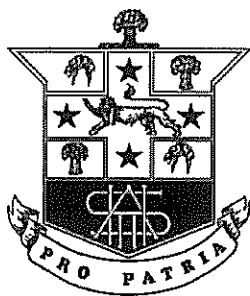


HURLSTONE AGRICULTURAL HIGH SCHOOL



MATHEMATICS

2013 HSC

ASSESSMENT TASK 1

Examiners ~ S Gee, G Rawson, B Morrison, S Faulds

GENERAL INSTRUCTIONS

- | | |
|---|--|
| <ul style="list-style-type: none">• Reading Time – 3 minutes.• Working Time – 40 MINUTES.• Attempt all questions.• All necessary working should be shown in every question.• This paper contains three (3) questions. | <ul style="list-style-type: none">• Marks may not be awarded for careless or badly arranged work.• Board approved calculators and Mathematical templates may be used.• Each question is to be started on a new piece of paper.• This examination paper must NOT be removed from the examination room. |
|---|--|

STUDENT NAME: _____

TEACHER: _____

Clearly label and place the correct response for each multiple choice question at the start of your answer booklet.

- (a) The line perpendicular to $4x - 3y + 5 = 0$ is:

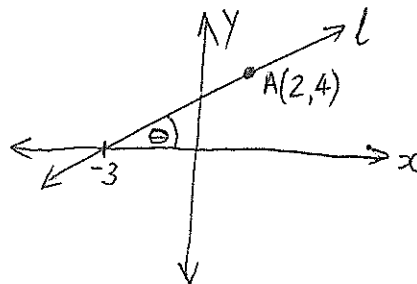
A: $4x - 3y + 8 = 0$

B: $4x + 3y - 7 = 0$

C: $3x - 4y = 0$

D: $3x + 4y + 1 = 0$

- (b) The diagram below shows a student's sketch of a straight line, l .



What is the size of the angle θ to the nearest degree? (The diagram is not to scale.)

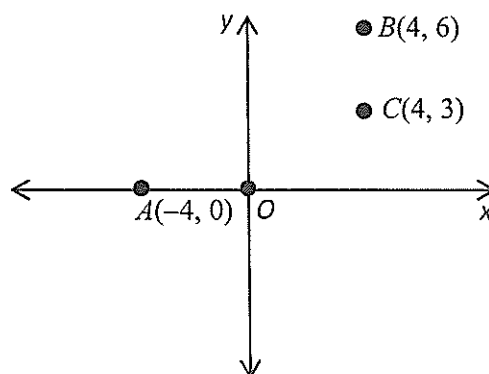
A: 16°

B: 39°

C: 74°

D: 143°

- (c) The number plane below shows the points $A(-4, 0)$, $B(4, 6)$, $C(4, 3)$ and the origin O .



- (i) Calculate the length of the intervals AB and OC .

2

- (ii) Show that the line AB has equation $3x - 4y + 12 = 0$.

2

- (iii) Show that $AB \parallel OC$. What type of quadrilateral is $OCBA$?

2

- (iv) Find the perpendicular distance of C from the line AB .

2

- (v) Calculate the area of the quadrilateral $OABC$.

2

Clearly label and place the correct response for each multiple choice question at the start of your answer booklet.

- (a) The quadratic equation $x^2 + 3x - 1 = 0$ has roots α and β .
What is the value of $\alpha\beta + (\alpha + \beta)$?

A: -4 B: -2 C: 2 D: 4

- (b) What is the value of k if the equation $2x^2 + 3x + k = 0$ has -3 as one of its roots?

A: -27 B: -9 C: 9 D: 27

- (c) (i) Solve the equation $2u^2 - 17u + 8 = 0$ 1

- (ii) Hence or otherwise, solve $2 \times 2^{2x} - 17(2^x) + 8 = 0$ 2

- (d) Find the values of m for which $m^2 - 4m - 12 < 0$. 2

- (e) For what values of k does $3x^2 - kx + 3 = 0$ have real roots? 2

- (f) For the quadratic equation $2x^2 - 3x + 4 = 0$ with roots α and β .
Find the value of :

- (i) $\alpha + \beta$ and $\alpha\beta$ 1

- (ii) $\alpha^2 + \beta^2$ 1

- (iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ 1

Clearly label and place the correct response for each multiple choice question at the start of your answer booklet.

(a) Which of the following parabolas has $x = 2$ as its axis of symmetry?

A: $(x + 2)^2 = 8(y - 3)$

B: $(y + 2)^2 = 8(x - 3)$

C: $(x - 2)^2 = 8(y + 3)$

D: $(y - 3)^2 = 8(x - 2)$

(b) The locus of a point that moves so that it is always equidistant from the points $(-2, 4)$ and $(4, 2)$ is given by the equation:

A: $3y - x = 0$

B: $3x - y = 0$

C: $3x - y + 2 = 0$

D: $3y - x + 2 = 0$

(c) Show that $x^2 - 4x + 12 = (x - 2)^2 + 8$

2

Hence or otherwise, by expressing the parabola $x^2 - 4x + 12 = 8y$ in the form

$$(x - h)^2 = 4a(y - k)$$

Find

(i) The vertex

1

(ii) The focal length

1

(iii) The focus

1

(iv) The equation of the directrix

1

(d) It is given that the point $P(x, y)$ moves so that it is always twice the distance from a point $M(2, 1)$ as it is from the point $N(5, 1)$.

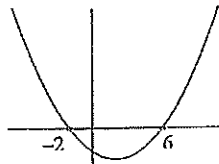
(i) Draw a clear diagram marking in all of the information.

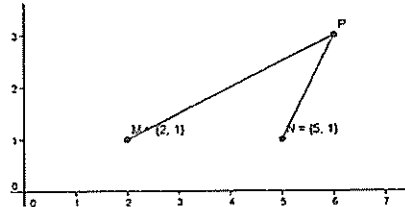
1

(ii) Find the locus of the point P .

3

Year 12 Mathematics Task 1 2013 (Term 4 2012)		
Question No. 1		Solutions and Marking Guidelines
Outcomes Addressed in this Question		
H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems		
Outcome	Solutions	Marking Guidelines
H5	(a) D	1 mark Correct answer
H5	(b) B	1 mark Correct answer
H5	(c) (i) $d_{AB} = \sqrt{8^2 + 6^2}$ $= \sqrt{100}$ $= 10 \text{ units}$ $d_{OC} = \sqrt{4^2 + 3^2}$ $= \sqrt{25}$ $= 5 \text{ units}$	2 marks Correct solution showing both distances correctly 1 mark Only one of the two distances calculated correctly
H5	(ii) $m_{AB} = \frac{6}{8}$ $= \frac{3}{4}$ Equation of AB $y - y_1 = m(x - x_1)$ $y - 0 = \frac{3}{4}(x + 4)$ $y = \frac{3}{4}x + 3$ $3x - 4y + 12 = 0$ as required	2 marks Correct solution 1 mark Substantial progress towards correct solution, including gradient of line AB.
H5	(iii) $m_{AB} = \frac{3}{4} \quad \text{Shown above}$ $m_{OC} = \frac{\text{rise}}{\text{run}}$ $= \frac{3}{4}$ $\therefore AB \parallel OC$ (gradients are equal) Quadrilateral OCBA is a trapezium (1 pair of opposite sides are parallel.)	2 marks Correct solution showing lines are parallel AND correctly names quadrilateral. 1 mark Shows only one part of the correct solution ie. Parallel lines OR name of quad.
H5	(iv) $d_{\text{perp.}} = \frac{ Ax_1 + By_1 + C }{\sqrt{A^2 + B^2}}$ $= \frac{ 3 \times 4 - 4 \times 3 + 12 }{\sqrt{3^2 + 4^2}}$ $= \frac{12}{5}$	2 marks Correct solution 1 mark Incorrect answer but correct substitution into correct formula
H5	(v) Area of Trapezium $A = \frac{1}{2}h(a+b)$ $= \frac{1}{2} \times \frac{12}{5} (10+5)$ $= \frac{12}{10} \times 15$ $= 18 \text{ units}^2$	2 marks Correct solution OR correct solution obtained from incorrect values from earlier parts of the question 1 mark Incorrect answer but correct substitution into correct formula

Year 11	Mathematics	2012	HSC Assessment Task 1
Question No. 2	Solutions and Marking Guidelines		
Outcomes Addressed in this Question			
P3 performs routine arithmetic and algebraic manipulation			
P4 chooses and applies appropriate arithmetic, algebraic, graphical and geometric techniques			
Outcome	Solutions	Marking Guidelines	
	(a) A	1 mark – correct answer	
	(b) B	1 mark – correct answer	
	(c) (i) $2u^2 - 17u + 8 = 0$ $(2u-1)(u-8) = 0$ $u = \frac{1}{2}, 8$	1 mark – correct solution	
	(ii) $2 \times 2^{2x} - 17(2^x) + 8 = 0$ $2u^2 - 17u + 8 = 0$ where $u = 2^x$ $u = 2^x = \frac{1}{2}, 8$ $x = -1, 3$	2 marks Correct solution 1 mark Partially correct solution	
	(d) $m^2 - 4m - 12 < 0$ $(m-6)(m+2) < 0$ $-2 < m < 6$ 	2 marks Correct solution 1 mark Partially correct solution	
	(e) $\Delta = b^2 - 4ac$ $= (-k)^2 - 4(3)(3)$ $= k^2 - 36 \geq 0$ for real roots ie $k \leq -6$ or $k \geq 6$	2 marks Correct solution 1 mark Partially correct solution	
	(f) (i) $\alpha + \beta = -\frac{b}{a} = \frac{3}{2}$ and $\alpha\beta = \frac{c}{a} = 2$	1 mark – correct answer	
	(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= \left(\frac{3}{2}\right)^2 - 2(2)$ $= -\frac{7}{4}$	1 mark – correct answer	
	(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$ $= \frac{3}{2} \div 2$ $= \frac{3}{4}$	1 mark – correct answer	

Year 12	Mathematics Extension 1	Assessment Task 1 2b.
Question No. 3	Solutions and Marking Guidelines	
Outcomes Addressed in this Question: H2 P4 H5		
Outcome	Solutions	Marking Guidelines
	(a) C: $(x-2)^2 = 8(y+3)$	<u>1 mark for C</u>
	(b) B: $3x - y = 0$	<u>1 mark for B</u>
	(c) $x^2 - 4x + 12 = x^2 - 4x + 4 + 8$ $= (x-2)^2 + 8$	<u>1 mark for first lin3</u> <u>2 marks for solution</u>
	(c) $x^2 - 4x + 12 = 8y$ becomes $(x-2)^2 + 8 = 8y$ and $(x-2)^2 = 8(y-1)$ (i) vertex = (2,1) (ii) focal length = 2 (iii) focus = (2,3) (iv) directrix is $y = -1$	<u>1 mark</u> <u>1 mark</u> <u>1 mark</u> <u>1 mark</u>
	(d) 	<u>1 mark</u>
	$ PM = 2 \times PN $ $\therefore PM ^2 = 4 PN ^2$ $(x-2)^2 + (y-1)^2 = 4\{(x-5)^2 + (y-1)^2\}$ $x^2 - 4x + 4 + y^2 - 2y + 1 = 4x^2 - 40x + 100 + 4y^2 - 8y + 4$ $0 = 3x^2 - 36x + 3y^2 - 6y + 99$ $0 = x^2 - 12x + y^2 - 2y + 33$ $0 = (x-6)^2 + (y-1)^2 - 4$ $(x-6)^2 + (y-1)^2 = 4$ Circle, centre (6,1) and radius 2	<u>1 mark</u> <u>1 mark</u> <u>1 mark</u> (for any version of the equation)