Student Name: ______________________________

Teacher Name: ______________________________

2023

Hurlstone Agricultural High School
Year 12
HSC Assessment Task 2

Mathematics Advanced

General Instructions

- **Senior Examiner:** G. Huxley
- Reading time – 5 minutes
- Working time – 60 minutes
- Write using black or blue pen
- NESA approved calculators may be used
- A Mathematics reference sheet is provided at the back of this paper
- Your handwritten notes on one A4 double sided page is permitted
- For all Questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 45

Section I – 6 marks

- Attempt Questions 1 – 6
- Allow about 10 minutes for this section

Section II – 39 marks

- Attempt Questions 7 – 9 in the space provided
- Allow about 50 minutes for this section

Section I

6 marks
Attempt Questions 1 – 6
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 6.

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1. What is the range of the function $y = 2 + 4\sin\left(3x - \frac{\pi}{4}\right)$?

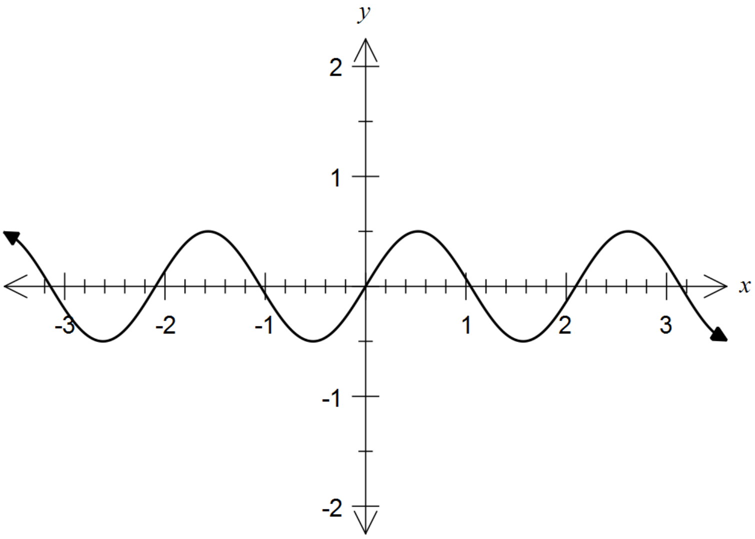
A. $-2 \leq y \leq 2$

B. $-4 \leq y \leq 4$

C. $-1 \leq y \leq 5$

D. $-2 \leq y \leq 6$

2. What function would describe the graph below?



A. $y = \frac{1}{2}\cos(3x)$

B. $y = \frac{1}{2}\sin(3x)$

C. $y = \frac{1}{2}\tan(3x)$

D. $y = \frac{1}{3}\sin(2x)$

Section I continues on next page ...

3. What is the primitive of $\frac{1}{x^4}$?

A. $-\frac{4}{x^5}+C$

B. $-\frac{1}{3x^4}+C$

C. $-\frac{1}{3x^3}+C$

D. $-\frac{1}{x^3}+C$

4. Let $g(x)=1-f(x)$. If $\int_1^4 f(x)dx=7$. What is the value of $\int_1^4 g(x)dx$?

A. -6

B. -4

C. 8

D. 10

5. Given $f(x)=\sin 3x$, the value of $f'\left(\frac{\pi}{9}\right)$ is given by which of the following?

A. $-\frac{3}{2}$

B. $-\frac{3\sqrt{3}}{2}$

C. $\frac{3\sqrt{3}}{2}$

D. $\frac{3}{2}$

6. $\int \cos x^o\, dx$ is given by which of the following expressions?

A. $\sin x^o + c$

B. $\frac{180}{\pi}\sin x^o + c$

C. $\frac{\pi}{180}\sin x^o + c$

D. $\pi \sin x^o + c$

~ End of Section I ~

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

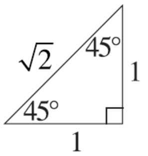
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

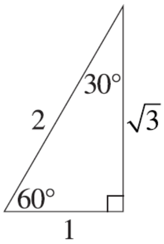
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

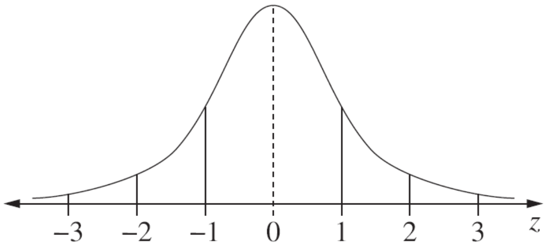
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n - r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n - x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} $$

$$ \binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!} $$

$$ (x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n} $$

Vectors

$$ \left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}} $$

$$ \underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2}, $$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$ \underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b} $$

Complex Numbers

$$ \begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned} $$

$$ \begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned} $$

Mechanics

$$ \frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right) $$

$$ x = a\cos(nt+\alpha) + c $$

$$ x = a\sin(nt+\alpha) + c $$

$$ \ddot{x} = -n^{2}(x-c) $$

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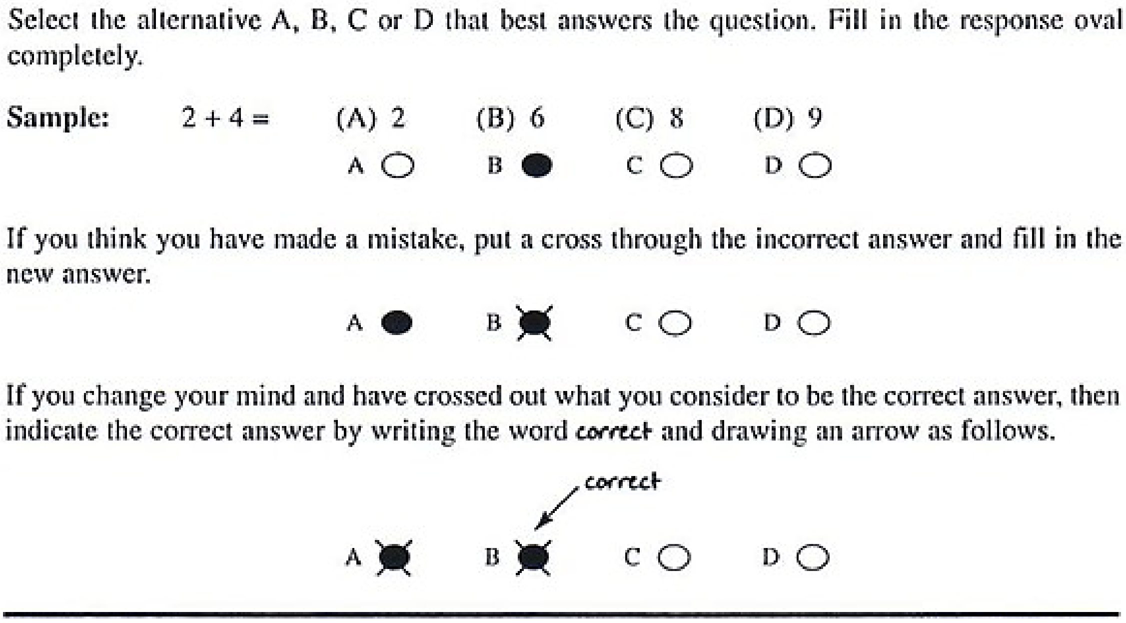
HSC Assessment Task 2

Mathematics Advanced

Multiple-Choice Answer Sheet



Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ☒ B ☒ C ○ D ○

Question

1. **A** ○ **B** ○ **C** ○ **D** ○

2. **A** ○ **B** ○ **C** ○ **D** ○

3. **A** ○ **B** ○ **C** ○ **D** ○

4. **A** ○ **B** ○ **C** ○ **D** ○

5. **A** ○ **B** ○ **C** ○ **D** ○

6. **A** ○ **B** ○ **C** ○ **D** ○

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Hurlstone Agricultural High School
Year 12
HSC Assessment Task 2

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Mathematics Advanced
Section II

Question 7
(13 Marks)

Applications of Differentiation and Graphing Trigonometric Functions

- Answer each question in the space provided.
- Your responses should include relevant mathematical reasoning and/or calculations.

(a)

The function $f(x) = x\,e^{-2x} + 1$

has first derivative $f'(x) = e^{-2x} - 2xe^{-2x}$

and second derivative $f''(x) = 4xe^{-2x} - 4e^{-2x}$.

(i) Find the value of x for which $y = f(x)$ has a stationary point. **1**

(ii) Find the values of x for which $f(x)$ is increasing. **2**

(iii) Find the value of x for which $y = f(x)$ has a point of inflection. **1**

(b) A function is defined as $f(x) = \sin x - \cos x$ for the domain $0 \leq x \leq 2\pi$.

It has x intercepts at $\left(\frac{\pi}{4}, 0\right)$ and $\left(\frac{5\pi}{4}, 0\right)$.

(i) Find the y intercept for the graph $y = f(x)$ **1**

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(ii) Use calculus to determine the co-ordinates of all stationary points in the given domain. **3**

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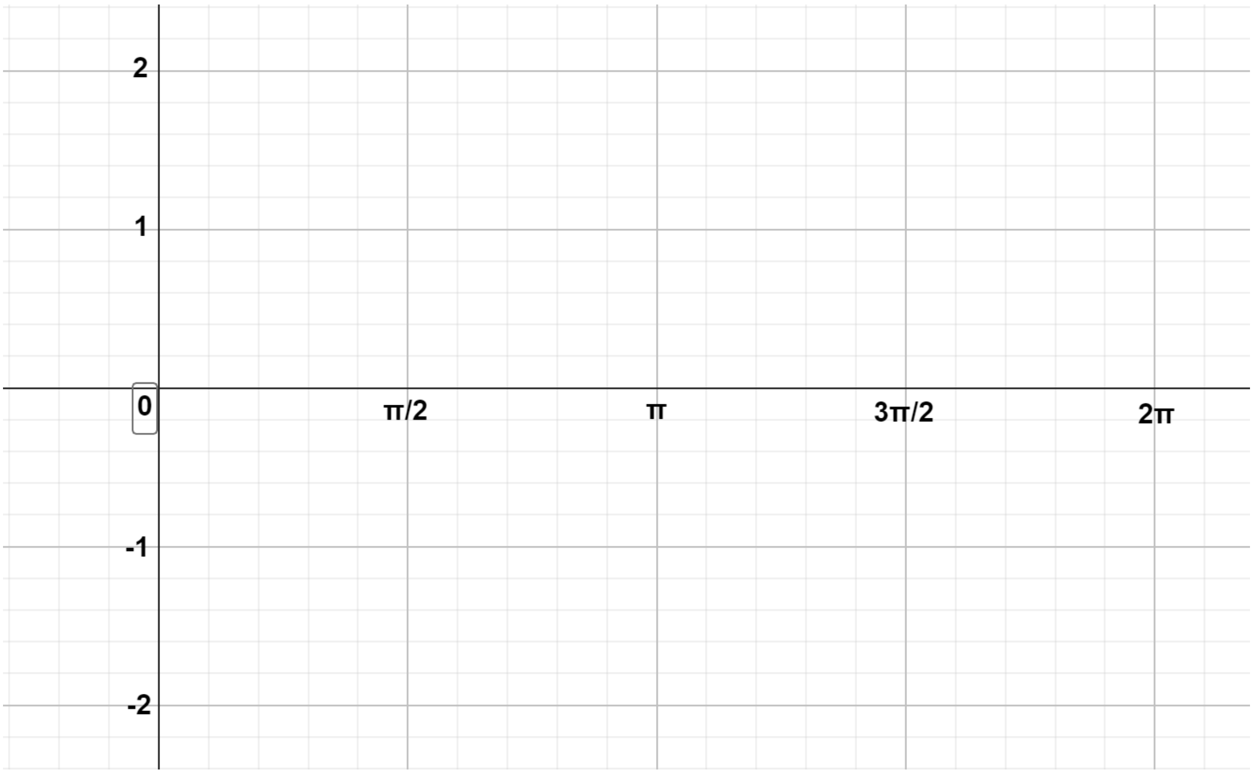
Ques 7(b) continues over the page

(iii) On the number plane supplied below, sketch the graph of $y = f(x)$ **3**



(c) Let m be a negative number. Include a diagram to help show that the equation : $\sin x = mx$ has $x = 0$ as it's **only** solution for $-\pi \leq x \leq \pi$. **2**

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End of Question 7

Question 7. Extra Writing Space. Please label the questions clearly.

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Hurlstone Agricultural High School
Year 12
HSC Assessment Task 2

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Mathematics Advanced
Section II

Question 8
(13 Marks)

Integral Calculus

- Answer each question in the space provided.
- Your responses should include relevant mathematical reasoning and/or calculations.

(a)

(i) Complete the table of values for the function $y = \sqrt{1 - x^2}$ **1**

Answer to 3 decimal places where required.

x	0	0.125	0.250	0.375	0.5
y		0.992	0.968		0.866

(ii) Use the trapezoidal rule with 4 sub intervals to estimate the area of the integral **2**

$$\int_0^{\frac{1}{2}} \sqrt{1 - x^2}$$

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(b) Find $\int (3x - 5)^5 \, dx$ **1**

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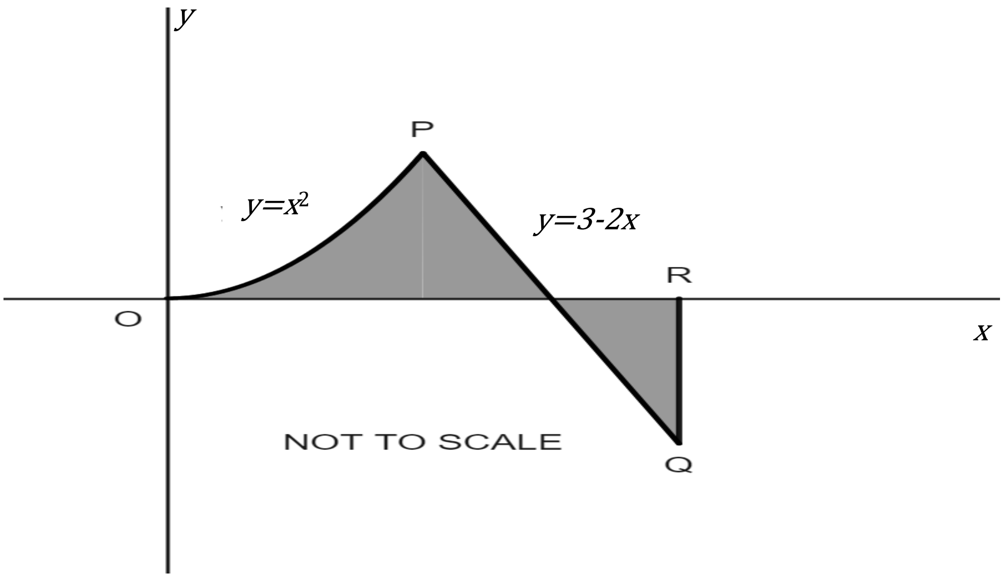
(c) Given $\int_{-1}^{2}(kx + 2)dx = 4$, what is the value of k ? **2**

(d) The curve $y = f(x)$ has the gradient function $\frac{dy}{dx} = 3x^2 + 4x - 1$. **2**

The curve passes through the point $P(1,0)$.

Find the equation of the curve $y = f(x)$

(e) The shaded region $OPQR$ is bounded by the lines $x = 0, x = 2$, the curve $y = x^2$, the line $y = 3 - 2x$ and the x-axis.



(i) Find the coordinates of P **2**

(ii) What is the area of the shaded region $OPQR$? **3**

End of Question 8

Question 8. Extra Writing Space. Please label the questions clearly.

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Hurlstone Agricultural High School
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HSC Assessment Task 2

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Mathematics Advanced
Section II

Question 9
(13 Marks)

Calculus of Trigonometric and Exponential Functions

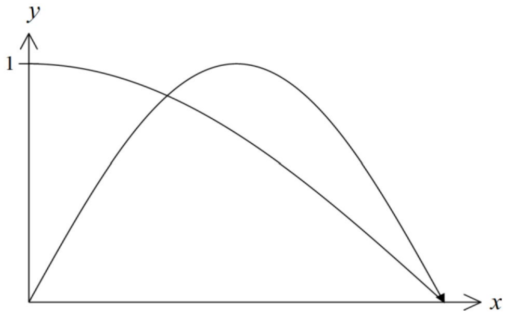
- Answer each question in the space provided.
- Your responses should include relevant mathematical reasoning and/or calculations.

(a) Evaluate $\int_0^1 e^{-x} dx$ leaving your answer in simplest exact form. **2**

(b) Given $y = e^{3x}$, show that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$ **2**

(c) Differentiate $\frac{x}{\tan x}$ **2**

(d) The diagram below shows the curves $y = \sin 2x$ and $y = \cos x$ for $0 \leq x \leq \frac{\pi}{2}$.



(i) It is given that $\sin 2x = 2\ \sin x \cos x$. Show that the x values where the curves intersect are $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$. **2**

(ii) Calculate the area bounded by $y = \sin 2x$, $y = \cos x$ and the x axis. **3**

(e) (i) Find $\frac{d}{dx}(\cos^2 x)$ **1**

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(ii) Hence, find $\int \sin x \cos x \, dx$ **1**

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End of Question 9

Question 9 Extra Writing Space. Please label the questions clearly.

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END OF TASK

2023 Yr12 Advanced Mathematics HSC AT2

Multiple Choice Solutions and Marking Guidelines

Outcomes can be matched to those named in Section 2 Q7, 8, 9.

Outcome	Solutions	Marking Guidelines
Q1 MA12-5	The function has a mean value of 2 and an amplitude equal to 4. Hence the range will go from 4 units below 2 up to 4 units above 2. $-2 \leq y \leq 6$	**Q1-6:** **1 mark:** Correct response **1 D** **2 B** **3 C** **4 B** **5 D** **6 B**
Q2 MA12-5	The graph has an amplitude of ½ (eliminate C, D). It is a sine graph. (Eliminate A). Answer can be checked by noticing that there are 3 periods from $-\pi$ to π, which is 3 times as many as a normal sine graph.	
Q3 MA12-7	Index form: x^{-4}. Hence the primitive is $-\frac{1}{3}\,x^{-3}$	
Q4 MA12-7	$\int_1^4 g(x)dx = \int_1^4 1dx - \int_1^4 f(x)dx$ $= x\}_1^4 \; - 7$ $= 4 - 1 - 7 = -4$	
Q5 MA12-6	. $f(x) = \sin 3x$ $f'(x) = 3\cos 3x$ $f'\left(\frac{\pi}{9}\right) = 3\cos\frac{\pi}{3}$ $= \frac{3}{2}$	
Q6 MA12-7	$\int \cos x^{\circ}\, dx \;=\; \int \cos\frac{\pi x}{180}\, dx$ $(\pi^c = 180°, \frac{\pi}{180} = 1°, \frac{\pi x}{180} = x^{\circ})$ $= \frac{1}{\frac{\pi}{180}} \sin\frac{\pi x}{180} \; + \text{c}$ $= \frac{180}{\pi}\sin x° + \text{c}$	

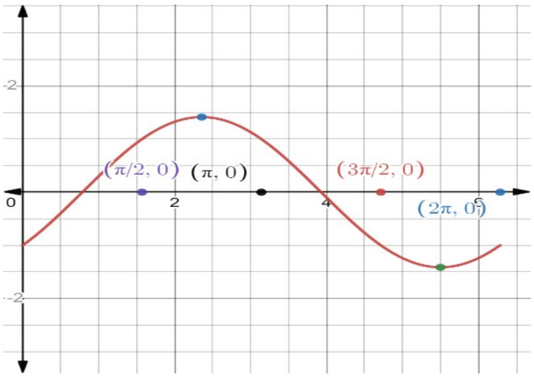
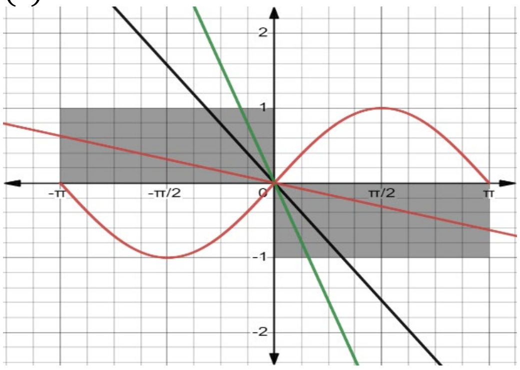
2023 Yr12 Advanced Mathematics HSC AT2

Question No. 7 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA12-3	Applies calculus techniques to model and solve problems.
MA12-5	Applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs

<table>
<tr><th>Outcome</th><th>Solutions</th><th>Marking Guidelines</th></tr>
<tr><td>12-3</td><td>(a) (i)
$$f'(x) = 0 \ \rightarrow e^{-2x}(1-2x) = 0$$ $$\therefore 1 - 2x = 0$$ $$x = \frac{1}{2}$$</td><td>(a)(i) 1 mark: Correct answer.
0 marks: extra incorrect answer.</td></tr>
<tr><td>12-3</td><td>(ii)
<table>
<tr><td>x</td><td>0</td><td>$\frac{1}{2}$</td><td>1</td></tr>
<tr><td>$f'(x)$</td><td>1>0</td><td>0</td><td>$\frac{-1}{e^2}$<0</td></tr>
</table>
We are looking for $f'(x)$ >0, which therefore occurs when $x < \frac{1}{2}$.
<i>Note: For those who chose to solve an inequation instead of testing a table of values:</i>
<i>You must <u>communicate</u> in your solution that:</i>
<i>Since</i> $e^{-2x} > 0$ <i>for all values of x, then 1-2x>0.</i>
<i>Stating a product being positive implies that both its factors must be positive was not sufficient for full marks.</i></td><td>(ii) 2 marks Correct answer with justification or table of values.
1 mark: Some relevant progress.</td></tr>
<tr><td>12-3</td><td>(iii) $f''(x) \ = \ 0 \ \rightarrow 4e^{-2x}(x-1) = 0$ $$\therefore x = 1$$</td><td>(iii) 1 mark: Correct answer.
0 marks: extra incorrect answer.</td></tr>
<tr><td>12-5</td><td>(b) (i)
$$f\ (0) = sin0 - cos0 = -1$$ y-intercept = $(0, -1)$</td><td>(b) (i) 1 mark: Correct y value.</td></tr>
<tr><td>12-3</td><td>(ii)
$$f'(x) = 0 \ \rightarrow \cos x + \sin x = 0$$ $$\therefore \tan x = \ -1 \quad (\text{Since } \cos x \neq 0)$$ In the given domain, $x = \frac{3\pi}{4}, \frac{7\pi}{4}$
Substituting into $f(x)$ gives stationary points at:
$$\left(\frac{3\pi}{4}, \sqrt{2}\right) \text{ and } \left(\frac{7\pi}{4}, -\sqrt{2}\right)$$</td><td>(ii) 3 marks: Correct values, with working. for derivative, x-values and function values at stationary points.
2 marks: One concept of solution incomplete.
1 mark: Considerable relevant progress.</td></tr>
</table>

12-5	(iii) 	**b(iii) 3 marks:** Diagram featuring labelled evidence of correct phase, period and amplitude. The end points of the curve had to be correct, and each had to be on increasing parts of the curve. **2 marks:** One concept of solution incomplete. **1 mark:** Considerable relevant progress.
12-5	(c)  Any line with a negative gradient will pass through the shaded areas (some examples of $y=mx$ have been drawn) In the given domain, $y=\sin x$ does not pass through the shaded areas, and as can be seen in the diagram, the only intersection between the straight lines and the trig function is at the origin, where $x=0$.	**(c) 2 marks:** Correct justification with diagram included as per the question's instructions. **1 mark:** Considerable progress, with one concept of explanation incomplete.

Year 12 Mathematics Advanced Assessment Task 2 2023

Question No. 8 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA12-7 applies the concepts and techniques of indefinite and definite integrals in the solution of problems

Solutions	Marking Guidelines

(a)

(i)

x	0	0.125	0.25	0.375	0.5
y	1	0.992	0.968	0.927	0.866

1 mark for completely correct table.

(ii)

$$A\sim\frac{0.5-0}{2\times4}[1+0.866+2(0.992+0.968+0.927)]$$

$$A\sim0.4775 \text{ units}^2$$

2 Marks for correct solution

1 Mark for substantial progress towards the correct solution.

b)

$$\int(3x-5)^5dx=\frac{1}{18}(3x-5)^6+C$$

Where C is a constant.

1 Mark for correct solution.
No deduction for missing 'C'

(c)

$$\int_{-1}^{2}(kx+2)dx=\left[\frac{kx^2}{2}+2x\right]_{-1}^{2}=4$$

$$\frac{k(2)^2}{2}+2(2)-\left(\frac{k(-1)^2}{2}+2(-1)\right)=4$$

$$\frac{4k}{2}+4-\frac{k}{2}+2=4$$

$$\frac{3k}{2}+6=4$$

$$3k=-4$$

2 marks for the correct value of k.

1 mark for correct integral.

$k = -\frac{4}{3}$	
(d) $f(x) = x^3 + 2x^2 - x + C$ Substitute point $P(1,0)$ into $f(x)$: $0 = (1)^3 + 2(1)^2 - (1) + C$ $0 = 1 + 2 - 1 + C$ $C = -2$ $\therefore f(x) = x^3 + 2x^2 - x - 2$	**2 marks** for correct solution of $f(x)$. **1 mark** for substantial progress towards the correct solution.
(e) (i) $x^2 = 3 - 2x$ $x^2 + 2x - 3 = 0$ $(x + 3)(x - 1) = 0$ $x = 3 \; or \; x = 1$ $\therefore x = 1$ (since, $x > 0$) Substitute $x = 1$ into $y = 3 - 2x$ $y = 1$ $\therefore P(1,1)$	**2 marks** for correct values of x and y. **1 mark** for one correct value of x or y.
(ii) From $y = 3 - 2x$, the x-intercept is $\frac{3}{2}$ Splitting the area into 3 sections, A_1, A_2, A_3, where A_2 and A_3 are triangles. $A_1 = \int_0^1 x^2 dx$	**3 marks** for correct shaded area. **2 marks** for correct area of the region $y = x^2$ from $x = 0$ to $x =$

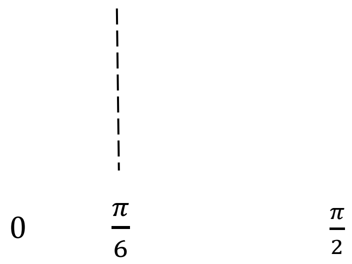
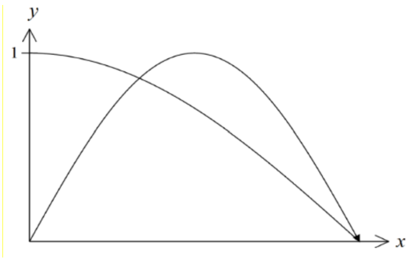
$= \left[\frac{x^3}{3}\right]_0^1$ $= \frac{1}{3}$ $A_2 = \frac{1}{2} \times \frac{1}{2} \times 1$ $= \frac{1}{4}$ $A_3 = \frac{1}{2} \times \frac{1}{2} \times 1$ $= \frac{1}{4}$ $\therefore$ Shaded Area $= \frac{1}{3} + \frac{1}{4} + \frac{1}{4}$ $= \frac{5}{6}$ units^2 Alternate solution: Shaded Area $= \int_0^1 x^2 dx + \int_1^{1.5} 3 - 2x\, dx + \left\lvert \int_{1.5}^2 3 - 2x\ \ dx \right\rvert$ $= \frac{1}{3} + \frac{1}{4} + \frac{1}{4}$ $= \frac{5}{6}$ units^2	1 and one other section or substantial progress towards the correct solution. **1 mark** for correct area of only one section or some progress towards the correct solution.

Year 12 Task 2 Mathematics Advanced 2023
Question No. 9 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA12-6 Applies appropriate differentiation methods to solve problems.
MA12-7 Applies the concepts and techniques of indefinite and definite integrals in the solution of problems.

Outcome	**Solutions**	**Marking Guidelines**
MA12-7	(a) $\int_0^1 e^{-x}dx = -\int_0^1 -e^{-x}dx$ $= -[e^{-x}]_0^1$ $= [e^{-x}]_1^0$ $= 1 - e^{-1}$ or $1 - \frac{1}{e}$ or $\frac{e-1}{e}$	2 marks : correct solution 1 mark : substantial progress towards correct answer
MA12-6	(b) $y = e^{3x}$ $\frac{dy}{dx} = 3e^{3x}$ $\frac{d^2y}{dx^2} = 9e^{3x}$ $\therefore \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 9e^{3x} - 4 \times 3e^{3x} + 3e^{3x}$ $= 9e^{3x} - 12e^{3x} + 3e^{3x}$ $= 0.$ **Note: setting out by many students needs a lot more attention. Marks were not deducted this time, but may be in future tasks. You should start with one side of the statement to be proven, clearly indicating which side you have started with.**	2 marks : correct solution 1 mark : substantial progress towards correct answer
MA12-6	(c) $\frac{d}{dx}\left(\frac{x}{\tan x}\right) = \frac{\tan x \times 1 - x \times sec^2 x}{\tan^2 x}$ $= \frac{\tan x - x sec^2 x}{\tan^2 x}$	2 marks : correct solution 1 mark : substantial progress towards correct answer
MA12-7	(d)(i) $y = \sin 2x$ and $y = \cos x$ meet when $\sin 2x = \cos x$, hence, when $2 \sin x \cos x = \cos x$ Solving $2 \sin x \cos x - \cos x = 0,$ $\cos x\ (2 \sin x - 1) = 0,$ $\cos x = 0$ and $2 \sin x - 1 = 0$, i.e $\sin x = \frac{1}{2}$ $\therefore x = \frac{\pi}{2}$ and $x = \frac{\pi}{6}$ for $0 \leq x \leq \frac{\pi}{2}.$	2 marks: correct solution 1 mark: substantial progress towards correct solution 3 marks : correct solution 2 marks : near correct solution with minor error

	Note: As question asked you to justify where the curves intersected, it was required that you state the coordinates lie on both curves & so they **intersect** there.	1 mark : substantial progress towards correct answer
MA12-7	(ii)  	
	 	1 mark : correct answer 1 mark : correct answer
MA12-6	Required area is area under $y = \sin 2x$ between $x = 0$ and $\frac{\pi}{6}$ plus area under $y = \cos x$ between $\frac{\pi}{6}$ and $\frac{\pi}{2}$. $A = \int_0^{\frac{\pi}{6}} \sin 2x\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x\,dx$ $= -\frac{1}{2}[\cos 2x]_0^{\frac{\pi}{6}} + [\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $=-\frac{1}{2}\left[\cos\frac{\pi}{3} - \cos 0\right] + \sin\frac{\pi}{2} - \sin\frac{\pi}{6}$ $= -\frac{1}{2}\left[\frac{1}{2} - 1\right] + 1 - \frac{1}{2}$ $= \frac{3}{4}\,\text{u}^2$	
	(e) (i) $\frac{d}{dx}(\cos^2 x) = \frac{d}{dx}(\cos x)^2$	

	$= 2(\cos x)^1 \times -\sin x$ $\therefore \frac{d}{dx}(\cos^2 x) = -2\cos x \sin x$ (ii) $\therefore \int -2\cos x \sin x \ dx = \cos^2 x + c$ $\therefore -2\int \cos x \sin x \ dx = \cos^2 x + c$ $\therefore \int \cos x \sin x \ dx = -\frac{1}{2}\cos^2 x + c$	