Student Name: ___________________________

Teacher Name: ___________________________

2024

Hurlstone Agricultural High School
Year 12
HSC Assessment Task 2

Mathematics Advanced

General Instructions

- **Senior Examiner:** G. Huxley
- Reading time – 5 minutes
- Working time – 60 minutes
- Write using black or blue pen
- NESA approved calculators may be used
- A Mathematics reference sheet is provided on pages 6 -9 of this section
- Your handwritten notes on one A4 double sided page are permitted
- For all Questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 50

Section I – 6 marks

- Attempt Questions 1 – 6 on the answer page provided on p11
- Allow about 8 minutes for this section

Section II – 44 marks

- Attempt Questions 7 – 9 in the space provided
- Allow about 52 minutes for this section

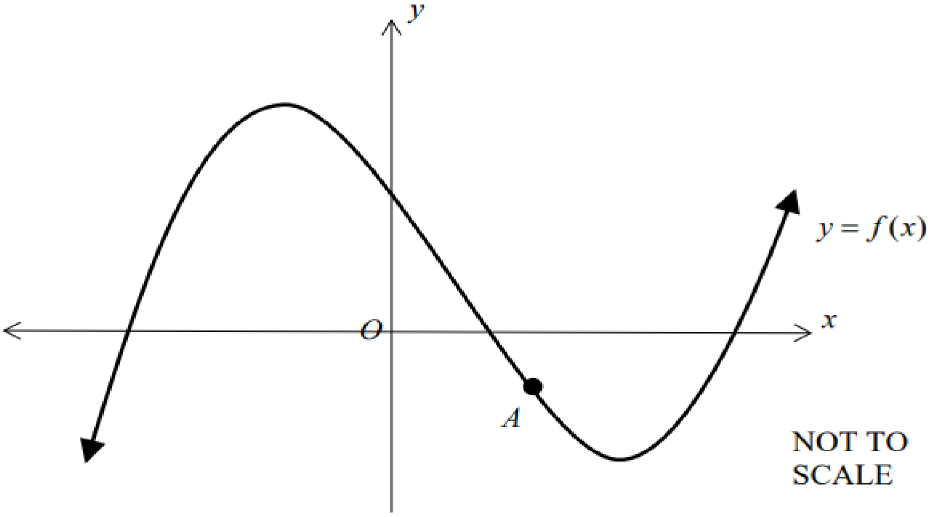
Section I

6 marks
Attempt Questions 1 – 6
Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 6.

---

1. The graph of the relation $y = f(x)$ is shown below:



What statement is correct at point A?

A. $f'(x) > 0$ and $f''(x) < 0$

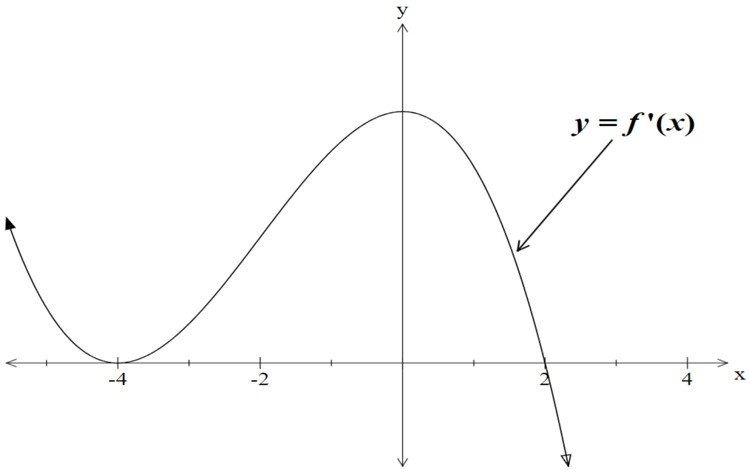
B. $f'(x) < 0$ and $f''(x) < 0$

C. $f'(x) > 0$ and $f''(x) > 0$

D. $f'(x) < 0$ and $f''(x) > 0$

Section I continues on next page

2. The diagram below represents the gradient function of the curve $y = f(x)$.



Which of the following is a true statement about the curve $y = f(x)$?

A. A minimum turning point occurs at $x = -4$.

B. A horizontal point of inflection occurs at $x = -4$.

C. A horizontal point of inflection occurs at $x = 2$

D. A minimum turning point occurs at $x = 2$

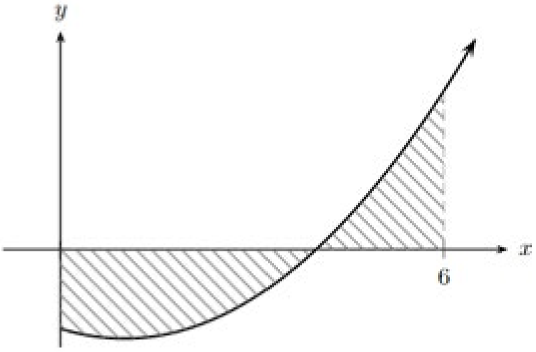
3. What is the value of the definite integral $\int_{-4}^{4} x^3\, dx$?

A.	128	B.	-128
C.	0	D.	64

Section I continues on next page

4. The diagram below shows the graph of $y = x^2 - 2x - 8$.

What is the correct expression for A, the shaded area shown?



A. $A = \int_0^5 (x^2 - 2x - 8)\, dx + \left| \int_5^6 (x^2 - 2x - 8)\, dx \right|$

B. $A = \int_0^4 (x^2 - 2x - 8)\, dx + \left| \int_4^6 (x^2 - 2x - 8)\, dx \right|$

C. $A = \left| \int_0^5 (x^2 - 2x - 8)\, dx \right| + \int_5^6 (x^2 - 2x - 8)\, dx$

D. $A = \left| \int_0^4 (x^2 - 2x - 8)\, dx \right| + \int_4^6 (x^2 - 2x - 8)\, dx$

5. An infinite geometric series has a first term of 12 and a limiting sum of 36.
What is the common ratio?

A. $\frac{2}{3}$

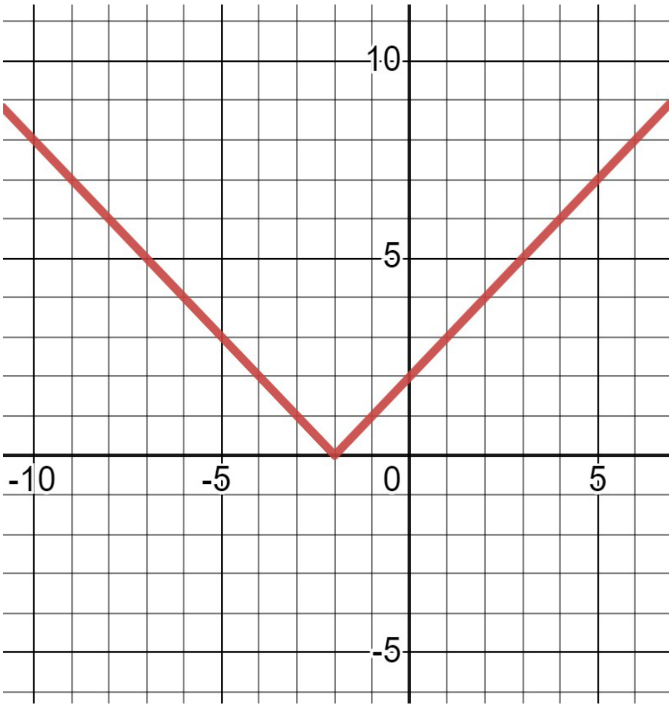
B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{3}{4}$

Section I continues on next page

6. A portion of the graph of $y = |x+2|$ is shown below.



Which of the following gives the solution to the equation $|x+2| \geq 3$?

A.	$x \geq 1$	B.	$-5 \leq x \leq 1$
C.	$x \leq -5$	D.	$x \leq -5$ or $x \geq 1$

~ End of Section I ~

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

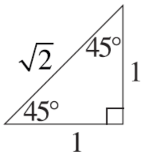
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

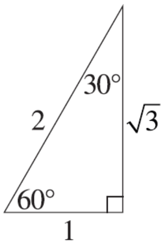
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

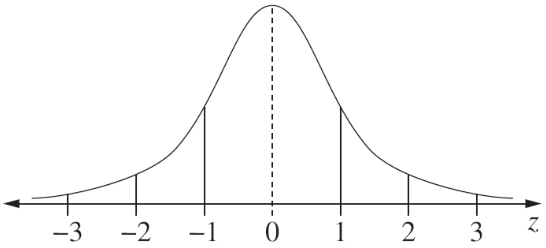
An outlier is a score

less than $Q_1 - 1.5 \times IQR$

or

more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	**Derivative**
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1 + \left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {^nC_r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2\,,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$

BLANK PAGE

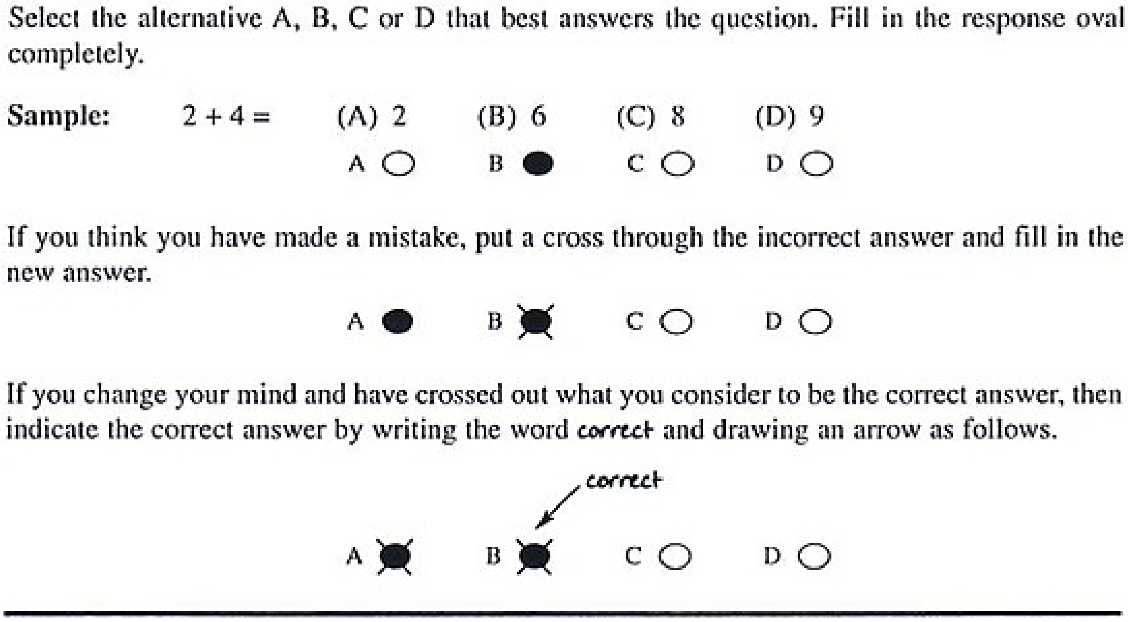
Students Name: ______________________________

Teachers Name: ______________________________

2024 Hurlstone Agricultural High School
Year 12
HSC Assessment Task 2

Mathematics Advanced

Multiple-Choice Answer Sheet



Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

Question

1. **A** ○ **B** ○ **C** ○ **D** ○

2. **A** ○ **B** ○ **C** ○ **D** ○

3. **A** ○ **B** ○ **C** ○ **D** ○

4. **A** ○ **B** ○ **C** ○ **D** ○

5. **A** ○ **B** ○ **C** ○ **D** ○

6. **A** ○ **B** ○ **C** ○ **D** ○

BLANK PAGE

Students Name: ____________________________

Teachers Name: ____________________________

2024

Hurlstone Agricultural High School
Year 12
HSC Assessment Task 2

/15

Mathematics Advanced
Section II

Question 7
(15 Marks)

Applications of Differentiation

- Answer each question in the space provided.
- Your responses should include relevant mathematical reasoning and/or calculations.

(a) The curve $y = x^3 + mx + n$ has a stationary point at $P(1,5)$. **2**
Find the values of the constants m and n.

(b) Consider the curve given by the equation $y = -x^3 + 12x + 1$

(i) Find the coordinates of the stationary points and determine their nature. **4**

Question 7 (b) continues on the next page

(ii) Find any points of inflection **2**

………

………

………

………

………

………

………

(iii) Sketch the curve, showing all necessary features including any stationary point(s) or points of inflection. (Do not find the x-intercepts). **2**

Question 7 continues on the next page

(c) A cylindrical container closed at both ends is made from thin sheet metal. The container is to have a radius of r cm and height of h cm, such that its volume is $1024\pi\ cm^3$.

(i) Show that the area of sheet metal required to make the container is **2**

$\left(2\pi r^2 + \frac{2048\pi}{r}\right) cm^2$

(ii) Hence find the minimum area of sheet metal required to make the container. **3**

Answer as an exact value.

End of Question 7

Question 7. Extra Writing Space. Please label the questions clearly.

BLANK PAGE

Students Name: ______________________________

Teachers Name: _____________________________

2024

Hurlstone Agricultural High School
Year 12
HSC Assessment Task 2

/15

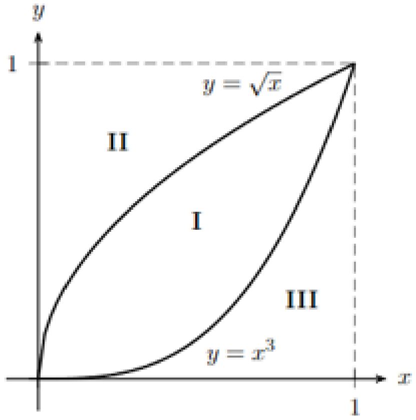
Mathematics Advanced
Section II

Question 8
(15 Marks)

Integral Calculus

- Answer each question in the space provided.
- Your responses should include relevant mathematical reasoning and/or calculations.

(a) The diagram below shows a unit square target for shooting on the Cartesian Plane. The target is divided into three regions, I, II and III by the curves $y = \sqrt{x}$ and $y = x^3$. **2**



Calculate the sum of areas I and II.

………

………

………

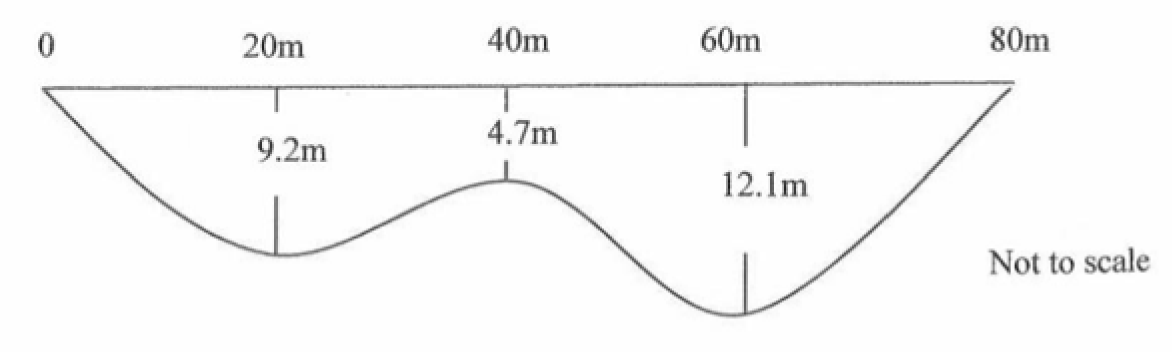
………

………

………

Question 8 continues on the next page

(b) The following diagram represents a cross-section through a river.
The depth of the river is marked every 20 metres.



Use the trapezoidal rule with 5 function values to estimate the area of the cross-section. **3**

……

……

……

……

(c) (i) Find: $\frac{d}{dx}\,(x^3+8)^3$ **2**

……

……

……

……

(ii) Hence evaluate $\int_1^2 x^2(x^3+8)^2\ dx$ **3**

……

……

……

……

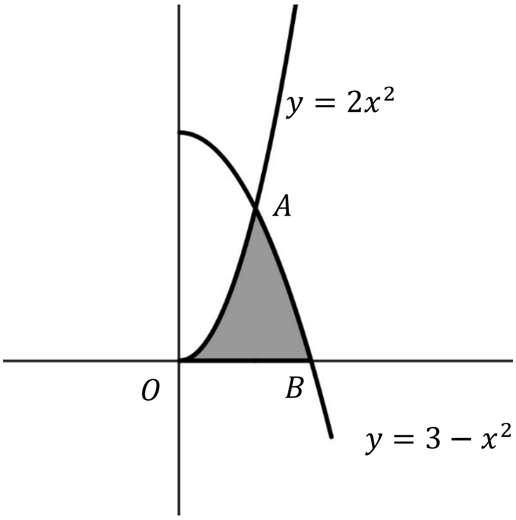
……

……

……

Question 8 continues on the next page

(d)



The shaded region OAB is bounded by the parabolas $y = 2x^2$, $y = 3 - x^2$ and the x-axis. Point A is the intersection of the two parabolas and point B is the x-intercept of the parabola $y = 3 - x^2$.

(i) Show that the x-coordinates of A and B are 1 and $\sqrt{3}$ respectively. **2**

………

………

………

………

………

(ii) Find the exact value of the shaded region. **3**

………

………

………

………

………

………

………

End of Question 8

Question 8 Extra Writing Space. Please label the questions clearly.

BLANK PAGE

Students Name: ____________________________

Teachers Name: ___________________________

2024

Hurlstone Agricultural High School
Year 12
HSC Assessment Task 2

/14

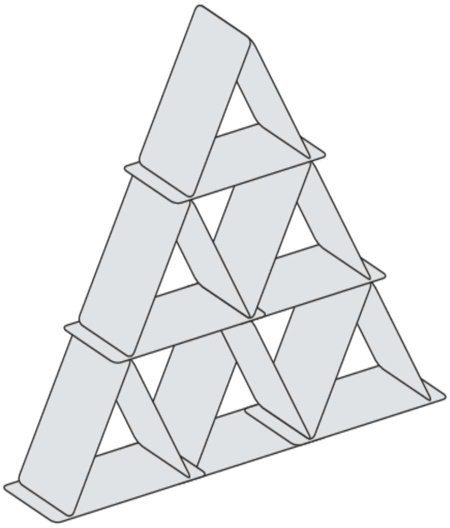
Mathematics Advanced
Section II

Question 9
(14 Marks)

Sequences and Series, Graphing Trigonometric Functions, Graphical Techniques

- Answer each question in the space provided.
- Your responses should include relevant mathematical reasoning and/or calculations.

(a) Cards are stacked to build a "house of cards". A house of cards with 3 rows is shown. A house of cards requires 3 cards in the top row, 6 cards in the next row, and each successive row has 3 more cards than the previous row.



Another house of cards has 234 cards. How many rows of cards must it have? **3**

(b) An amount of \$1800 is to be divided into 4 shares that form a geometric sequence. Given that the largest share is 8 times as large as the smallest share, determine the 4 amounts that make up the sequence. **2**

(c) The graph of the function $f(x) = x^2$ is translated m units to the right, and dilated vertically by a scale factor of k. **2**
The equation of the transformed function is $g(x) = 4x^2 - 24x + 36$.
Find the values of m and k.

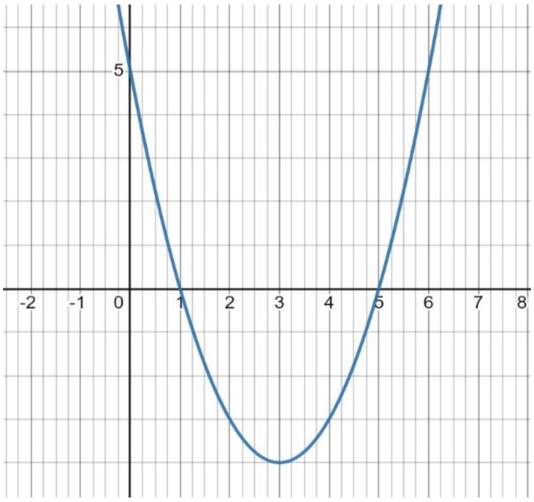
....

....

....

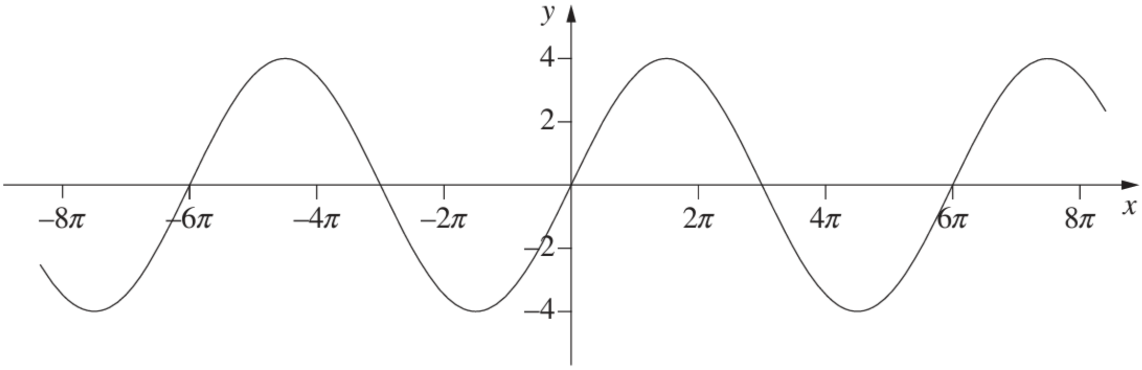
....

(d) The figure below shows the graph of the equation $y = f(x)$. **2**



The curve crosses the x-axis at $(1, 0)$ and $(5, 0)$, and the y-axis at $(0, 5)$.
The curve has a minimum turning point at $(3, -4)$.
Sketch in the space provided below, the graph of $y = f(x + 1)$.
Your sketch must include the coordinates of any points where the graph crosses the coordinate axes and the new coordinates of the minimum turning point.

(e) The graph of $y = k\sin(px)$ is shown.



What are the values of k and p ? **2**

……………………………………………………………………………………………………

……………………………………………………………………………………………………

(f) The depth of water in a bay rises and falls with the tide. On a particular day the depth of the water, d metres, can be modelled by the equation:

$$d = 1.8 - 1.3\,cos\left(\frac{4\pi}{25}\,t\right) \qquad \text{where } t \geq 0,$$

and t is the time in hours since low tide.

(i) What will the depth of the water be at the first high tide after $t = 0$? **1**

……………………………………………………………………………………………………

……………………………………………………………………………………………………

(ii) How many hours are there until that first high tide? **2**

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

End of Question 9

END OF TASK

Question 9. Extra Writing Space. Please label the questions clearly.

BLANK PAGE

Year 12	Mathematics Advanced	Assessment Task 2 2024
Multiple Choice	Solutions and Marking Guidelines	

Outcomes Addressed in these Questions

MA12-1: Uses algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar concepts.
MA12-4: Applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems.
MA12-6 Applies appropriate differentiation methods to solve problems.
MA12-7 Applies the concepts and techniques of indefinite and definite integrals in the solution of problems

Question	Solutions	Outcome
1	On the curve y=f(x), there is a point of inflection at A. It is decreasing, hence there is a turning point below of the x-axis. Hence, y=f'(x)<0. From y=f'(x), there is a minimum turning point which results in a positive gradient at y=f''(x), hence f''(x)>0. **Option D**	**12-6**
2	In the gradient function, there is a minimum turning point at x=-4. As such, when sketching the function y=f(x) results in a point of inflection at x=-4. **Option B**	**12-6**
3	Using $\int_{-a}^{a} odd\ function\,.dx = 0$ Then $\int_{-4}^{4} x^3\,.dx = 0$ **Option C**	**12-7**
4	Solving for the x-intercept $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ $x = 4, \quad x > 0$ Combined with, the area to the left of $x = 4$ is <u>below</u> the x axis and the area to the right of $x = 4$ is <u>above</u> the x axis **Option D**	**12-7**
5	Using reference sheet formula: $36 = \frac{12}{1-r}$ gives $r = \frac{2}{3}$. **Option A**	**12-4**
6	Rule the line $y = 3$ through the graph and you can see that the red function is above this line to the outer sides of $x = -5$ and 1. (When the absolute value is greater than a constant, the solution involves 2 regions, which only applies to one of the given responses.) **Option D.**	**12-1**

Year 12	Mathematics Advanced	Assessment Task 2 2024
Question No. 7 Solutions and Marking Guidelines		

Outcomes Addressed in this Question

MA12-6 applies appropriate differentiation methods to solve problems

Part	Solutions	Marking Guidelines

(a)

$$y = x^3 + mx + n$$

$$y' = 3x^2 + m$$

Stationary points occur at $y' = 0$

$$3x^2 + m = 0$$

When $x = 1, 3(1)^2 + m = 0$

$$m = -3$$

For $P(1,\ 5)$:

$$5 = (1)^3 - 3(1) + n$$

$$n = 7$$

2 marks
Correct value of m AND n

1 mark
Correct value of either m or n

(b)

(i)

$$y = -x^3 + 12x + 1$$

$$y' = -3x^2 + 12$$

Stationary points occur at $y' = 0$

$$-3x^2 + 12 = 0$$

$$3(x^2 - 4) = 0$$

$$x = \pm 2$$

When $x = -2, y = -15$

When $x = 2, y = 17$

Therefore stationary points occur at $(-2, -15)$ and $(2,17)$

Using the first derivative test:

x	-3	-2	-1	1	2	3
y'	-15	0	9	9	0	-15

$(-2, -15)$ is a local minimum stationary point and $(2,17)$ is a local maximum stationary point.

Note: When testing, it is necessary to include the values of y' or y'' and not just use only signs.

4 marks
Finds the 1st derivative and equates to zero AND correctly determines both coordinates of the stationary points AND correctly determines their nature either using the first derivative or second derivative test.

3 marks
Substantial progress towards the correct solution with only ONE error.

2 marks
Some progress towards the correct solution with at most TWO errors

1 mark
Some progress towards the correct solution.

(ii)

$$y'' = -6x$$

Points of inflection occur at $y'' = 0$

$$-6x = 0$$

$$x = 0$$

When $x = 0, y = 1$

A possible point of inflection occurs at $(0,1)$

Testing for change in concavity:

x	-1	0	1
y''	6	0	-6

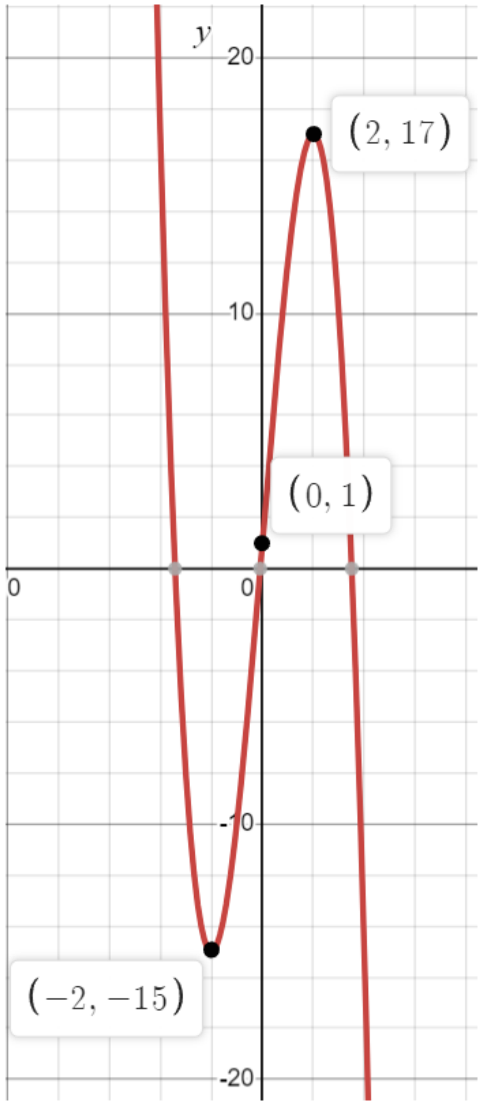
Change in concavity exists, hence the point of inflection is $(0,1)$

Note: It is not a horizontal point of inflection.

2 marks
Shows a possible point of inflection AND tests for change in concavity.

1 mark
Only shows possible point of inflection.

(iii)



2 marks
Correctly sketches a neat curve with correct shape and labels all stationary points and point of inflection.

1 mark
Correct shape, however, curve is messy or does not include relevant points or scales.

(c) **(i)**	$1024\pi = \pi r^2 h$ $h = \frac{1024\pi}{\pi r^2}$ $h = \frac{1024}{r^2}$ $SA = 2\pi r^2 + 2\pi rh$ $SA = 2\pi r^2 + 2\pi r\left(\frac{1024}{r^2}\right)$ $SA = 2\pi r^2 + \left(\frac{2048\pi}{r}\right) cm^2$ Common mistake: Many students used $h = \frac{1024}{\pi r^2}$,substituted incorrectly and just wrote it as $2\pi r^2 + \left(\frac{2048\pi}{r}\right) cm^2$ (proven). This resulted in no mark being awarded. If students substituted correctly and cancelled out terms accordingly, 1 mark was awarded.	**2 marks** Correctly writes an expression of h or equivalent AND shows the correct substitution to find the area of sheet metal. **1 mark** Writes the correct expression of h OR shows the correct substitution using incorrect value.
(ii)	$\mathrm{A} = 2\pi r^2 + 2048\pi r^{-1}$ $\frac{dA}{dr} = 4\pi\mathrm{r} - 2048\pi r^{-2}$ $= 4\pi r - \frac{2048}{r^2}$ When $\frac{dA}{dr} = 0,$ $4\pi r^3 - 2048\pi{=}0$ $r^3 - 512 = 0$ $r = 8$ $A = 2\pi(8)^2 + \frac{2048\pi}{8}$ $A = 384\pi\ cm^2$	**3 marks** Finds the correct value of $\frac{dA}{dr}$ AND finds the correct radius r AND finds correct area in exact value. **2 marks** Correctly determines TWO of the criterions above. **1 mark** Correctly determines ONE of the criterions above.

Year 12	Mathematics Advanced	Assessment Task 2 2024
Question No. 8	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MA12-7 applies the concepts and techniques of indefinite and definite integrals in the solution of problems

Part	Solutions	Marking Guidelines
(a)	$Area$ $= Area\ of\ the\ square - Area\ III$ $= 1 \times 1 - \int_0^1 x^3\,dx$ $= 1 - \frac{1}{4}[x^4]_0^1$ $= 1 - \frac{1}{4}$ $= \frac{3}{4}\ units^2$	**2 marks** For correct solution **1 mark** Substantial progress towards the correct solution.
(b)	$Area\ of\ the\ cross - section$ $\approx \frac{20}{2}[0 + 2(9.2 + 4.7 + 12.1) + 0]$ $= 520\,m^2$	**2 marks** For correct solution **1 mark** Substantial progress towards the correct solution.
(c) **(i)**	$\frac{d}{dx}(x^3+8)^3$ $= 3(x^3+8)^2 \times 3x^2$ $= 9x^2(x^3+8)^2$	**2 marks** For correct solution **1 mark** Substantial progress towards the correct solution.
(ii)	$From\ part\ (i)$ $\int_1^2 9x^2(x^3+8)^2dx = [(x^3+8)^3]_1^2$ $\int_1^2 x^2(x^3+8)^2dx = \frac{1}{9}[(x^3+8)^3]_1^2$ $= \frac{1}{9}[16^3 - 9^3]$ $= \frac{3367}{9}\ or\ 374\frac{1}{9}$	**3 marks** For correct solution **2 marks** Substantial progress towards the correct solution. **1 mark** Some progress towards the correct solution.
(d)		

(i)	$Solving\ for\ A$: $2x^2 = 3 - x^2 \quad (both = y)$ $3x^2 = 3$ $x^2 = 1$ $x = 1 \quad (x > 0)$ $Solving\ for\ B$ $0 = 3 - x^2 \quad (x - intercept)$ $x^2 = 3$ $x = \sqrt{3} \qquad (x > 0)$ $\therefore the\ x - coordinates\ of\ A\ and\ B\ are\ 1\ and\ \sqrt{3}\ respectively$	**2 marks** For correct solution **1 mark** Substantial progress towards the correct solution.
(ii)	$Area$ $= \int_0^1 2x^2\,dx + \int_1^{\sqrt{3}} 3 - x^2\,dx$ $= \frac{2}{3}[x^3]_0^1 + [3x - \frac{x^3}{3}]_1^{\sqrt{3}}$ $= \frac{2}{3}[1 - 0] + \left[\left(3\sqrt{3} - \frac{3\sqrt{3}}{3}\right) - (3 - \frac{1}{3})\right]$ $= \frac{2}{3} + \left[\left(3\sqrt{3} - \sqrt{3}\right) - 2\frac{2}{3}\right]$ $= \frac{2}{3} + 2\sqrt{3} - 2\frac{2}{3}$ $= 2\sqrt{3} - 2 \quad units^2$	**3 marks** For correct solution **2 marks** Substantial progress towards the correct solution. **1 mark** Some progress towards the correct solution.

Year 12	Mathematics Advanced	Assessment Task 2 2024
Question No. 9	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MA12-1: Uses algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar concepts.
MA12-4: Applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems.
MA12-5: Applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs.

Part/ Outcome	Solutions	Marking Guidelines
(a) **12-4**	Arithmetic Series with a=d=3 is described. $234 = \frac{n}{2}\left(2(3) + (n-1)3\right)$ $\rightarrow 468 = 3n^2 + 3n$ $n^2 + n - 156 = 0$ $(n+13)(n-12) = 0$ $n = -13, 12$ But n must be positive, so the only solution is n=12 rows. *Note: Most students misread/misunderstood the question. The "House of Cards" is explained, but most people calculated a term instead of the sum.*	**(a)3 marks:** Correct solution. **2 marks** Considerable progress including creation of quadratic equation. **1 mark** Substitution of correct *a* and *d* into a formula pertaining to an arithmetic series.
(b) **12-4**	Geometric Series is described. $ar^3 = 8a \rightarrow r = 2$ The sum is therefore $a + ar + ar^2 + ar^3 = 1800$ $15a = 1800$ $\therefore a = 120$ Hence the 4 amounts are $120, $240, $480, $960.	**(b) 2 marks:** Complete solution. **1 mark:** Correct sum equation including *r*=2, or 4 correct amounts from an incorrect calculation of *a*.
(c) **12-1**	$g(x) = 4(x^2 - 6x + 9) = 4(x-3)^2$ Therefore $k = 4,\ m = 3.$	**(c) 2 marks:** Correct *k* and *m*. **1 mark:** 1 Value correct, or correct factorisation if both values incorrect.
(d) **12-1**	*f(x+1)* w moves the graph 1 unit to the left. Hence you needed to draw a parabola shape with labelled *x*-intercepts (0,0) and (4,0), and labelled vertex at (2, -4).	**(d) 2marks:** All required features included and labelled. **1 mark:** One feature omitted.
(e) **12-5**	Amplitude equals 4, so k= 4. (Vertical stretch factor). Period is equal to 6π, so a horizontal stretch factor of 3 gives $p = \frac{1}{3}$	**(e) 2 marks:** Correct *k* and *p*. **1 mark:** 1 Value correct,

(f) **12-5**	(i) HIGH tide is the maximum height this graph will reach. It has a median value of 1.8 and an amplitude of 1.3, so the high tide must equal 3.1 + 1.8 = 3.1 metres. *Note: This question was very poorly read, with many people just substituting t=0. The question tells you that t represents the time elapsed since low tide, so there is no possibility that both tides happen at t=0.* **(ii)** The negative sign before the cosine ratio tells us that the shape of the graph is upside down in relation to a normal cos graph, so the highest point will be halfway through the period (which makes sense since low tide must be at the beginning and therefore the end of the period.) Hence we have: $\cos\left(\frac{4\pi}{25}t\right) = -1$ $\frac{4\pi}{25}t = \pi$ $t = \frac{25}{4} = 6\frac{1}{4}$ hours.	**(f)** **(i) 1 mark:** Correct answer. **(ii) 2 marks:** Correct solution. **1 mark:** Considerable relevant progress.