

Student Name: _____

Teacher: _____



HURLSTONE AGRICULTURAL HIGH SCHOOL

2021

Year 12
TASK 1

Mathematics Advanced

Examiners Mr. S. Faulds, Mr. D. Potaczala, Ms L. Yuen

General

Instructions

- Reading time – 3 minutes
- Working time – 40 minutes
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- A multiple choice answer sheet is provided for your use. This is included at the end of the booklet.

Total

marks: 33

Section 1 – 3 marks

- Attempt Questions 1-3
- Answer on the Multiple Choice answer sheet provided at the back of the question booklet.
- The Multiple Choice answer sheet may be detached from the question booklet and handed in separately.
- Allow about 5 minutes for this section.

Section 2 – 30 marks

- Attempt Questions 4-6
- In Questions 4-6, answer in the spaces provided, showing relevant mathematical reasoning and/or calculations
- Allow about 35 minutes for this section

Written by 2021 HAHS Maths Faculty, rewritten by Ariq Abdullah

Section 1

Attempt Questions 1-3

Allow about 5 minutes from this section.

Question 1

The following probability distribution has a mean of 1

X	0	1	2
$Pr(X = x)$	a	b	0.4

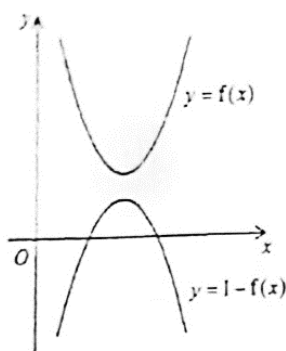
What are the values of a and b ?

- a. $a = 0.5$ and $b = 0.1$
- b. $a = 0.2$ and $b = 0.4$
- c. $a = 0.4$ and $b = 0.2$
- d. $a = 0.1$ and $b = 0.5$

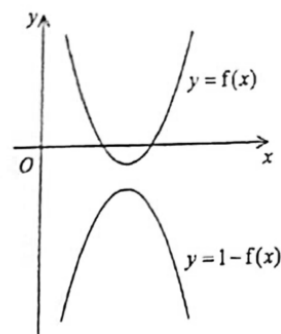
Question 2

Which of the following may represent the graph of $y = f(x)$ and the graph of $y = 1 - f(x)$ on the same cartesian plane?

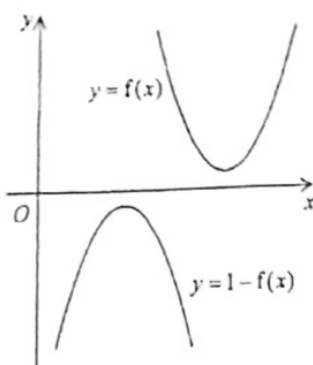
a.



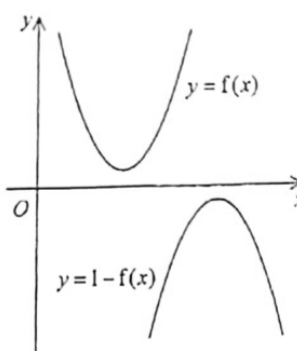
c.



b.



d.



Question 3

The formula for loudness is $L = 10 \log_{10} \left(\frac{I}{I_0} \right)$ where I_0 is the intensity of the threshold sound and L is measured in decibels (dB).

What is the intensity of the sound (I) in terms of I_0 if loudness is measured as 25dB?

- a. $0.003I_0$
- b. $0.04I_0$
- c. $25I_0$
- d. $316I_0$

Section 2

30 marks

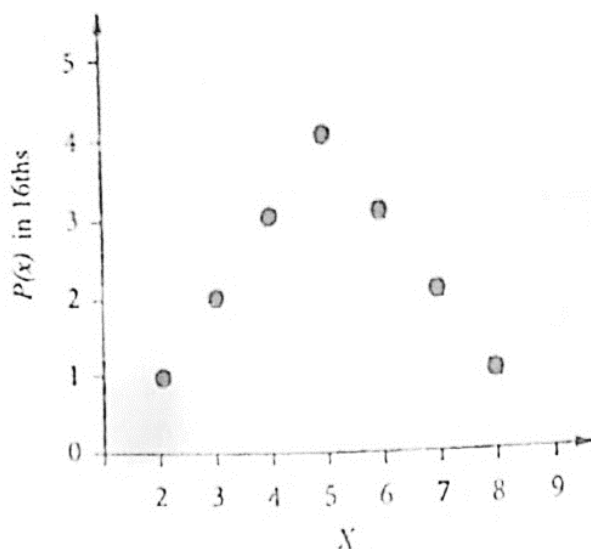
Attempt Questions 4-6

Allow about 35 minutes for this section

Question 4 (10 Marks)

Marks

- (a) Explain the difference between the sample mean and population mean. 1
- (b) Find the expected value and standard deviation from the discrete probability distribution shown below. 3



- (c) A random variable X is defined by:

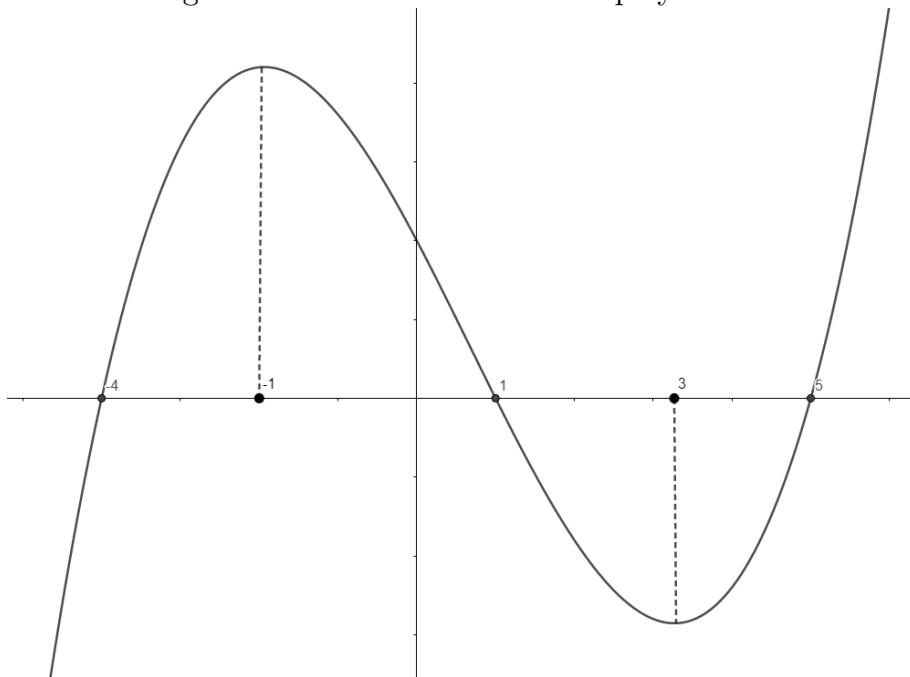
$$P(X = x) = \begin{cases} \frac{1}{3} & \text{if } x = -1 \\ \frac{1}{2} & \text{if } x = 1 \\ \frac{1}{6} & \text{if } x = 2 \end{cases}$$

Find:

- (i) $E(X)$ 1
- (ii) $E(3X - 4)$ 1
- (iii) $E(X^2)$ 1
- (d) An insurance company covers a \$20 000 sapphire ring against theft and loss. A payout is \$8000 is made to the owner if the ring is lost and if the ring is stolen it is paid out in full. From data collected, in a twelve-month period, there is a 0.25% chance the ring will be stolen and a 3% chance it will be lost.
- (i) Find the probability distribution of the cost of a payout. 1
- (ii) Using part (i) or otherwise, find how much the insurance company should charge to have an expected return of \$100. 2

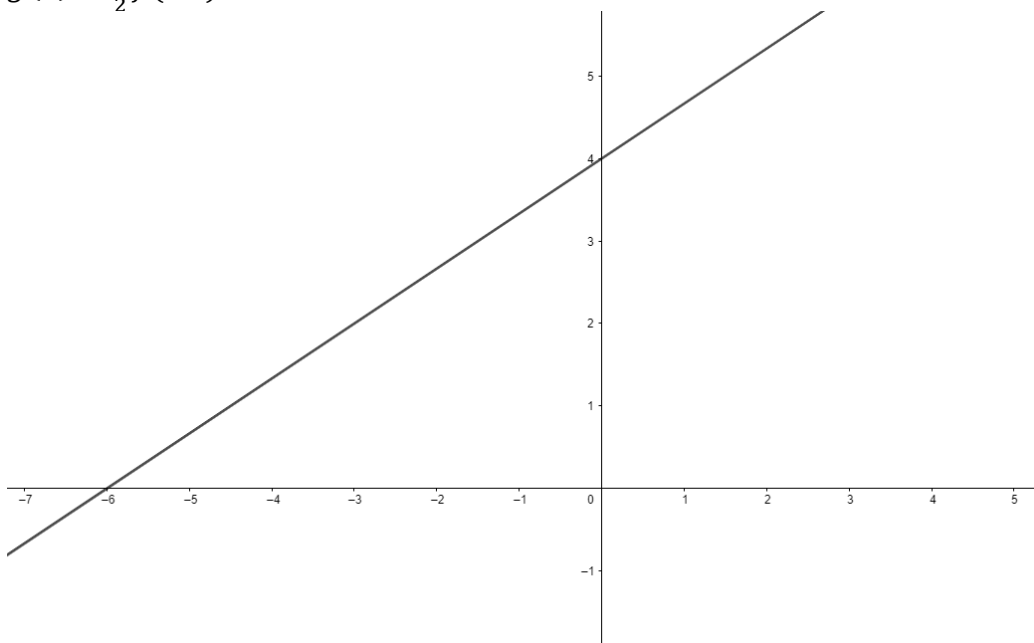
Question 5 (10 marks)**Marks**

- (a) The following function sketched below is a polynomial.

1

The transformation $f(x - 1)$ is applied. Sketch the transformed function on the same Cartesian number plane above.

- (b) The point $A(-8,0)$ lies on the graph of the function $y = f(x)$. A transformation maps the graph of $y = f(x)$ to the graph of $y = g(x)$, where $g(x) = \frac{1}{2}f(-x)$.



- (i) Sketch the graph of the function $y = g(x)$ on the Cartesian number plane above.
- (ii) The same transformation maps the point A to the point P . Find the coordinates of the point P .

1**1****Question 5 Continued On Next Page**

- (c) Let $f(x) = -x^2$ and $g(x) = x - 2$.
- (i) Draw an accurate graph of $y = f(x)$ and $g(x)$ between $-3 \leq x \leq 3$. **2**
- (ii) By using the graph in part (i), find when $f(x) > g(x)$. **1**
- (d) For the function $f(x) = \frac{x+1}{x}$.
- (i) Show that $f(x) = 1 + \frac{1}{x}$ **1**
- (ii) Hence, sketch $y = f(x)$, showing all important features. **2**
- (iii) Using (ii), find the number of solution(s) to $1 + \frac{1}{x} = -x$. Justify your answer. **1**

Question 6 (10 marks)

Marks

- (a) Solve $\log_4(7 - x) - \log_4(2 - x) = \frac{1}{2}$ for x , where $x < 2$. **2**
- (b) Differentiate the exponential function $f(x) = e^{4x+12}$, and hence, evaluate $f'(-3)$. **2**
- (c) If $f(x) = 1 + e^x$, show that $f(2x - 1) \times f(1 - 2x) = f(2x - 1) + f(1 - 2x)$. **2**
- (d) The temperature inside an industrial oven increases according to the formula $T = 15 + 10e^{1.6t}$, where T is the temperature in degrees Celsius and t is the time in hours since the oven was started.
- (i) What is the initial temperature of the oven? **1**
- (ii) What is the temperature in the oven after 2 hours, correct to 2 decimal places? **1**
- (iii) When the oven temperature reaches 500°C , the oven shuts down to cool. **2**
After how many hours does this happen? (Answer to the nearest 0.1h)

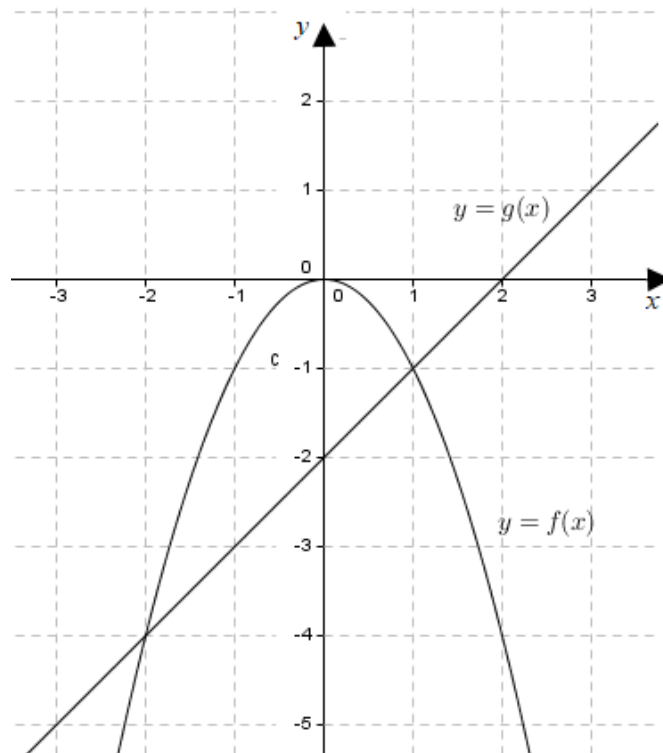
End of Exam

Year 12 Mathematics HSC Assessment Task 1 2021	
Multiple Choice Questions No. 1, 2, 3	Solutions
Outcomes Addressed in these Questions	
MA11-6	manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems (Question 3)
MA11-7	uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions. (Question 1)
MA12-1	uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts. (Question 2)
1. C	
2. A	
3. D	

Year 12	Mathematics Advanced	Task 1	HSC 2021
Question No.4	Solutions and Marking Guidelines		
MA11-7 uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions.			
Part	Solutions	Marking Guidelines	
(a)	The population mean is the expected value of a dataset of a population. While sample mean is the expected value of a dataset that is a sample of the population; it is an approximation of the population mean.	1 mark Complete correct definition	
(b)	$E(X) = 2 \times \frac{1}{16} + 3 \times \frac{2}{16} + 4 \times \frac{3}{16} + 5 \times \frac{4}{16} + 6 \times \frac{3}{16} + 7 \times \frac{2}{16} + 8 \times \frac{1}{16}$ $= \frac{80}{16}$ $= 5$ $V(X) = E(X^2) - E(X)^2$ $= 2^2 \times \frac{1}{16} + 3^2 \times \frac{2}{16} + 4^2 \times \frac{3}{16} + 5^2 \times \frac{4}{16} + 6^2 \times \frac{3}{16} + 7^2 \times \frac{2}{16} + 8^2 \times \frac{1}{16} - 5^2$ $= \frac{5}{2}$ $\sigma = \sqrt{V(X)}$ $= \sqrt{2.5}$ $= 1.58$	3 marks Correct solution 2 marks Substantially correct 1 mark Demonstrated correct calculation of part of the solution	
(c) i	$E(X) = (-1) \times \frac{1}{3} + 1 \times \frac{1}{2} + 2 \times \frac{1}{6}$ $= \frac{1}{2}$	1 mark Correct solution	
ii	$E(3X - 4) = (3(-1) - 4) \times \frac{1}{3} + (3(1) - 4) \times \frac{1}{2} + (3(2) - 4) \times \frac{1}{6}$ $= -\frac{5}{2}$	1 mark Correct solution	
iii	$E(X^2) = (-1)^2 \times \frac{1}{3} + 1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{6}$ $= \frac{3}{2}$	1 mark Correct solution	

(d) i	<table><tr><td>x</td><td>0</td><td>8000</td><td>20000</td></tr><tr><td>$P(X = x)$</td><td>0.9675</td><td>0.03</td><td>0.0025</td></tr></table>	x	0	8000	20000	$P(X = x)$	0.9675	0.03	0.0025	1 mark Correct solution
	x	0	8000	20000						
$P(X = x)$	0.9675	0.03	0.0025							
	Note: Random variables have numbers associated with outcomes (i.e. x should equal a number). Also, a probability distribution has all outcomes listed and the sum of their probabilities equal one.									
ii	Expected cost of an insurance policy $E(X) = 0 \times 0.9675 + 8000 \times 0.030 + 20000 \times 0.0025$ $= 290$ [In other words, on average an insurance policy will cost the insurance company \$290 (some policies costs more, some cost less)] To get an expected return of \$100 the insurance company needs to charge \$390.	2 marks Correct solution 1 mark Substantially correct								

Year 12 HSC 2021		Mathematics Advanced	HSC Task 1
Question No. 5		Solutions and Marking Guidelines	
Outcomes Addressed in this Question			
MA12-1 uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts			
Outcome	Solutions		Marking Guidelines
MA12-1	a)		1 mark for correct graphing
MA12-1			1 mark for correct graphing
MA12-1	b i)		1 mark for correct answer
MA12-1			2 marks for correct graphing 1 mark for substantial progress
MA12-1	ii) $P(8,0)$		1 mark for correct solution
	c i)		1 mark for correctly shown



2 marks for
correct
graphing

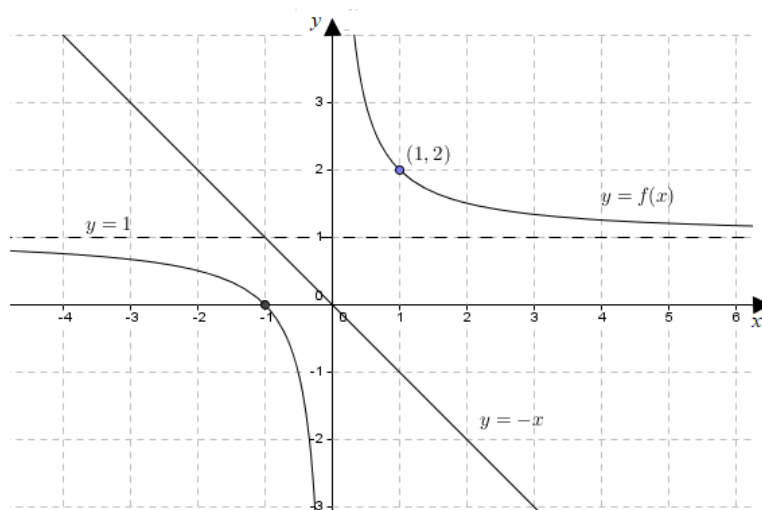
1 mark for
substantial
progress

1 mark for
correct
justification

ii) $-2 < x < 1$

$$\begin{aligned} \text{d i)} \quad f(x) &= \frac{x+1}{x} \\ &= \frac{x}{x} + \frac{1}{x} \\ &= 1 + \frac{1}{x} \end{aligned}$$

ii)



iii) zero number of solutions as the two graphs have no intersecting points. (Q asked for number of solution(s)), therefore there is a difference between zero and no solution)

Year 12 Mathematics HSC Assessment Task 1 2021		
Question No. 6 Solutions and Marking Guidelines		
Outcomes Addressed in this Question		
MA11-6 manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems		
Outcome	Solutions	Marking Guidelines
MA11-6	(a) $\log_4(7-x) - \log_4(2-x) = \frac{1}{2}$ $\log_4\left(\frac{7-x}{2-x}\right) = \frac{1}{2}$ $\frac{7-x}{2-x} = 4^{\frac{1}{2}}$ $7-x = 2(2-x)$ $= 4-2x$ $x = -3$	2 marks Correct solution. 1 mark Substantial progress towards correct solution.
H5	(b) Using the rule $\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$, $f(x) = e^{4x+12}$ $f'(x) = 4e^{4x+12}$ $f'(-3) = 4e^{4 \times -3 + 12}$ $= 4e^0$ $= 4$	2 marks Correct solution. 1 mark Correctly differentiates the function OR evaluates derivative correctly from an incorrect derivative of equal difficulty.
H5		
H6	(c) $L.H.S = (1+e^{2x-1})(1+e^{1-2x})$ $= 1+e^{1-2x}+e^{2x-1}+e^0$ $= 2+e^{1-2x}+e^{2x-1}$ $R.H.S = 1+e^{2x-1}+1+e^{1-2x}$ $= 2+e^{1-2x}+e^{2x-1}$ $= L.H.S$ as required	2 marks Correct solution. 1 mark Substantial progress towards correct solution.
H5	(d)(i) $T = 15 + 10e^{1.6t}$ when $t = 0$ $T = 15 + 10e^{1.6 \times 0}$ $= 15 + 10$ $= 25^\circ C$	1 mark Correct answer.
H5	(ii) After 2 hours $T = 15 + 10e^{1.6t}$ $= 15 + 10e^{1.6 \times 2}$ $= 15 + 10e^{3.2}$ $= 260.33^\circ C$	1 mark Correct answer.
H5		

H5 H8 H8	(iii) When $T = 500^\circ$, $500 = 15 + 10e^{1.6t}$ $485 = 10e^{1.6t}$ $48.5 = e^{1.6t}$ $\ln 48.5 = \ln e^{1.6t}$ $\ln 48.5 = 1.6t$ $t = \frac{\ln 48.5}{1.6}$ $= 2.4\text{h} \quad (\text{to nearest } 0.1\text{h})$	2 marks Correct solution. 1 mark Substantial progress towards correct solution.
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