STUDENT'S NAME: _____

TEACHER'S NAME: _____

2023

HURLSTONE AGRICULTURAL HIGH SCHOOL

YEAR 11 ASSESSMENT TASK 1

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour
- Write using a black or blue pen
- NESA approved calculators may be used
- A reference sheet is provided in the Section I booklet
- For questions in Section II, show all relevant mathematical reasoning and/or calculations
- This examination paper is not to be removed from the examination centre

Total marks: 45

Section I – 6 marks (pages 2 – 5)

- Attempt Questions 1 – 6. The multiple-choice answer sheet has been provided.
- Allow about 8 minutes for this section

Section II – 39 marks (pages 11– 22)

- Attempt Questions 7 – 9, write your solutions in the spaces provided.
- There are 3 separate question/answer booklets. Extra working pages are available if required.
- Allow about 52 minutes for this section.

SECTION I (6 marks)

Attempt Questions 1 - 6

Allow about 8 minutes on this section.

Use the multiple-choice answer sheet provided for Questions 1 – 6.

1. What is $\frac{3\sqrt{5}}{\sqrt{15}}$ written in simplest form with a rational denominator?

   (A) 3

   (B) $\frac{3}{\sqrt{3}}$

   (C) $\sqrt{3}$

   (D) $\frac{\sqrt{3}}{3}$

2. What is $\frac{n^2-9}{n^2-n-12} \div \frac{n^2-3n}{n^2-9n+20}$ written in simplest form?

   (A) $\frac{n}{n-5}$

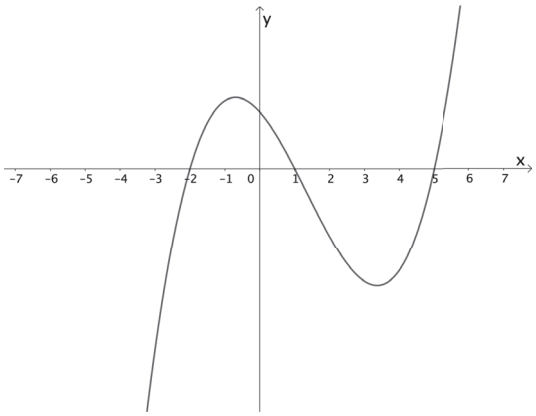
   (B) $\frac{n-5}{n}$

   (C) $\frac{(n-5)(n-4)^2}{n(n-3)^2}$

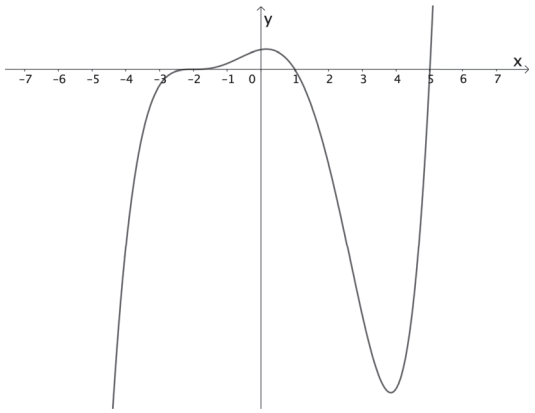
   (D) n

3. Which one of the diagrams below shows a sketch of the graph of the function $f(x) = (x+5)^3(x+1)^2(x-2)$?

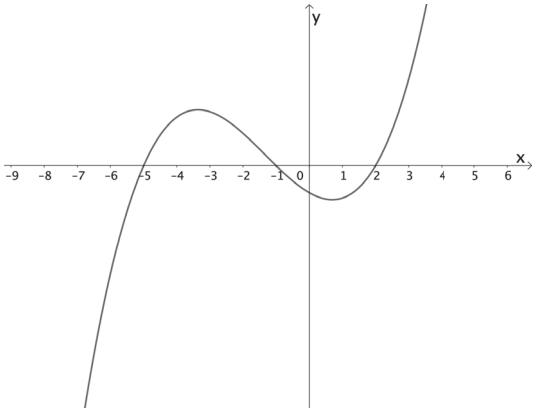
(A)



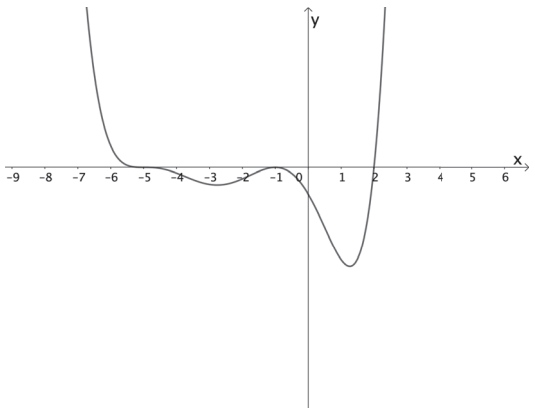
(B)



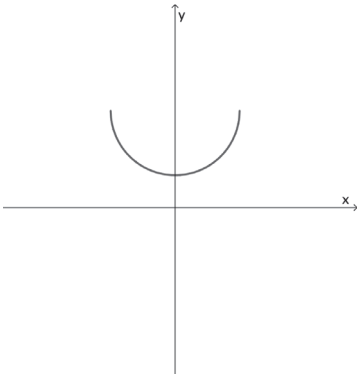
(C)



(D)

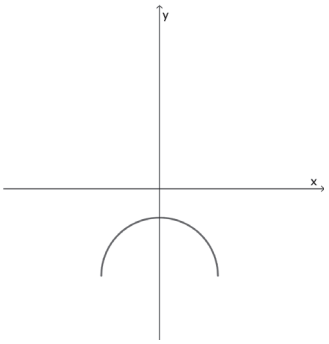


4. The graph of a function $y = g(x)$ is shown below.

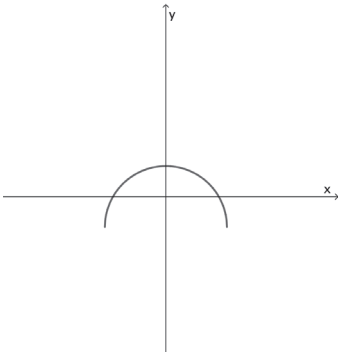


Which one of the following shows the graph of $y = -f(-x)$?

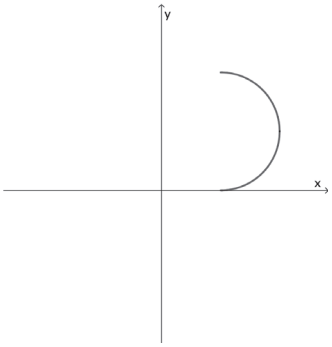
(A)



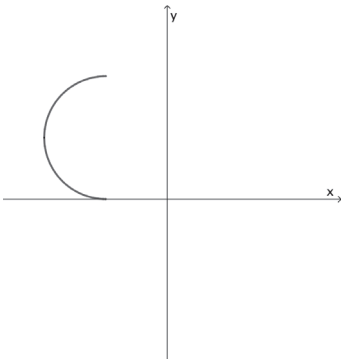
(B)



(C)



(D)



5. A car used 12 L of petrol and travelled a total distance of 290 km.
The cars fuel usage is 25 km/L on a highway and 20 km/L in the city.
Let h represent the number of L used on the highway.
Which of the following equations can be used to find the value of h?

(A) $25h + 20(12 - h) = 290$

(B) $25(12 - h) + 20h = 290$

(C) $25(12 + h) + 20h = 290$

(D) $25h + 20(12 + h) = 290$

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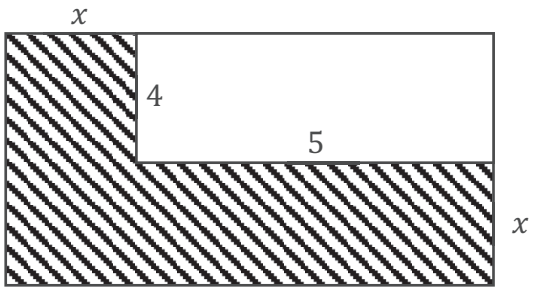
6. The area of the shaded figure in the diagram is 22 cm^2.



Units are in centimetres.
Diagram is not to scale.

Which one of the following equations should be solved to find the value of x?

(A) $x^2 + 9x - 20 = 0$

(B) $4x + 18 = 0$

(C) $9x - 20 = 22$

(D) $x^2 + 9x - 22 = 0$

End of Section I

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

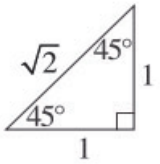
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

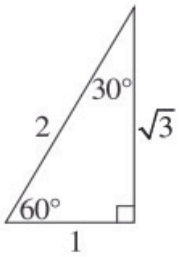
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

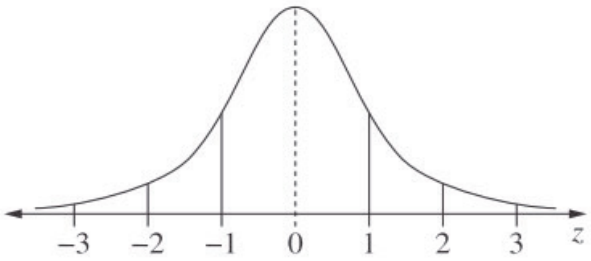
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c \quad \text{where } n \neq -1$$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

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SECTION II

Name: _____

39 marks

Attempt Questions 7 - 9

Allow about 52 minutes on this section.

Answer each question in the spaces provided. Extra working space is available after each question. If you need to use this extra space, you must clearly indicate this in the main solution space, and then clearly indicate the question number and part that you are answering in the extra space.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

2023 Year 11 Mathematics Advanced Task 1 Section II

Question 7 (13 marks) **Marks**

(a) (i) Write down the equation of the axis of symmetry of the parabola $y = 5 + 4x - 2x^2$ and hence find the coordinates of the vertex. **2**

(ii) Using interval notation, state the domain and range of this function. **2**

(b) Consider the function $f(x) = \frac{x}{x^2-1}$. **2**
Is this function ODD, EVEN or NEITHER? (Justify your answer with appropriate working.)

Question 7 continues on page 12...

Marks

(c) Show that the function $y = 2x^3 - 4x^2 + x$ has three x-intercepts and state their values. **3**

(d) Given $f(x) = 2x - 2$ and $g(x) = x^3$:

(i) Evaluate $f\big(g(-2)\big)$ **1**

(ii) Evaluate $g(f(3))$ **1**

(iii) Draw a neat sketch of the function $y = f\big(g(x)\big)$, showing all x and y intercepts. **2**

End of Question 7

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: _____

Question 8 (13 marks) **Marks**

(a) For the equation $x^2 + 3x - 2k = 0$ find the values of k for which the equation has real roots. **2**

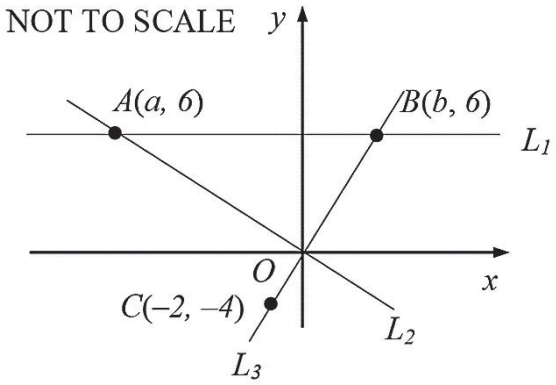
(b) Given the quadratic equation $y = -x^2 + 3x - 6$ and the line $y = mx - 2$, Find the values of m for which the line is tangent to the curve. **2**

(c) Solve the linear inequality $0 \leq \frac{2x-1}{4} - \frac{x}{3} < 2$. **2**

Question 8 continues on page 13...

Marks

(d)



On the diagram above line L_1 is parallel with the x-axis and crosses the y-axis at 6. Lines L_2 and L_3 each pass through the origin, O, and intersect with the line L_1 at the points $A(a,\ 6)$ and $B(b,\ 6)$ respectively.
The point $C(-2,\ -4)$ lies on the line L_3.

(i) Find the equation of the line L_3 . **1**

(ii) Find the x value for the coordinate of point B. **1**

(iii) Find the exact length of interval BC. **1**

(iv) Find the coordinates of point A such that $\angle AOB$ is a right angle. **1**

Question 8 continues on page 17...

Marks

(e) Find the coordinates of the centre and the length of the radius of the following circle: **3**

$$2x^2 + 2y^2 - 8x + 5y + 3 = 0$$

End of Question 8

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: _____

Question 9 (13 marks) **Marks**

(a) Two art exhibits, Aditify and Svaray, opened at the NSW Art Gallery on the same day.

Onnjishuist, the statistician, calculated that the attendance for Aditify can be modelled by the equation:

$$y = x^2 - 4x + 35$$

and the attendance for Svaray can be modelled by the equation:

$$y = 7x + 5$$

where y is the number in attendance and x is the day number.

(i) Using the equations, determine on which day/s since opening, the attendance for both exhibits was the same. **3**

(ii) What was the attendance for each exhibit on these days? **1**

Question 9 continues on page 20...

Marks

(b) In an experiment it is found that at a constant temperature of 100°C, the volume (V) of a gas varies inversely as the pressure (P), ie. $PV = k$

(i) If 2 litres of argon gas is at a pressure of 15.28 atmospheres, then find the value of k. **1**

(ii) If the temperature was maintained at 100°C and the volume was expanded to 4 litres, then what would be the expected pressure? **1**

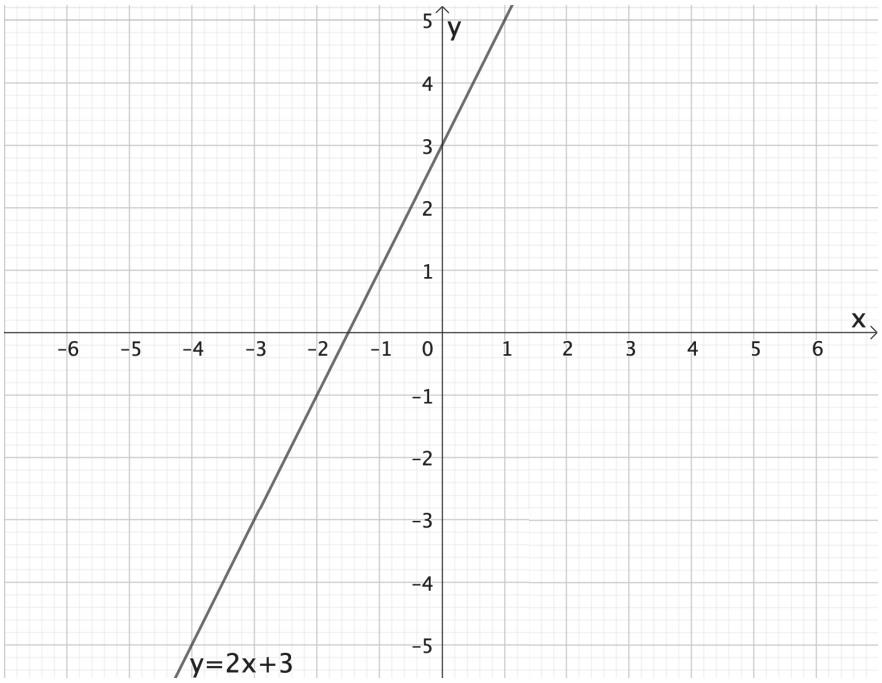
(iii) Draw a neat sketch of the graph $PV = k$, using the value of k found in part (i), showing any important features. Consider V as the independent variable (ie. V on the horizontal axis). **2**

Question 9 continues on page 21...

Marks

(c) Solve the equation: $\left|\frac{x}{2} - 3\right| = 11$ **3**

(d) The diagram below shows a graph of the function $y = 2x + 3$.



(i) On the same set of axes, draw a neat sketch of the graph $y = |x - 3|$. **1**

(ii) How many solutions are there for the equation $|x - 3| = 2x + 3$? Use the graph to justify your answer. **1**

End of Question 9

End of Task 1

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

HURLSTONE
AGRICULTURAL
HIGH
SCHOOL
Mathematics

Student Name _____

Teacher_____

Section I - MULTIPLE-CHOICE ANSWER SHEET

2023 Year 11 Mathematics Advanced Task 1

Allow about 8 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ● C ○ D ○

1.	**A** ○	**B** ○	**C** ○	**D** ○
2.	**A** ○	**B** ○	**C** ○	**D** ○
3.	**A** ○	**B** ○	**C** ○	**D** ○
4.	**A** ○	**B** ○	**C** ○	**D** ○
5.	**A** ○	**B** ○	**C** ○	**D** ○
6.	**A** ○	**B** ○	**C** ○	**D** ○

Year 11	Mathematics Advanced	Half Yearly Assessment 2023
Multiple Choice Question No.s 1-6		Solutions and Marking Guidelines

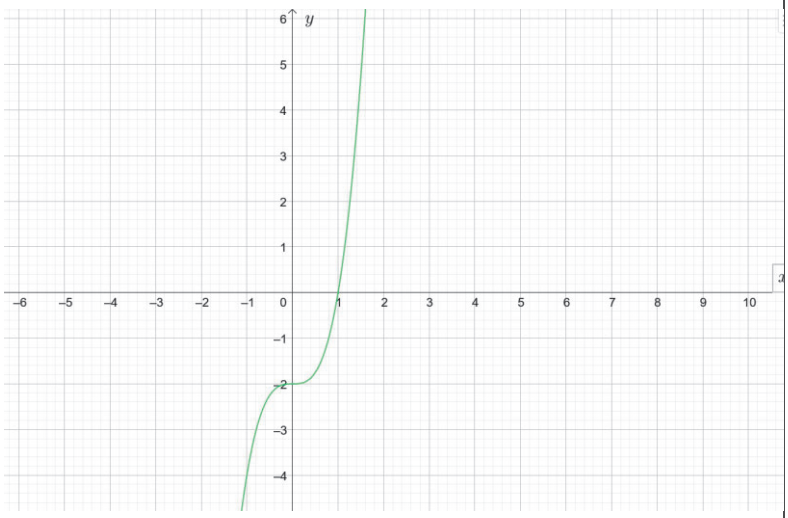
Outcomes Addressed in this Question

MA11-1	uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems
MA11-2	uses the concepts of functions and relations to model, analyse and solve practical problems

Outcome	Solutions	Marking Guidelines
MA11-1	**1. Answer: C** $\frac{3\sqrt{5}}{\sqrt{15}} = \frac{3\sqrt{5}}{\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}}$ $= \frac{3\sqrt{75}}{15}$ $= \frac{3\times5\sqrt{3}}{15}$ $= \frac{15\sqrt{3}}{15}$ $= \sqrt{3}$	1 mark
MA11-1	**2. Answer: B** $\frac{n^2-9}{n^2-n-12} \div \frac{n^2-3n}{n^2-9n+20}$ $= \frac{(n+3)(n-3)}{(n-4)(n+3)} \div \frac{n(n-3)}{(n-4)(n-5)}$ (Factorising) $= \frac{(n+3)(n-3)}{(n-4)(n+3)} \times \frac{(n-4)(n-5)}{n(n-3)}$ ("invert and multiply) $= \frac{n-5}{n}$ (cancelling common factors)	1 mark
MA11-1	**3. Answer: D** Polynomial $f(x) = (x+5)^3(x+1)^2(x-2)$ is of degree 6 with a positive leading term (x^6), hence, both tails of the graph should be heading towards $+\infty$. **D** is the only graph with this characteristic. **OR** Polynomial has a triple root at $x = -5$ ("wobble" through the x axis), a double root at $x = -2$ ("bounce" of the x axis) and a single root at $x = 2$ (passes through the x axis). Again, **D** is the only graph with these characteristics.	1 mark
MA11-1	**4. Answer: A** Breaking down the transformation $f(x) = -f(-x)$ into two parts: $f(x) = f(-x)$ transforms the function by reflecting it in the y axis. Since the function is already symmetrical about the y axis, there is no change to the graph of the function. $f(x) = -f(-x)$ then transforms the function by reflecting it in the x axis. Hence, **A** is the correct graph.	1 mark

MA11-2	**5. Answer: A** The car uses 12 L of petrol for this trip. If h L are used on the highway, $(12 - h)$ L must be used in the city. Car will travel $25h$ km on the highway and $20(12 - h)$ km in the city (using the given fuel usage rates). If the total distance travelled is then 290km, the correct equation is then $25h + 20(12 - h) = 290$.	**1 mark**
MA11-2	**6. Answer: D** Algebraically, the area of the shaded area is: $(x + 4)(x + 5) - 20 = x^2 + 9x$ Equating this with the actual area of the rectangle: $x^2 + 9x = 22$ Hence, the value of x can be found by solving: $x^2 + 9x - 22 = 0$	**1 mark**

Year 11 Mathematics Advanced Assessment Task 1 2023		
Question No. 7	Solutions and Marking Guidelines	
Outcomes Addressed in this Question		
MA11-1 uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems		
Outcome	**Solutions**	**Marking Guidelines**
MA11-1	a) i) Axis of symmetry $x = \frac{-b}{2a}$ $x = \frac{-4}{2(-2)}$ $\therefore x = 1$ Vertex is at (1, 7) ii) Domain $(-\infty, \infty)$ Range $(-\infty, 7]$	**2 marks** for the correct solution. **1 mark** for substantial progress towards the solution
MA11-1	b) $f(-x) = \frac{(-x)}{(-x)^2-1}$ $= \frac{-x}{x^2-1}$ $-f(x) = \frac{-x}{x^2-1}$ $\therefore f(-x) = -f(x)$ This is an odd function.	**2 marks** for both intervals shown correctly **1 mark** for one interval shown correctly **2 marks** for the correct solution. **1 mark** for substantial progress towards the solution
MA11-1	c) $y = 2x^3 - 4x^2 + x$ $y = x(2x^2 - 4x + 1)$ $\therefore x = 0,\ 2x^2 - 4x + 1 = 0$ By quadratic formula, solve $2x^2 - 4x + 1 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$ $x = \frac{4 \pm \sqrt{8}}{4}$ $\therefore x = \frac{2 \pm \sqrt{2}}{2}$ $\therefore$ The function has three *x*-intercepts at $x = 0, \quad \frac{2+\sqrt{2}}{2}, \quad \frac{2-\sqrt{2}}{2},$	**3 marks** for the complete correct solution **2 marks** for substantial progress towards correct solution. **1 mark** for some progress towards the solution

	d) i) $f(g(-2))$ $= f((-2)^3)$ $= f(-8)$ $= 2(-8) - 2$ $= -18$ ii) $g(f(3))$ $= g(2(3) - 2)$ $= g(4)$ $= 4^3$ $= 64$	
MA11-1	iii) $f(g(x)) = 2x^3 - 2$	
MA11-1		**1 mark** for the correct value
MA11-1		**1 mark** for the correct value
		2 marks for a complete and correct graph and intercepts **1 mark** for substantial progress towards correct graph and/or intercepts

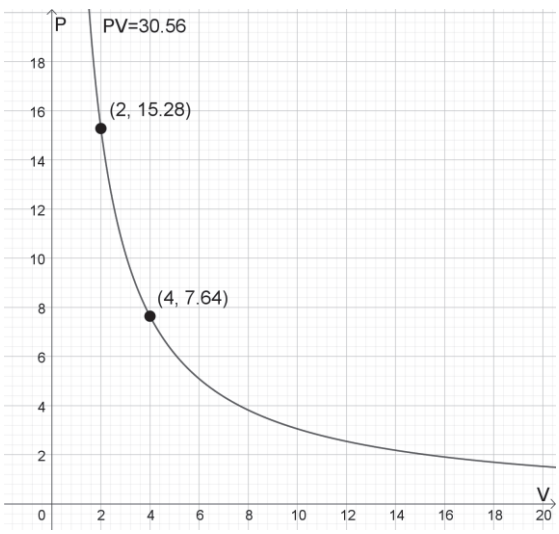
Year 11 Advanced Mathematics Task 1 2023		
Question 8		**Solutions and Marking Guidelines**
Outcome Addressed in this Question		
MA11-1	uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems	
Outcome	**Solutions**	**Marking Guidelines**
MA11-1	(a) $\Delta = b^2 - 4ac$ $\Delta = (3)^2 - 4(1)(-2k)$ $\Delta = 9 + 8k$ For quadratic to have real roots $\Delta \geq 0$. $9 + 8k \geq 0$ $k \geq -\frac{9}{8}$	2 marks for correct solutions 1 mark for some progress towards correct solution
MA11-1	(b) For the line to be a tangent to the quadratic they must meet at one point. $-x^2 + 3x - 6 = mx - 2$ $x^2 - (3 - m)x + 4 = 0$ For it to be a tangent $\Delta = 0$. $\Delta = (3 - m)^2 - 4\,(1)(4) = 0$ $9 - 6m + m^2 - 16 = 0$ $m^2 - 6m - 7 = 0$ $(m - 7)(m + 1) = 0$ $m = 7$ or $m = -1$	2 marks for correct solutions 1 mark for some progress towards correct solution
MA11-1	(c) $0 \leq \frac{3(2x - 1) - 4x}{12} < 2$ $0 \leq 2x - 3 < 24$ $3 \leq 2x < 27$ $\frac{3}{2} \leq x < \frac{27}{2}$	2 marks for correct solutions 1 mark for some progress towards correct solution

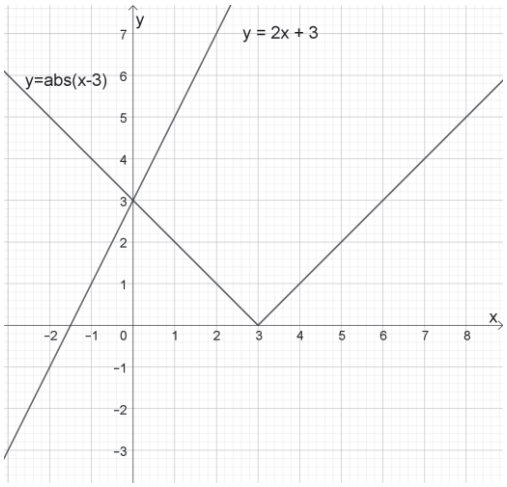
Question	Solutions	Marking Guidelines
MA11-1	(d) (i) $$m_{l3} = \frac{-4-0}{-2-0} = 2$$ $$y-(-4) = 2\big(x-(-2)\big)$$ $$y = 2x$$	1 mark for correct solution
MA11-1	(ii) Substitute $(b, 6)$ into equation of line $$6 = 2b$$ $$b = 3.$$	1 mark for correct solution
MA11-1	(iii) $$d_{BC} = \sqrt{(6+4)^2+(3+2)^2}$$ $$= \sqrt{125} = 5\sqrt{5} \text{ units}$$	1 mark for correct solution
MA11-1	(iv) Line L3 is perpendicular to L2, therefore $m_{l3} \times m_{l2} = -1$ $$\therefore\ m_{l2} = -\frac{1}{2}$$ $$m_{l2} = \frac{6-0}{a-0} = -\frac{1}{2}$$ $$\frac{6}{a} = -\frac{1}{2}$$ $$a = -12$$ $$\therefore\ A(-12, 6)$$	1 mark for correct solution
MA11-1	(e) $$2x^2 - 8x + 2y^2 + 5y = -3$$ $$x^2 - 4x + y^2 + \frac{5}{2}y = -\frac{3}{2}$$ $$(x-2)^2 + \left(y+\frac{5}{4}\right)^2 = -\frac{3}{2} + 4 + \frac{25}{16}$$ $$(x-2)^2 + \left(y+\frac{5}{4}\right)^2 = \frac{65}{16}$$ $$\therefore\ C\left(2, -\frac{5}{4}\right) \text{ and } r = \frac{\sqrt{65}}{4}$$	3 marks for correct solutions 2 mark for substantial progress towards correct solution 1 mark for some progress towards correct solution

Year 11	Mathematics Advanced	Half Yearly Assessment 2023
Question No. 9	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MA11-1 uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems
MA11-2 uses the concepts of functions and relations to model, analyse and solve practical problems
MA11-9 provides reasoning to support conclusions which are appropriate to the context

Outcome	Solutions	Marking Guidelines
MA11-2 MA11-9	(a)(i) Attendance is equal when: $x^2 - 4x + 35 = 7x + 5$ $x^2 - 11x + 30 = 0$ $(x - 6)(x - 5) = 0$ $x = 5, 6$ ie. 5th and 6th days have equal attendance.	**3 marks** Correct solution that equates both equations, simplifies, factorises and solves resulting equation to find two correct answers. **2 marks** Substantial progress towards correct solution. **1 mark** Some progress towards correct solution.
MA11-2 MA11-9	(ii) Substituting to find attendance (either equation may be used) Day 5 attendance $= 7x + 5$ $= 7 \times 5 + 5$ $= 40$ Day 6 attendance $= 7 \times 6 + 5$ $=47$	**1 mark** Both answers correct
MA11-2 MA11-9	(b)(i) $PV = k$ $15.28 \times 2 = k$ $k = 30.56$	**1 mark** Correct value of k found
MA11-2 MA11-9	(ii) $PV = 30.56$ when $V = 4$, $P \times 4 = 30.56$ $P = \frac{30.56}{4}$ $= 7.64$	**1 mark** Correct value of P found
MA11-2	(iii)  	**2 marks** Correctly drawn graph: - in first quadrant only - at least one point labelled - approaching the asymptotes **1 mark** Correctly drawn graph with only one of the above characteristics absent

MA11-1 **MA11-9**	**(c)** $\left\lvert\frac{x}{2}-3\right\rvert = 11$ $\frac{x}{2}-3=11$ OR $-\left(\frac{x}{2}-3\right)=11$ $\frac{x}{2}=14$ $-\frac{x}{2}+3=11$ $x=28$ $-\frac{x}{2}=8$ $x=-16$	**3 marks** Correct solution with both equations shown and solved. **2 marks** Substantial progress towards correct solution **1 mark** Some progress towards correct solution
MA11-1	**(d)(i)**  	**1 mark** Correctly drawn graph
MA11-1 **MA11-9**	(ii) $y=\lvert x-3\rvert$ intersects $y=2x+3$ once only. $\therefore \lvert x-3\rvert = 2x+3$ has only one solution.	**1 mark** Correct answer