + 700$ many did well.

(ii) Estimate the minimum number of people needed at the dance to cover the costs. **1**

87 people many did well

(iii) How much profit will be made if 150 people attend the dance? **1**

$P = I - C$

$P = 20 \times 150 - (12 \times 150 + 700)$ many did well.

$P = \$500$

(iv) Susan and Nick plan to sell 200 tickets. They want to make a profit of \$1 500. What should be the price of a ticket assuming all 200 tickets will be sold? **2**

let the ticket price be x many did well

$1500 = 200x - (12 \times 200 + 700)$

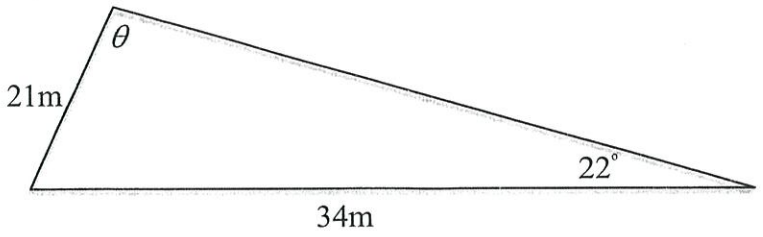
$200x = 4600$

$x = \frac{4600}{200}$

$x = \$23$

Question 10 continues on next page

(b) Calculate the value of θ, correct to the nearest minute. **3**



NOT TO SCALE

$$\frac{\sin\theta}{34} = \frac{\sin 22^\circ}{21}$$

$$\sin\theta = \frac{34\sin 22^\circ}{21}$$

$\theta = 37^\circ 20'$ or $142^\circ 40'$

- many forgot the ambiguous case
- some forgot to change the calculator mode to degree.

Question 11 (8 marks)

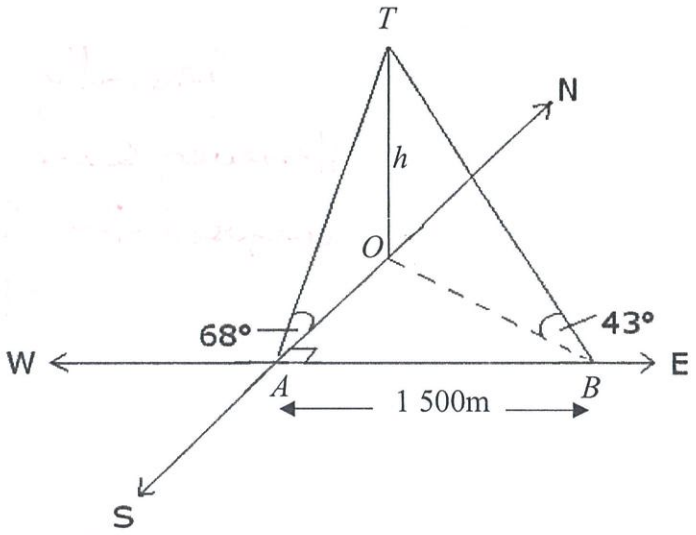
(a) Solve for x: $\cos x = -\frac{\sqrt{2}}{2}$ for $0° \leq x \leq 360°$. **2**

acute angle $x = 45°$

Q11 $= 180 - 45$ Q111 $= 180 + 45°$

$\therefore x = 135°$, $x = 225°$ Well done!

(b) A lighthouse of height h metres, is observed from two points, A and B, in the same horizontal plane as the base of the lighthouse. From point A, due south of the lighthouse, the angle of elevation to the top of the lighthouse is 68°. From point B, due east of point A, the angle of elevation to the top of the lighthouse is 43°. Points A and B are 1 500 metres apart.



NOT TO SCALE

(i) Show that $OA = h \tan 22°$ **2**

$\tan 68° = \frac{h}{OA}$

$OA = \frac{h}{\tan 68°}$

$OA = h \cot 68°$ Well done!

$\therefore OA = h \tan 22°$

Question 11 continues on next page

Question 11 (continued)

(ii) Show that $h = \dfrac{1\,500}{\sqrt{\tan^2 47° - \tan^2 22°}}$ **3**

$OB = h\tan 47°$ (similarly to $OA = h\tan 22°$)

$\therefore 1500^2 = (h\tan 47°)^2 - (h\tan 22°)^2$ (from Pythagoras Theorem)

$1500^2 = h^2(\tan^2 47° - \tan^2 22°)$

$h^2 = \dfrac{1500^2}{\tan^2 47° - \tan^2 22°}$

$\therefore h = \dfrac{1500}{\sqrt{\tan^2 47° - \tan^2 22°}}$, $h > 0$

Generally well done!
However, some students confused the Pythagoras Theorem!

(iii) Find the height of the tower correct to the nearest metre. **1**

$\therefore h = 1399.514865\ldots$

$\therefore h = 1400$ m (correct to nearest metre).

Generally well done!

END OF PAPER