

STUDENT NAME: _____

TEACHER: _____



HURLSTONE
AGRICULTURAL
HIGH
SCHOOL

2021

Year 11
Assessment Task 1

Mathematics Advanced

Examiners

- **Ms T Tarannum**, Ms M Sabah, Mr S Faulds.

**General
Instructions**

- Working time – 40 minutes
- Reading time – 3 minutes
- Write using blue or black pen
- NESA-approved calculator may be used
- A Reference sheet is provided
- In Questions 4 – 6, show relevant mathematical reasoning and/or calculations

**Total marks:
30****Section I – 3 marks**

- Attempt Questions 1 – 3
- Allow about 3 minutes for this section

Section II – 27 marks

- Attempt Questions 4 – 6
- Allow about 37 minutes for this section

SECTION I

3 marks

Attempt Questions 1 – 3

Allow about 3 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 3

Question 1

What is the value of $x + y$ if $5x + 8y = 67$ and $2x - y = 31$?

- A. 14
- B. 16
- C. 15
- D. 0

Question 2

If $\sin x = \frac{3}{4}$, then which of the following is the value of $\cos x$ for $0^\circ \leq x \leq 90^\circ$?

- A. $\frac{2}{3}$
- B. $\frac{\sqrt{3}}{2}$
- C. $\frac{\sqrt{7}}{4}$
- D. $\frac{1}{2}$

Question 3

Two functions are defined as follows:

$$g(x) = 5 - 4x$$

$$h(x) = (x - 1)^2$$

What is the correct expression for the composite function $g(h(x))$?

- A. $1 - 4x^2$
- B. $x^2 - x + 1$
- C. $1 + 8x - 4x^2$
- D. $16x^2 - 16x + 16$

**Mathematics Advanced**
Mathematics Extension 1
Mathematics Extension 2**REFERENCE SHEET****Measurement****Length**

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

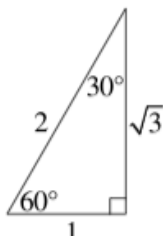
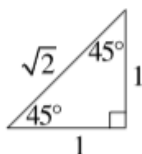
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1 + t^2}$$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

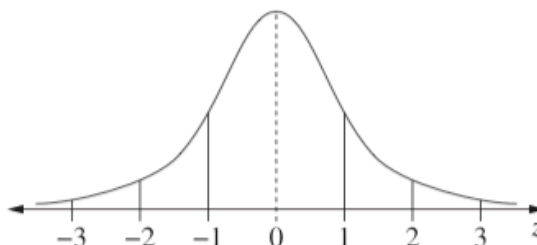
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z -scores between -1 and 1
- approximately 95% of scores have z -scores between -2 and 2
- approximately 99.7% of scores have z -scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function

Derivative

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

Integral Calculus

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \left\{ f(a) + f(b) + 2[f(x_1) + \dots + f(x_{n-1})] \right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^n P_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underline{u}| = |x\underline{i} + y\underline{j}| = \sqrt{x^2 + y^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1 x_2 + y_1 y_2,$$

$$\text{where } \underline{u} = x_1 \underline{i} + y_1 \underline{j}$$

$$\text{and } \underline{v} = x_2 \underline{i} + y_2 \underline{j}$$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

Complex Numbers

$$\begin{aligned} z &= a + ib = r(\cos \theta + i \sin \theta) \\ &= r e^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos \theta + i \sin \theta)]^n &= r^n (\cos n\theta + i \sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2 x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

Student Name _____

Teacher _____

Select the letter A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $3 + 5 =$ (A) 7 (B) 8 (C) 9 (D) 10

A B C D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A  B  C  D 

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word '*correct*' and drawing an arrow as follows:

A  B  C  D 

Question

1. **A**  **B**  **C**  **D** 
2. **A**  **B**  **C**  **D** 
3. **A**  **B**  **C**  **D** 

Question	Marks
MC	
4	
5	
6	
Total	30

STUDENT NAME: _____

TEACHER: _____

Q 4

SECTION II

27 marks

Attempt Questions 4 – 6

Allow about 37 minutes for this section.

In Questions 4 – 6, your responses should include relevant mathematical reasoning and/or calculations.

- **Write your name at the top of each booklet.** Answer the questions in the space provided. These spaces provide guidance for the expected length of response.
- Extra writing space is provided at the back of each question booklet. If you use this space, clearly indicate which part of the question you are answering.

Question 4 (8 marks)

- (a) Rationalise the denominator $\frac{3}{2-\sqrt{2}}$. **2**

- (b) Factorise completely $4x^2 - 12xy + 9y^2 - 49z^2$ **2**

Question 4 continues on page 10

(c) Solve by completing the square $2x^2 + 7x + 6 = 0$

3

(d) Given that $9^p \times 27^q = 3^n$. Express n in terms of p and q .

1

End of Question 4

Extra writing space.

If you use this space, clearly indicate which Question/part you are answering.

[illegible]

Extra writing space.

If you use this space, clearly indicate which Question/part you are answering.

[illegible]

STUDENT NAME: _____

Q 5

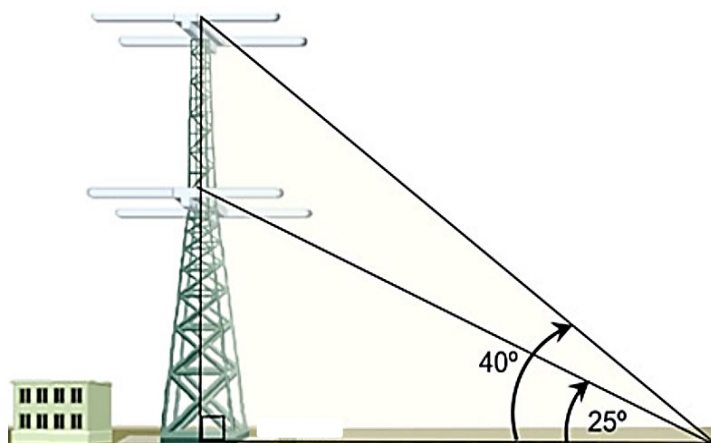
TEACHER: _____

- **Write your name at the top of each booklet.** Answer the questions in the space provided. These spaces provide guidance for the expected length of response.
- Extra writing space is provided at the back of each question booklet. If you use this space, clearly indicate which part of the question you are answering.

Question 5 (5 marks)

- (a) A radio station tower was built in two sections. From a point 87 m from the base of the tower, the angle of elevation of the top of the first section is 25° , and the angle of elevation of the top of the second section is 40° . To the nearest metre, what is the height of the top section of the tower?

3



87 m

Question 5 continues on page 14

(b) What is the **exact** value of $\operatorname{cosec}60^\circ - \cot 60^\circ$?

2

End of Question 5

Extra writing space.

If you use this space, clearly indicate which Question/part you are answering.

[illegible]

Extra writing space.

If you use this space, clearly indicate which Question/part you are answering.

[illegible]

STUDENT NAME: _____

TEACHER: _____

Q 6

- **Write your name at the top of each booklet.** Answer the questions in the space provided. These spaces provide guidance for the expected length of response.
- Extra writing space is provided at the back of each question booklet. If you use this space, clearly indicate which part of the question you are answering.

Question 6 (14 marks)

- (a) Express the quadratic function $y = x^2 - 8x + 10$ in the form $y = (x - a)^2 + b$. **2**

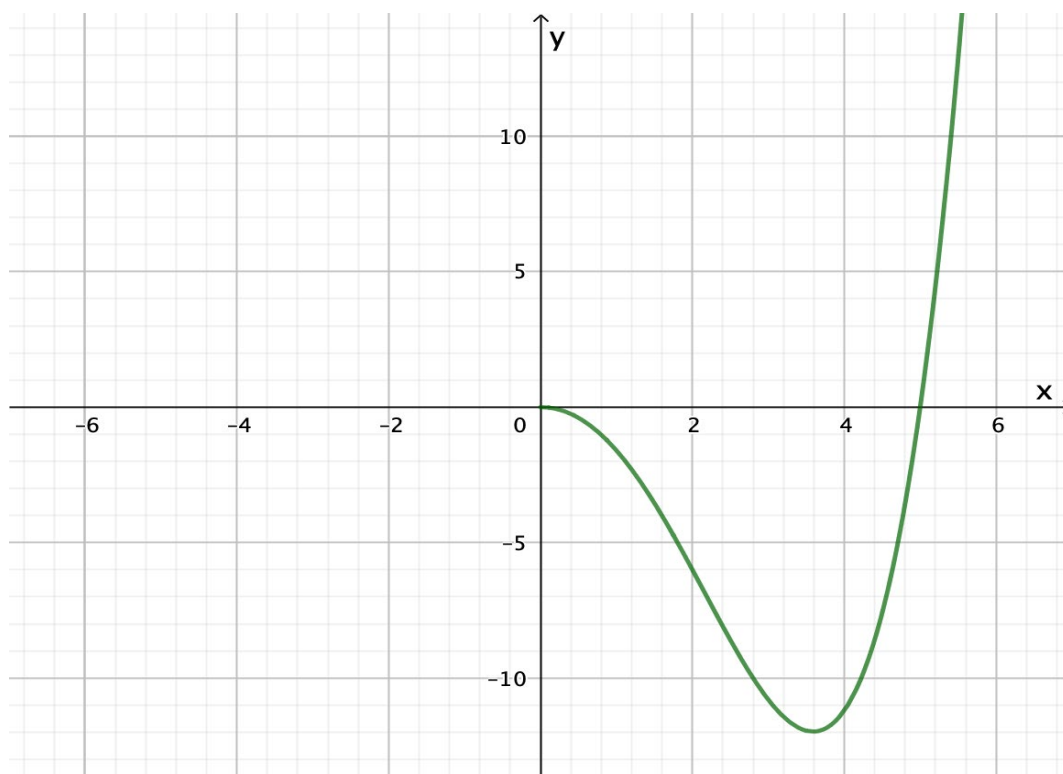
- (b) Neatly sketch the parabola from part (a) in the space below, clearly showing the coordinates of the vertex and y intercept. **2**

Question 6 continues on page 18

(c) Each of the diagrams below show an incomplete graph of a function. Complete each graph if the function is:

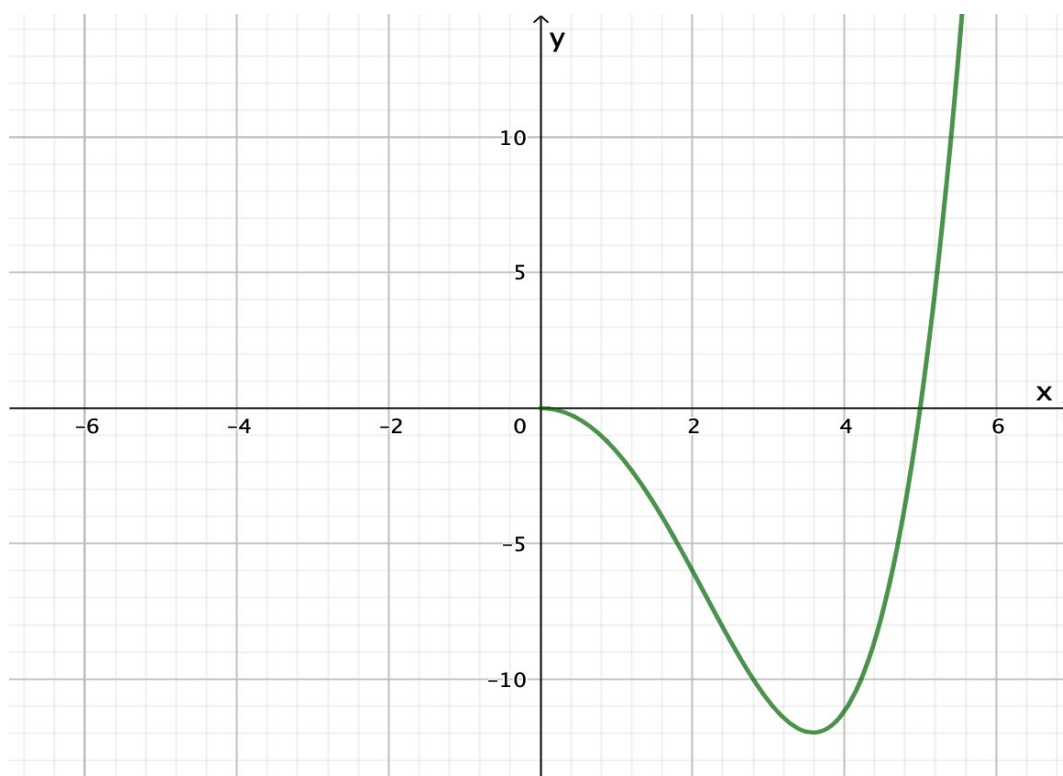
(i) odd

1



(ii) Even

1

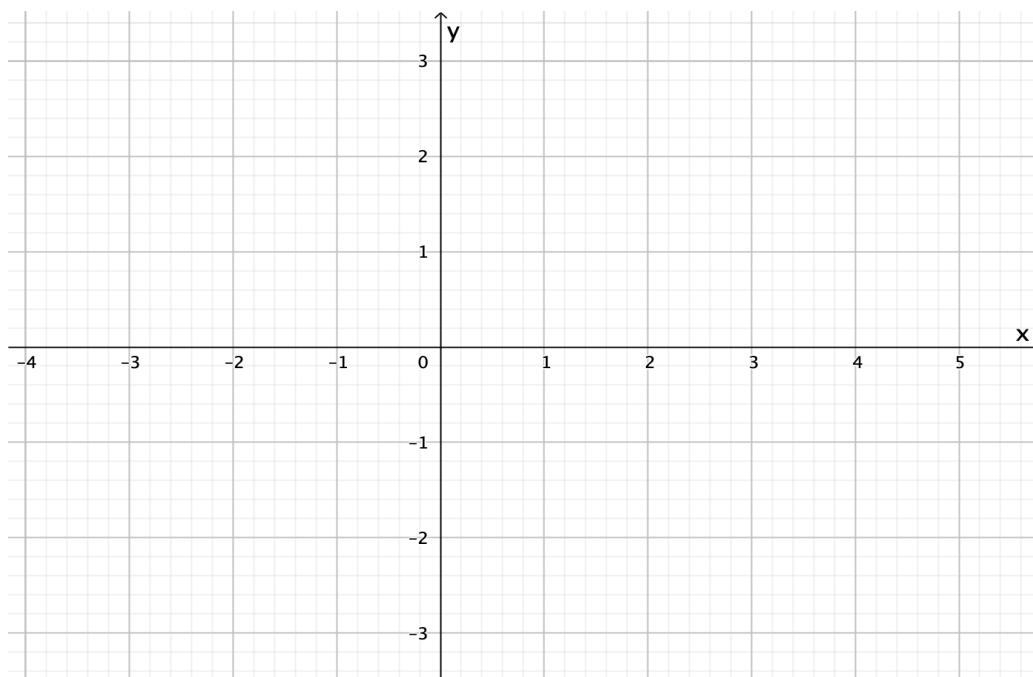


Question 6 continues on page 19

(d) Draw a neat sketch of each graph in the space provided, stating it's domain and range.

(i) $x^2 + y^2 = 4$

2

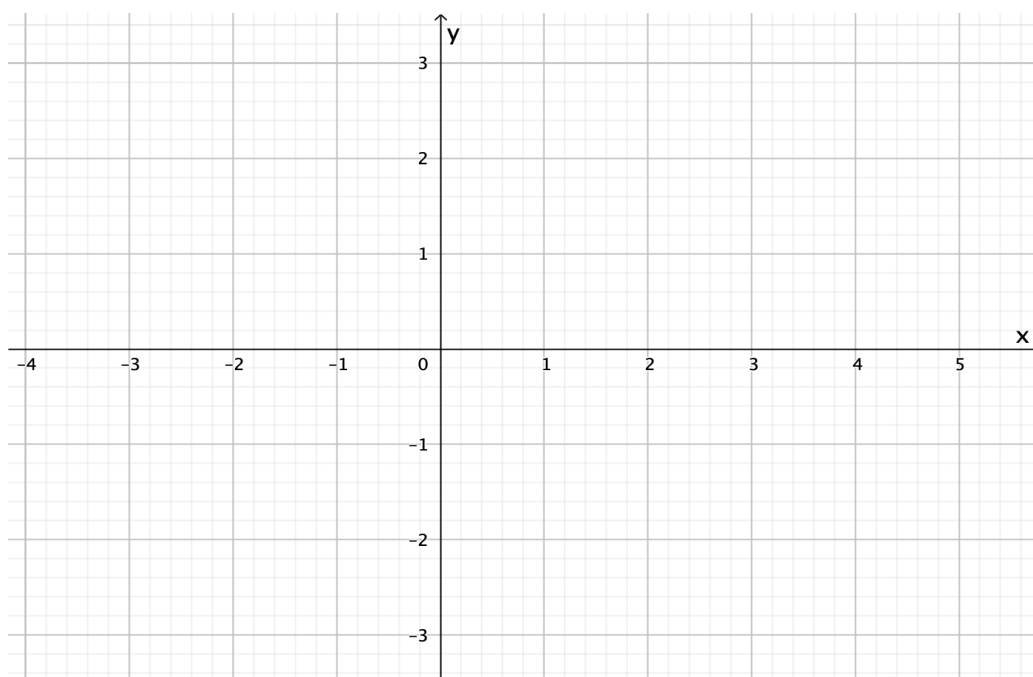


Domain:

Range:

(ii) $y = -\frac{2}{x}$

2



Domain:

Range:

Question 6 continues on page 20

- (iii) With reference to the graphs in (i) and (ii) above, explain why the graph in (i) represents a relation and the graph in (ii) represents a function.

2

- (e) Solve the equation $|2x - 8| = 6$.

2

End of Question 6

End of Paper

Extra writing space.

If you use this space, clearly indicate which Question/part you are answering.

[illegible]

Extra writing space.

If you use this space, clearly indicate which Question/part you are answering.

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Outcomes Addressed in this Question

MA11-1

uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems

Solutions	Marking Guidelines
a) $\frac{3}{2-\sqrt{2}}$ $= \frac{3}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}}$ $= \frac{3(2+\sqrt{2})}{4-2}$ $= \frac{3(2+\sqrt{2})}{2}$ b) $4x^2 - 12xy + 9y^2 - 49z^2$ $= (4x^2 - 12xy + 9y^2) - 49z^2$ $= (2x - 3y)^2 - 49z^2$ $= (2x - 3y)^2 - (7z)^2$ $= (2x - 3y - 7z)(2x - 3y + 7z)$ c) $2x^2 + 7x + 6 = 0$ $x^2 + \frac{7}{2}x = -3$ $x^2 + \frac{7}{2}x + \left(\frac{7}{4}\right)^2 = -3 + \left(\frac{7}{4}\right)^2$ $\left(x + \frac{7}{4}\right)^2 = \frac{1}{16}$ $x + \frac{7}{4} = \pm \sqrt{\frac{1}{16}}$ $x + \frac{7}{4} = \pm \frac{1}{4}$ $x = -\frac{7}{4} \pm \frac{1}{4}$ $x = -\frac{3}{2} \text{ or } x = -2$	Award 2 marks for the correct solution. Award 1 mark for substantial progress towards the solution Award 2 marks for the correct solution. Award 1 mark for substantial progress towards the solution Award 3 marks for the correct solution. Award 2 mark for substantial progress towards the correct solution. Award 1 mark for some progress towards the correct solution.

d) $9^p \times 27^q = 3^n$ $(3^2)^p \times (3^3)^q = 3^n$ $3^{2p} \times 3^{3q} = 3^n$ $3^{2p+3q} = 3^n$ $\therefore n = 2p+3q$	Award 1 mark for the correct solution.
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Multiple Choice Answers

1	A
2	C
3	C

Outcomes Addressed in this Question

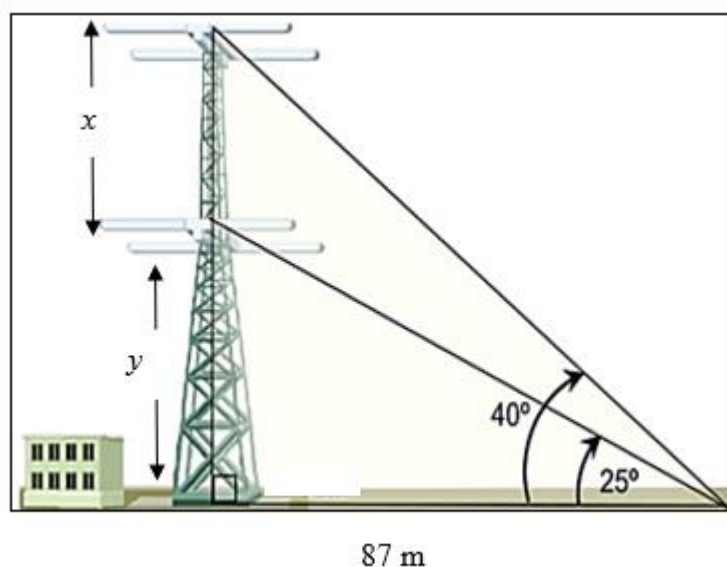
MA11-3

uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes

Solutions

Marking Guidelines

a)



Award 3 marks for the correct solution.

Award 2 mark for substantial progress towards the correct solution.

Award 1 mark for some progress towards the correct solution.

$$\frac{y}{87} = \tan 25^\circ$$

$$y = 87 \tan 25^\circ$$

$$\frac{x+y}{87} = \tan 40^\circ$$

$$x+y = 87 \tan 40^\circ$$

$$x = 87 \tan 40^\circ - y$$

$$x = 87 \tan 40^\circ - 87 \tan 25^\circ$$

$$x = 32.43290165$$

$$x = 32m \text{ (nearest metre)}$$

b)

$$\operatorname{cosec} 60^\circ - \cot 60^\circ$$

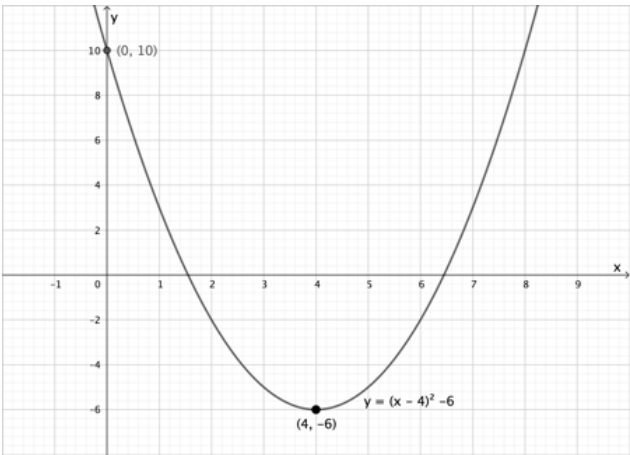
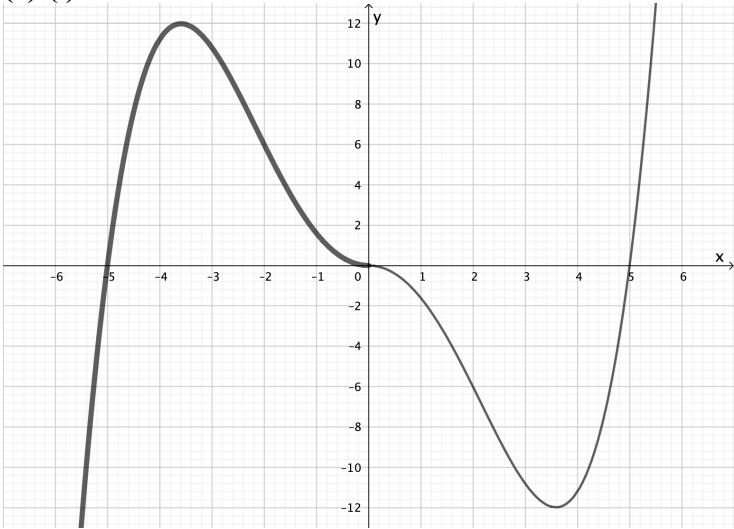
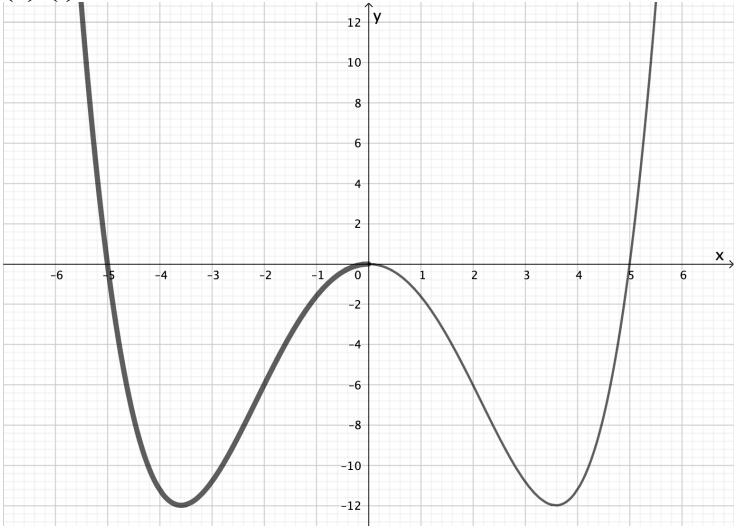
$$= \frac{1}{\sin 60^\circ} - \frac{1}{\tan 60^\circ}$$

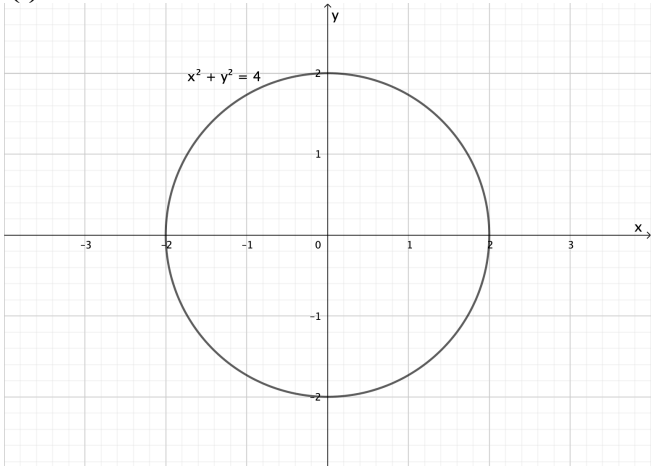
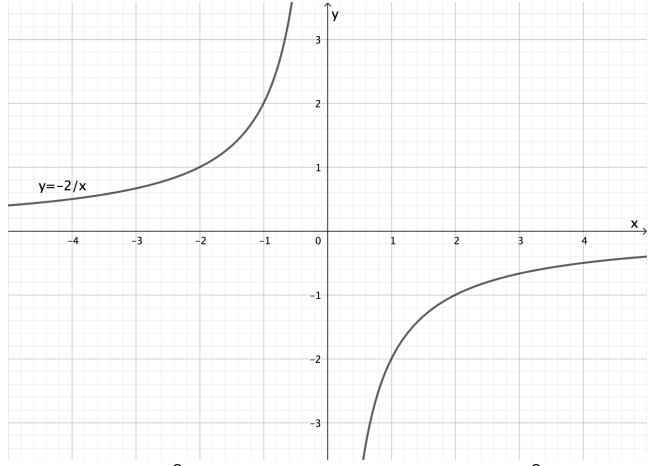
$$= \frac{1}{\frac{\sqrt{3}}{2}} - \frac{1}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{3}$$

Award 2 marks for the correct solution.

Award 1 mark for substantial progress towards the solution

Year 11 Mathematics Half Yearly Assessment (Task 1) 2021		
Question No. 9		Solutions and Marking Guidelines
Outcomes Addressed in this Question		
MA11-2 uses the concepts of functions and relations to model, analyse and solve practical problems		
Outcome	Solutions	Marking Guidelines
MA11-2	(a) $y = x^2 - 8x + 10$ $= (x^2 - 8x + 16) - 16 + 10$ $= (x - 4)^2 - 6$	2 marks Correct solution. 1 mark Attempts to complete the square or equivalent merit.
MA11-2	(b) 	2 marks Correct solution indicating y-intercept and vertex. 1 mark Correct graph only OR incorrect graph with correct vertex and y intercept.
MA11-2	(c) (i) 	1 mark Correct solution.
MA11-2	(c) (i) 	1 mark Correct solution.

MA11-2	(d) (i)  $-2 \leq x \leq 2$ $-2 \leq y \leq 2$ Domain: <i>OR</i> Range: <i>OR</i> $x : x \in [-2, 2]$ $y : y \in [-2, 2]$	2 marks Correct graph AND correct domain and range. 1 mark Incorrect graph but correct domain and range for drawn graph OR correct graph but incorrect domain and range.
MA11-2	(ii)  $x : x \neq 0$ $y : y \neq 0$ Domain: <i>OR</i> Range: <i>OR</i> $(-\infty, 0) \cup (0, \infty)$ $(-\infty, 0) \cup (0, \infty)$	2 marks Correct graph AND correct domain and range. 1 mark Incorrect graph but correct domain and range for drawn graph OR correct graph but incorrect domain and range.
MA11-2	(iii) Graph (i) is a relation because: * it fails the vertical line test * it is one-many * it is many-many Graph (ii) is a function because: * it satisfies the requirements of the vertical line test * it is one -one	2 marks Correct explanation for both. 1 mark Correct explanation for only one.
MA11-2	(f) $2x - 8 = 6$ $2x - 8 = 6$ $2x = 14$ $x = 7$ OR $2x - 8 = -6$ $2x = 2$ $x = 1$	2 marks Correct solution (both equations). 1 mark Only one correct solution OR attempts to apply correct process but makes error.