YEAR 11 - 2 UNIT ASSESSABLE -- NOVEMBER 1990

QUESTION 1 (10 marks)

(a) Solve for x : $2x + 7 < 3x + 10$

(b) What is the size of each interior angle of a regular pentagon?

(c) The point A is where the line $x + 2y = 8$ intersects the x-axis. Find the coordinates of A.

(d) Solve the equation : $2 \cos x = \sqrt{3}$ for $0° \leq x \leq 360°$.

(e) State the largest possible domain for the function $y = \dfrac{1}{x^2 - 4}$

QUESTION 2 (10 marks)

(a) Differentiate: (i) $3x^4 - 2x + \dfrac{1}{x^2} + 1$

(ii) $\dfrac{x}{1 + x}$

(iii) $(5x + 1)(2x + 3)^3$

(b) Find the equation of the tangent to the curve $y = 2x^2 + x + 1$ at the point where $x = -1$.

QUESTION 3 (10 marks)

(a) The function p(x) is defined by the rule: $p(x) = (x - 1)(x^2 - 5)$

(i) Find the coordinates of the turning points of p(x) and state whether they are maxima or minima.

(ii) Find the real roots of the equation $p(x) = 0$.

(iii) Draw a sketch graph of $y = (x - 1)(x^2 - 5)$ for the domain $-3 \leq x \leq 3$. Showing all essential features.

(b) Solve the quadratic inequality: $x^2 + 7x + 6 > 0$.

QUESTION 4 (10 marks)

(a) Find $\lim\limits_{h \to 0} \dfrac{h^2 + 5h}{h}$

(b) Find $\dfrac{ds}{dt}$ if $s = \sqrt{2t^2 - 3t}$

(c) Find k if $4x^2 - 5x + k = 0$ has equal roots.

(d) If α and β are the roots of the quadratic equation $2x^2 + 3x + 4 = 0$ find : (i) $\alpha + \beta$ (ii) $\alpha\beta$ (iii) $\alpha^2 + \beta^2$

Solutions to 2 unit Assessable – Nov '90

Question 1:

2 (a) $2x+7 < 3x+10$
$-x < 3$
$x > -3$.

2 (b) $\frac{(5-2)\times 180^\circ}{5} = 108^\circ$

2 (c) For A: $y=0$.
$\therefore x+0=8$
$x=8$
$\therefore A(8,0)$

2 (d) $\cos x = \frac{\sqrt{3}}{2}$
$\therefore x^\circ = 30^\circ$ and $360-30^\circ$
$= 30^\circ, 330^\circ$

2 (e) All real x except $x=2$ and $x=-2$.

Question 2:

2 (a) (i) $12x^3 - 2 - 2x^{-3} = 12x^3 - 2 - \frac{2}{x^3}$

2 (ii) $u = x \quad v = x+1$
$u' = 1 \quad v' = 1$

$$y' = \frac{(x+1)\times 1 - 1\times x}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$$

3 (iii) $u = 5x+1 \quad v = (2x+3)^3$
$u' = 5 \quad v' = 3(2x+3)^2\times 2$
$= 6(2x+3)^2$

$$y' = 5\times(2x+3)^3 + 6(2x+3)^2(5x+1)$$
$$= (2x+3)^2\left[5(2x+3) + 6(5x+1)\right]$$
$$= (2x+3)^2\left[10x+15+30x+6\right]$$
$$= (2x+3)^2(40x+21)$$

(b) $y = 2x^2+x+1$
$y' = 4x+1$ At $x=-1$ $y' = -4+1 = -3$

and at $x=-1$ $y = 2(-1)^2 - 1 + 1 = 2$.

$y-2 = -3(x--1)$
$y-2 = -3x-3$
$3x+y+1=0$.

Question 3:

4 (a) (i) $p(x) = (x-1)(x^2-5) = x^3-5x-x^2+5$
$= x^3-x^2-5x+5$

$p'(x) = 3x^2-2x-5$

For turning point $p'(x)=0$.

ie $3x^2-2x-5=0$
$3x \quad -5$
$x \quad 1$
$\therefore (3x-5)(x+1)=0$
$x=-1$ or $x=5/3$.

-1.1	-1	-0.9
+	0	−

$\therefore$ Max T.P.
$y = (-1-1)(1-5)$
$= -2\times -4$
$= 8$
$\therefore$ Maxima at $(-1,8)$.

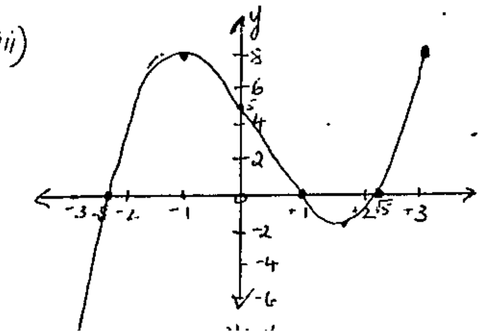
1	$5/3$	2
−	0	+

$\therefore$ Min T.P.
$y = (5/3-1)\left((\frac{5}{3})^2-5\right)$
$= -1\frac{13}{27}$
$\therefore$ Minima at $(5/3, -1\frac{13}{27})$

2 (ii) $p(x)=0$
$\therefore (x-1)(x^2-5)=0$.

$\therefore x=1$ or $\pm\sqrt{5}$.

3 (iii)



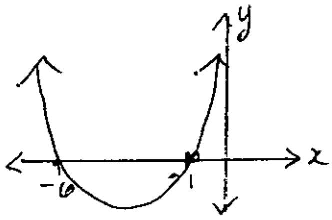
1 correct domain
1 x,y intercepts.
1 turning pts.

2 Question 3: (continued).

(b) $x^2 + 7x + 6 > 0$

$(x+1)(x+6) > 0$.

$\{x : x > -1\} \cup \{x : x < -6\}$



Question 4:

2 (a) $\lim_{h \to 0} \frac{h^2+5h}{h} = \lim_{h \to 0} \frac{\cancel{h}(h+5)}{\cancel{h}} = 5$

2 (b) $s = \sqrt{2t^2 - 3t} = (2t^2-3t)^{1/2}$

$$\frac{ds}{dt} = \frac{1}{2}(2t^2 - 3t)^{-1/2}(4t-3)$$

$$= \frac{4t-3}{2\sqrt{2t^2-3t}}$$

2 (c) Equal roots mean $\Delta = 0$

ie $b^2 - 4ac = 0$

$\therefore 25 - 4 \times 4 \times k = 0$

$25 - 16k = 0$

$16k = 25$

$k = 25/16 = 1\,9/16$.

1 (d)(i) $\alpha + \beta = -b/a = -3/2$.

1 (ii) $\alpha\beta = c/a = 4/2 = 2$.

2 (iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (-3/2)^2 - 2 \times 2$

$= \frac{9}{4} - 4$

$= -1\,3/4$.